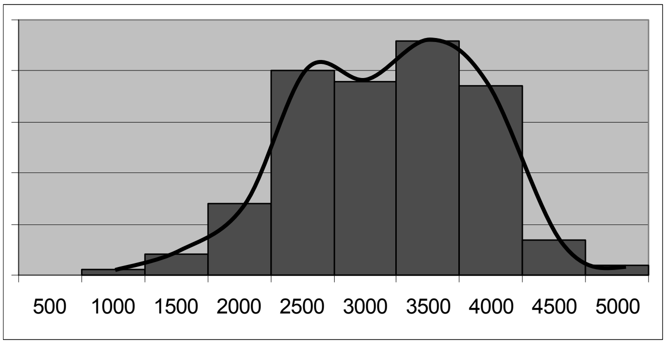
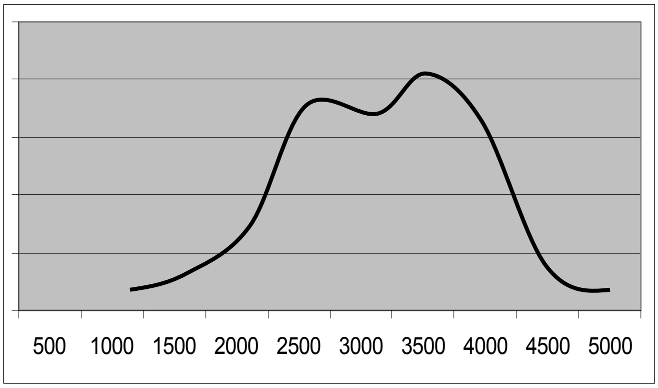


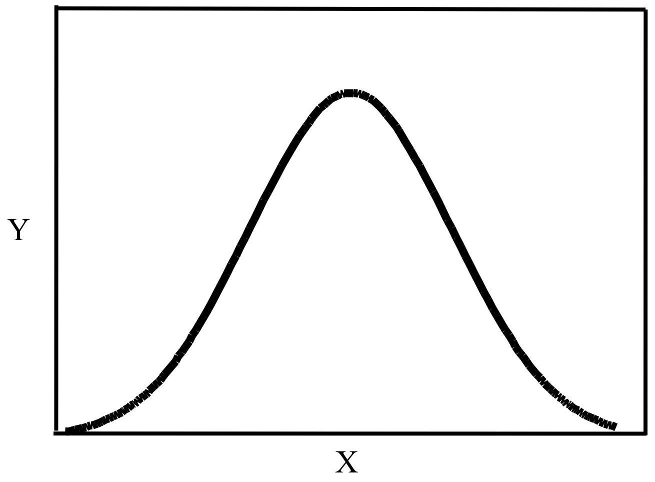
Probability density function - the curved line



The height of the curve --> density for a particular X

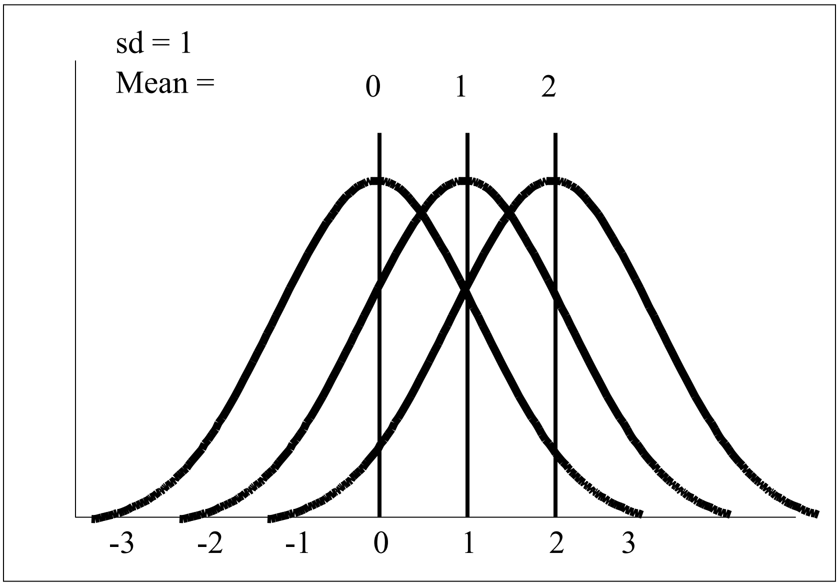
Density = relative concentration of observations

The Normal Distribution



$$Y_i = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X_i-\mu)^2}{2\sigma^2}}$$

The height of the curve at X_i

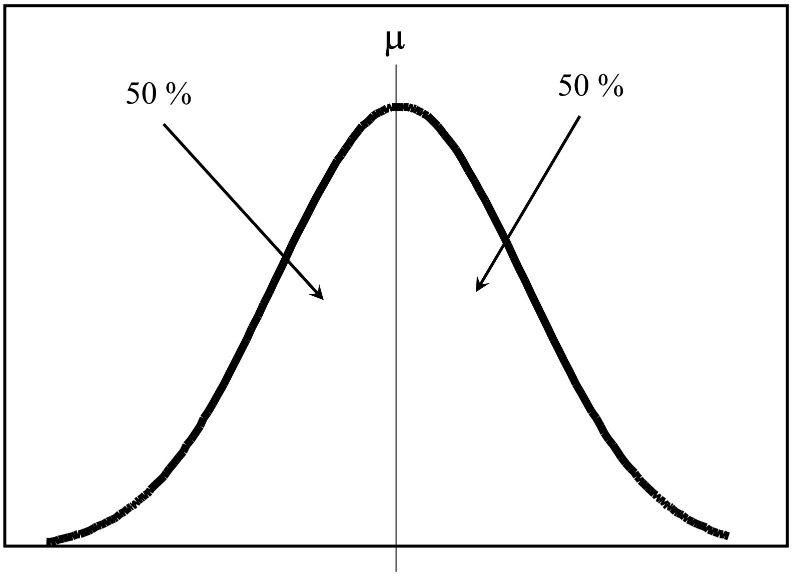


The Standardized Normal Curve

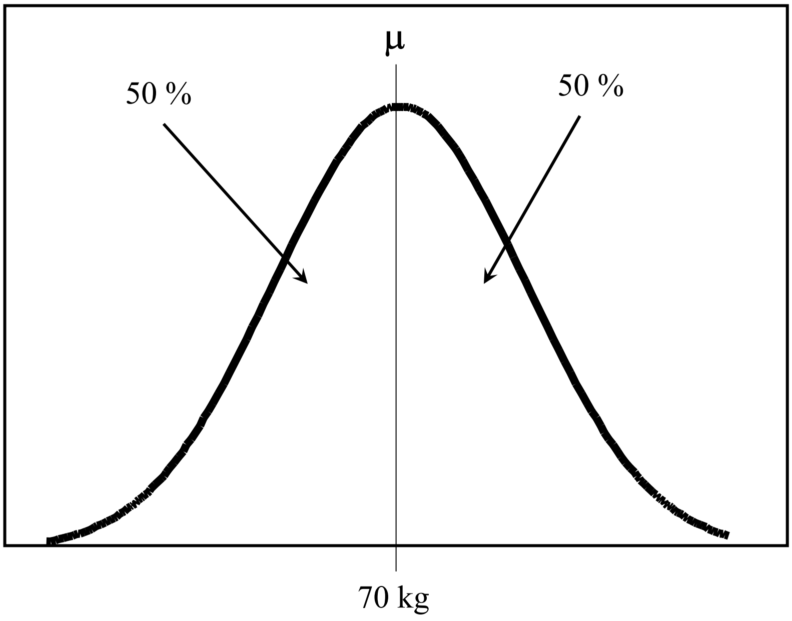
--> $\mu = 0$ and $\sigma = 1$

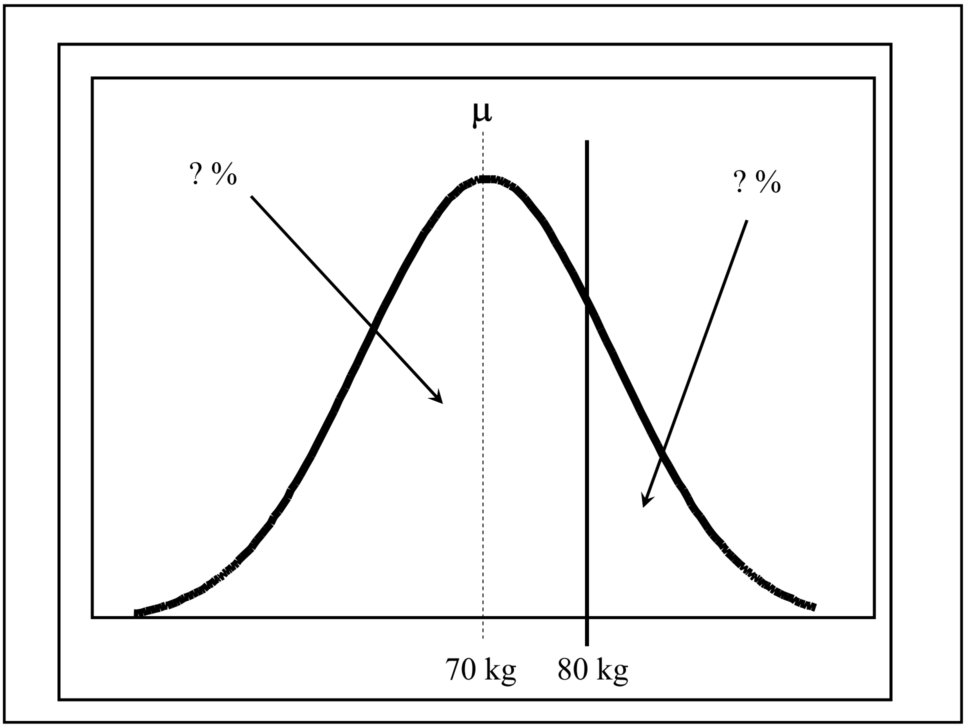
$$Y_i = \frac{1}{1\sqrt{2\pi}} e^{\frac{-(X_i - 0)^2}{2(1)^2}}$$

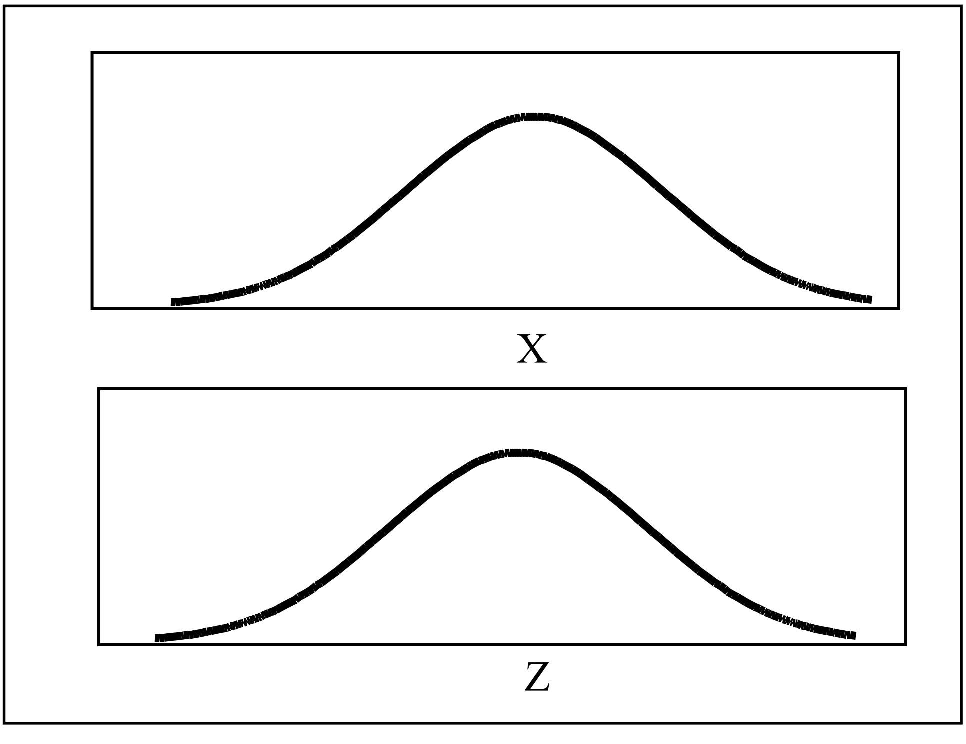
$$Y_i = \frac{1}{\sqrt{2\pi}} e^{\frac{-(X_i\,)^2}{2}}$$



Let's say you have a **population** with a mean of 70kg mass and a standard deviation of 10 kg.




μ
? %
? %
70 kg
80 kg


X
Z

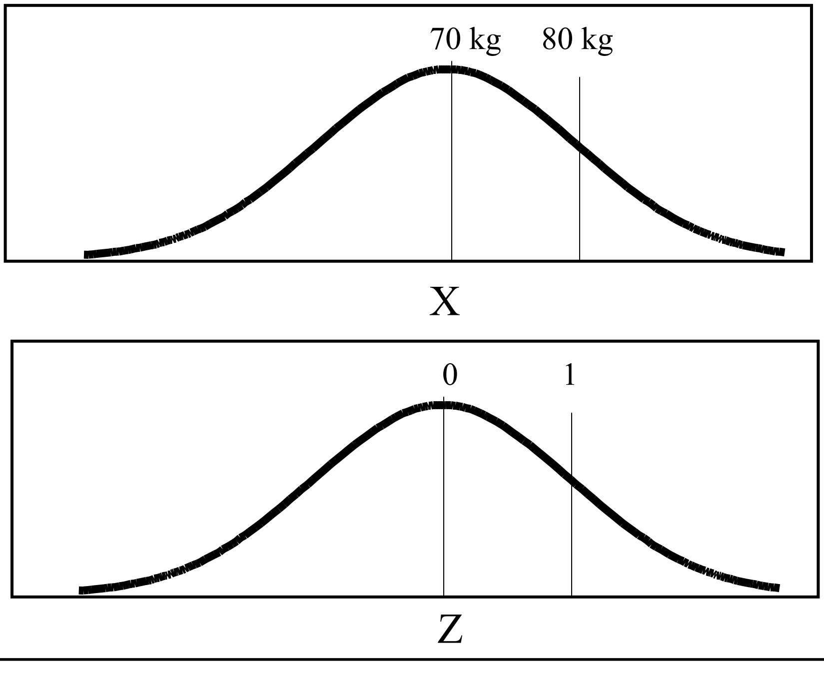
Standard Normal Deviate

$$\boxed{Z = \frac{X_i - \mu}{\sigma}}$$

$$\boxed{Z = \frac{80 - 70}{10} = 1}$$

What does Z= 1 mean?

Need to go to a table to get percent.



Statistical Table 3 in Samuels and Witmer (sort of)

Z

%

0.000	0.000	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090
0.000	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.100	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.200	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.300	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.400	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
0.500	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.600	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.700	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.800	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.900	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1.000	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
1.100	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.200	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
1.300	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.400	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.500	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.600	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.700	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.800	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.900	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.000	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.100	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.200	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.300	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.400	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.500	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.600	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.700	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.800	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.900	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.100	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Z=0

Z=1

0.000	0.000	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090
0.000	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.100	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0.200	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
0.300	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.400	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
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0.600	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.700	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.800	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.900	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1.000	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
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1.300	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.400	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.500	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
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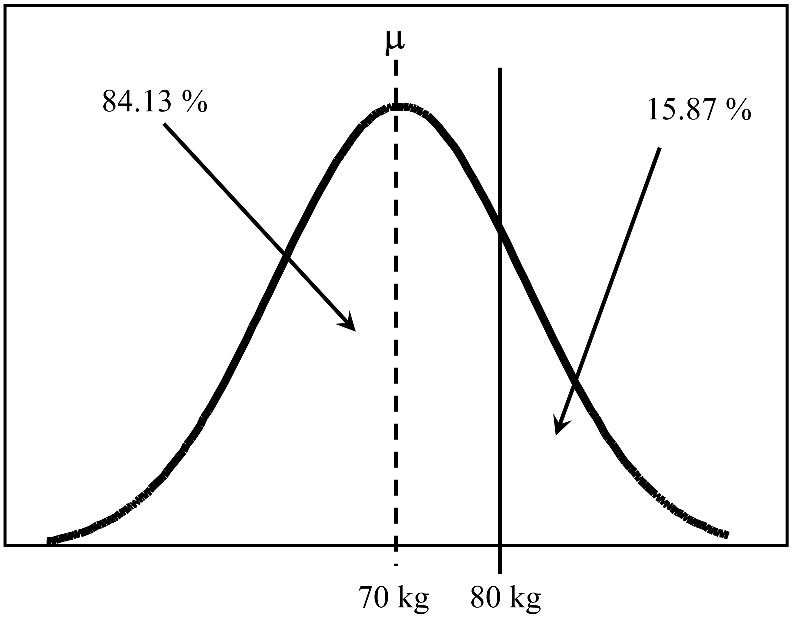
1.700	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.800	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.900	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.000	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.100	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.200	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.300	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.400	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.500	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.600	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.700	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.800	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.900	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.100	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

What can we say about this?

“Given a population with a mean of 70 kg and a standard deviation of 10 kg, the probability of finding an individual that is > 80 kg in a random sample is 0.1587 (or 15.87%).”

We can also say..

“Given a population with a mean of 70 kg and a standard deviation of 10 kg, the probability of finding an individual that is < 80 kg in a random sample is 1 - 0.1587 (or 84.13%).”



The CENTRAL LIMIT THEROEM

So far, we've been talking about populations.

If we collect a **BUNCH** of **SAMPLES** from a population having a normal distribution

→ the distribution of the MEANS of those samples will also have a normal distribution

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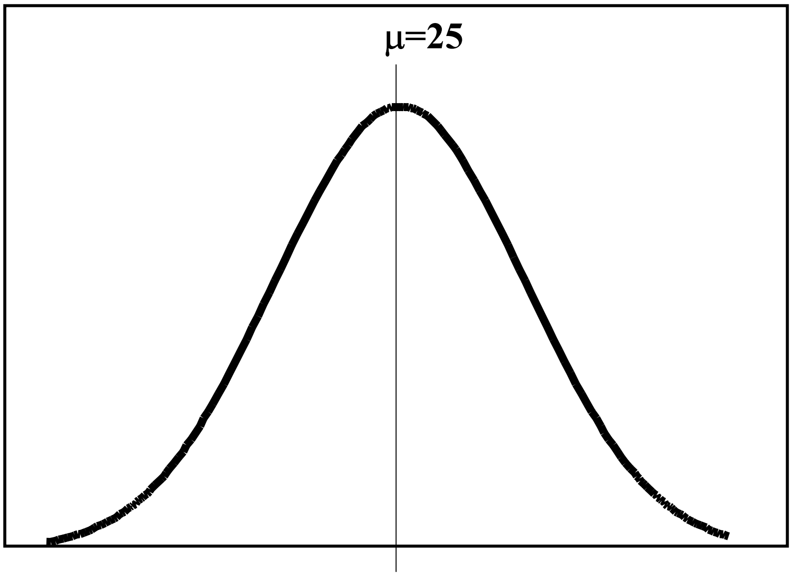
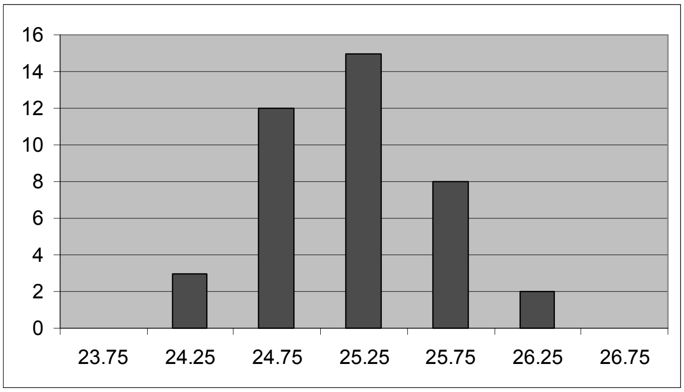
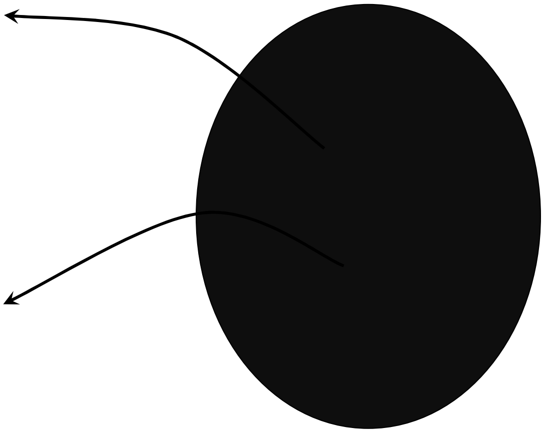


Frequency of means for forty samples of n = 15 taken from a population comprised of N = 5000 individuals having a mean of 25.



Also, as the size of the samples increases, the variance of the distributions will decrease.



n = 5
- 20 times

Mean = 24.951
StDev = 0.8686
Min = 23.77
Max = 26.98

n = 15
- 20 times

Mean = 24.963
StDev = 0.5454
Min = 23.96
Max = 25.94

Variance of the Mean

If I collected all possible samples of size n and calculated their means, the variance of the means would equal the population variance divided by n.

$$\sigma_{\overline{X}}^2 = \frac{\sigma^2}{n}$$

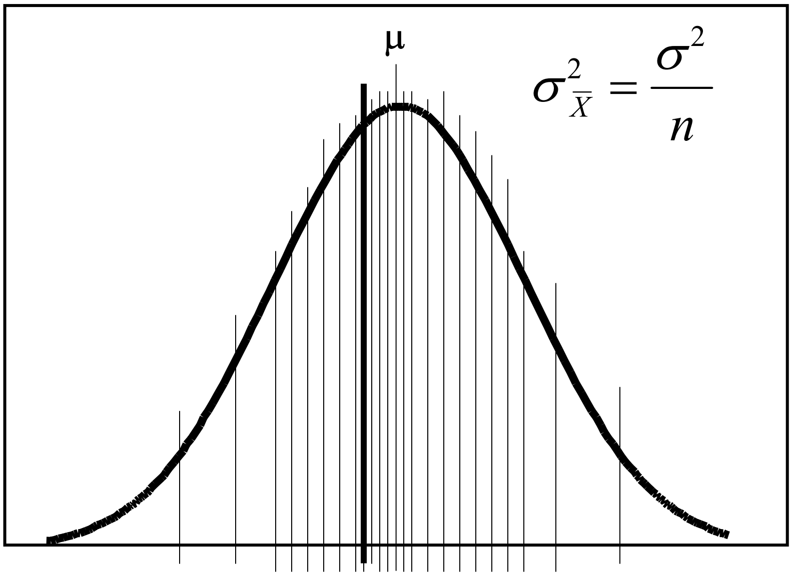
Standard Deviation of the Mean

$$\sigma_{\overline{X}} = \sqrt{\frac{\sigma^2}{n}}$$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

This value is most commonly referred to as the

Standard Error of the Mean



$$\sigma^2_{\overline{X}} = \frac{\sigma^2}{n}$$

$$Z = \frac{X_i - \mu}{\sigma}$$

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

So what?

Can answer:

What is the probability of collecting a random sample of 10 individuals that has a mean of greater than 80 kg in our population that has a mean of 70 kg and a standard deviation of 10 kg?

$$\sigma_{\overline{X}} = \frac{10kg}{\sqrt{10}} = 3.16kg$$

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{80 - 70}{3.16} = 3.16$$

0.000	0.000	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090
0.000	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
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0.600	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.700	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
0.800	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
0.900	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
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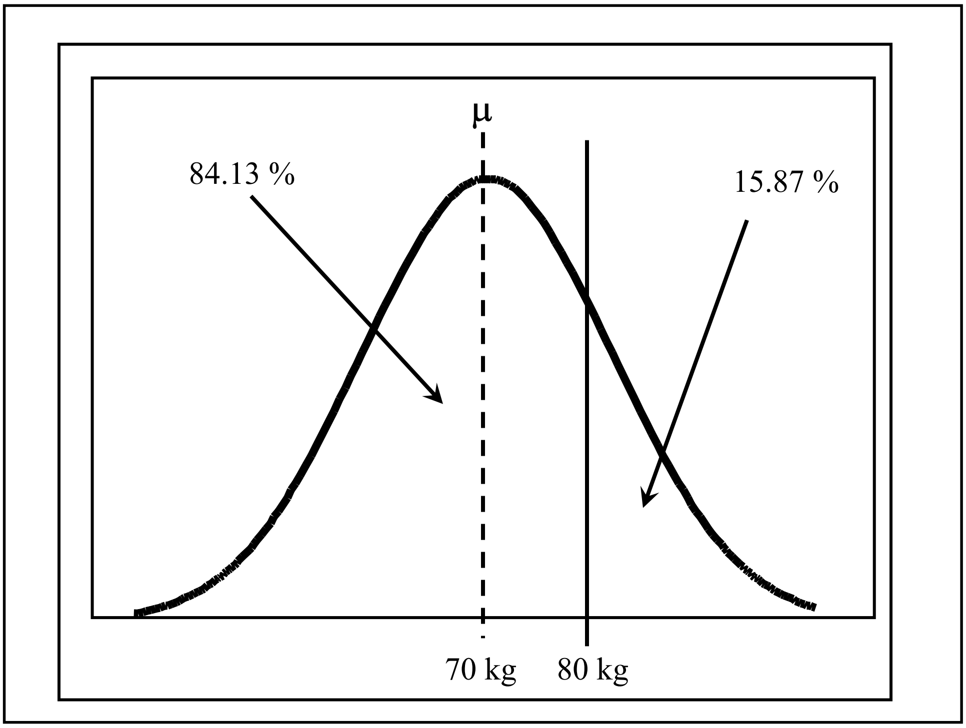
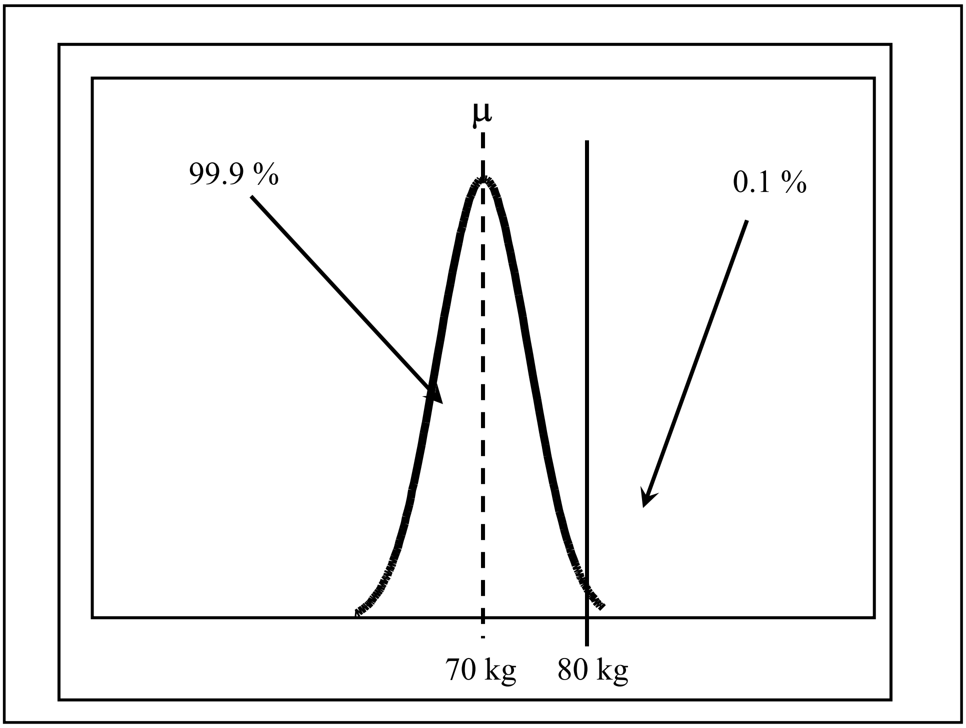
1.300	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
1.400	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.500	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.600	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.700	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
1.800	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
1.900	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2.000	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
2.100	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.200	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.300	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
2.400	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.500	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.600	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.700	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.800	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.900	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3.100	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001


μ
84.13 %
15.87 %
70 kg
80 kg


μ
99.9 %
0.1 %
70 kg
80 kg