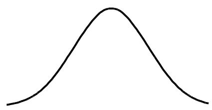
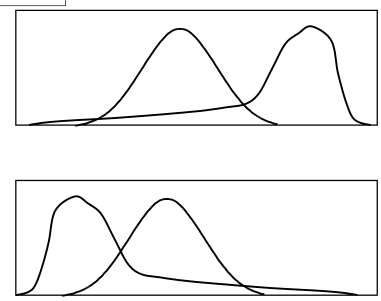
Assumptions of t-test and ANOVA

1. Normality

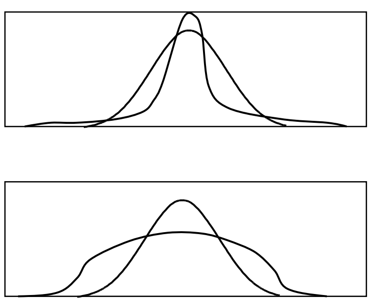


Both t-test and ANOVA require your data to fit a distribution that is normal.

Skewness



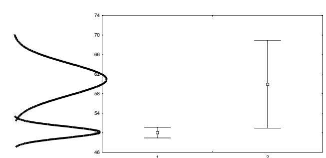
Kurtosis



Assumptions of t-test and ANOVA

2. Homogeneity of Variance (Equal variance).

Both t-test and ANOVA require your samples to have equal variance.



Assumptions of t-test and ANOVA

3. Independent observations.

More of an experimental design issue than an analysis issue but very important.

For example, Monoxidil experiment...

Departures from Normality.

--> there are specific tests determine if your data are normal or not.

However, the tests are not that good.

Luckily, t-test and ANOVA do just fine if this assumption is not violated too badly.

So, use your eye to determine if normal or not

Heterogeneity of Variance.

Again, t-test and ANOVA are relatively robust to violation of the Equal variance assumption

--> provided sample sizes are the same for all of your groups.

What to do.

t-test --> do a Variance Ratio Test

$$F = \frac{s_A^2}{s_B^2} \; or \; \frac{s_B^2}{s_A^2}$$

Depending on whether var A is larger or if var B is larger

Variance Ratio Test

H_o: $\sigma_A^2 = \sigma_B^2$

H_o: $\sigma_A^2 \neq \sigma_B^2$

Two samples, A and B
Sample size 12 and 10 respectively
and variance 45 and 32 respectively.

$$F_{observed} = \frac{s_A^2}{s_B^2} = \frac{45}{32} = 1.406$$

$$F_{0.05(2),12,10} = 3.62$$

Therefore, do not reject H_o, the variances are equal

For ANOVA, probably ok most of the time.

--> particularly true if sample with biggest variance also has biggest sample size

--> however, if other way round, big variance and small sample size - may want to do something besides ANOVA.

Effects of Non-normality on your analysis:

Inflate your probability of error (Types I and II)

Analyses we've discussed so far are robust to non-normality

BUT, sometimes just too much departure….

Then what?

Transform your data

Non-parametric methods for comparing samples

--> do not require the estimation of population mean and variance

--> do not require normality

--> do not require homogeneity of variance

	Parametric test	Non-parametric test
2 samples	t-test	Mann-Whitney
paired samples	paired t-test	Wilcoxon
3 or more samples	ANOVA	Kruskal-Wallace

Deal with Means and Variances

Deal with Ranks

The Mann-Whitney Test for comparing 2 samples

Hypotheses:

H_o: the two samples are the same

H_A: the two samples are not the same

Each observation --> assigned a rank from largest to smallest

- include observations from both samples

For example, suppose we had two samples of mice given different levels of growth hormone and we wanted to compare the number of days until adult weight is attained.

Group 1	Group 2
152	155
148	168
136	162
158	168
162	169

For example, suppose we had two samples of mice given different levels of growth hormone and we wanted to compare the number of days until adult weight is attained.

Group 1	Rank	Group 2	Rank
152	8	155	7
148	9	165	3
136	10	161	5
158	6	168	2
162	4	169	1

For example, suppose we had two samples of mice given different levels of growth hormone and we wanted to compare the number of days until adult weight is attained.

Group 1	Rank	Group 2	Rank
152	8	155	7
148	9	165	3
136	10	161	5
158	6	168	2
162	4	169	1
	37		18

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \qquad U' = n_2 n_1 + \frac{n_2(n_2+1)}{2} - R_2$$

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$= 5*5 + \frac{5(6)}{2} - 37 = 25 + 15 - 37$$

$$= 3$$

$$U' = n_2 n_1 + \frac{n_2(n_2+1)}{2} - R_2$$

$$= 5*5 + \frac{5*6}{2} - 18 = 25 + 15 - 18$$

$$= 22$$

If either U or U' is greater than the critical value of U, then you should reject the H_o

Critical value of U:

$$U_{Critical} = U_{0.05,(2),n_1,n_2} \quad \text{if } n_1 < n_2$$

$$U_{Critical} = U_{0.05,(2),n_2,n_1} \quad \text{if } n_1 > n_2$$

$$U_{Critical} = U_{0.05,(2),n_1,n_2} = U_{0.05,(2),5,5} = 23$$

U= 3 and *U'* = 22, therefore, do not reject H_o

Dealing With Tied Ranks

For example, suppose we had two samples of mice given different levels of growth hormone and we wanted to compare the number of days until adult weight is attained.

Group 1	Rank	Group 2	Rank
152	8	160	6
148	9	162	3.5
136	10	160	6
160	6	168	2
162	3.5	169	1

(3+4)/2=3.5
(5+6+7)/3=6

Assign ranks then proceed as in previous example

One-tailed Mann-Whitney U test.

Use U or U' depending on whether you expect sample 1 or sample 2 to be bigger

Table 8.2 of Zar

	H_o: $G_1 \geq G_2$ H_A: $G_1 < G_2$	H_o: $G_1 \leq G_2$ H_A: $G_1 > G_2$
Ranking is low to high	*U*	*U'*
Ranking is high to low	*U'*	*U*

Group 1	Rank	Group 2	Rank
152	8	160	6
148	9	162	3.5
136	10	160	6
160	6	168	2
162	3.5	169	1
			18.5

H_o: $G_1 \geq G_2$
H_A: $G_1 < G_2$

$$U' = n_2 n_1 + \frac{n_2(n_2+1)}{2} - R_2$$

$$= 5*5 + \frac{5*6}{2} - 18 = 25 + 15 - 18.5$$

$$= 21.5$$

$$U_{0.05,(1),5,5} = 21$$

$\therefore$ Reject H_o

Wilcoxon paired sample test

--> set-up same as paired sample t-test
--> calculate the difference between two measurements

Deer	Front Leg	Back Leg	Diff	Rank |d|	Signed Rank |d|
1	142	138	4		
2	140	136	4		
3	144	147	-3		
4	144	139	5		
5	142	143	-1		
6	146	141	5		
7	149	143	6		
8	150	145	5		
9	142	136	6		
10	148	146	2		

Next, assign ranks to the absolute values (low to high) of the differences, and then assign the corresponding sign to the rank

Deer	Front Leg	Back Leg	Diff	Rank |d|	Signed Rank |d|
1	142	138	4	4.5	4.5
2	140	136	4	4.5	4.5
3	144	147	-3	3	-3
4	144	139	5	7	7
5	142	143	-1	1	-1
6	146	141	5	7	7
7	149	143	6	9.5	9.5
8	150	145	5	7	7
9	142	136	6	9.5	9.5
10	148	146	2	2	2

Sum the positive ranks - $T_+ = 51$
Sum the negative ranks - $T_- = 4$

Sum the positive ranks - $T_+ = 51$
Sum the negative ranks - $T_- = 4$

If either T_+ or T_- is less than or equal to $T_{0.05,\,(2),n}$ then reject H_o

$T_{0.05,\,(2),10} = 8 > T_- = 4$, so reject H_o

Can also do these one tailed:

H_o: Measurement 1 ≤ Measurement 2
H_A: Measurement 1 > Measurement 2

--> reject H_o if $T_- \le T_{0.05,\,(1),n}$

H_o: Measurement 1 ≥ Measurement 2
H_A: Measurement 1 < Measurement 2

--> reject H_o if $T_+ \le T_{0.05,\,(1),n}$

Kruskal-Wallis test

Yields of corn under 4 fertilizer treatments

	Control	K+N	K+P	N+P
	40 (1)	82 (14)	57 (2)	71 (7)
	61 (4)	84 (15.5)	59 (3)	78 (11)
	72 (8.5)	96 (20)	63 (5)	79 (12)
	76 (10)	99 (21.5)	64 (6)	87 (17)
	84 (15.5)	104 (23)	72 (8.5)	91 (18)
	99 (21.5)	105 (23)	81 (13)	92 (19)
n	6	6	6	6
R	60.5	117	37.5	84

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	Control	K+N	K+P	N+P
	40 (1)	82 (14)	57 (2)	71 (7)
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n	6	6	6	6
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$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$$

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$$= \frac{12}{24(25)}\left(60.5^2 + 117^2 + 37.5^2 + 84^2\right) - 3(25)$$

$$= 0.02(3660.25 + 13689 + 1406.25 + 7056) - 75$$

$$= 516.23 - 75$$

$$= 441.23$$

Correction factor for tied ranks

$$C = 1 - \frac{\sum t}{N^3 - N}$$
$$= 1 - \frac{18}{24^3 - 24}$$
$$= 1 - 0.0013$$
$$= 0.9987$$

$$\sum t = \sum_{i=1}^{m} t_i^3 - t_i$$
$$= 2^3 - 2 + 2^3 - 2 + 2^3 - 2$$
$$= 18$$

$$H_c = \frac{H}{C} = \frac{441.23}{0.9987} = 441.80$$

Critical value:

$$\chi^2_{0.05,k-1} = \chi^2_{0.05,3} = 7.815$$

$$H_C > \chi^2_{0.05,3}$$

Reject Ho