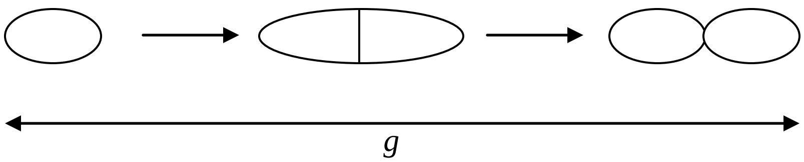
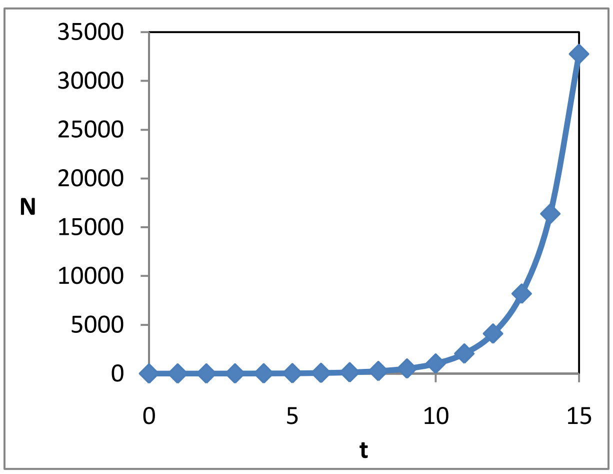
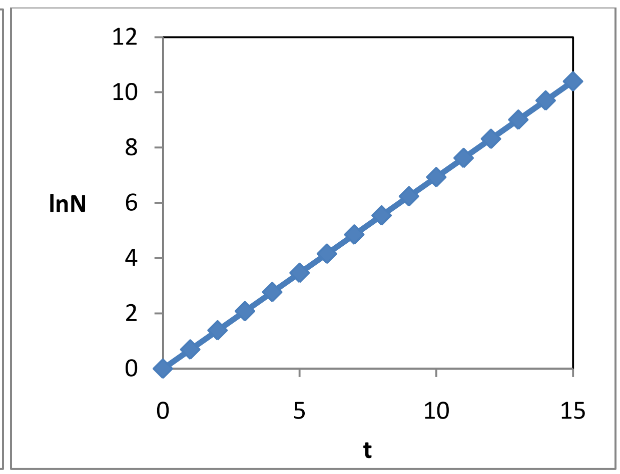
Microbial (binary) growth

Consider a population of a microorganism whose cell cycle requires time g.



This quantity, known as the generation time, is also the time required for a population to double in size. The rate at which a population grows is $\frac{dN}{dt}$, the slope of the graph on the left. This rate depends both on the intrinsic growth rate of the cells, r, and the number of cells present. The intrinsic growth rate can be determined by taking the slope of a semi-logarithmic plot, shown on the right.





$$r = \text{Intrinsic growth rate} = \frac{\ln N_2 - \ln N_1}{t_2 - t_1}$$

Given that g is the time required for each cell, and consequently the population, to double, when $t_2 - t_1 = g$, $N_2 = 2N_1$ and $r = \frac{\ln 2N_1 - \ln N_1}{g} = \frac{\ln 2}{g}$.

As the rate of population increase depends on both the rate of growth of each cell and the number of cells present at any given time, the slope of the graph on the left is also rN, such that $\frac{dN}{dt} = rN$. Rearranging, we see that $r = \frac{dN}{Ndt}$. Accordingly, the value of N at any time is $N = \frac{dN}{rdt}$. For small increases in N, $dN = N_t - N_0$. Solving for N_t, we can determine the number of cells present at any time as $N_t = N_0 e^{rt}$, where t is the time elapsed between the two measurements. $r = \frac{\ln N_t - \ln N_0}{t} \quad \Rightarrow \quad \ln N_t = \ln N_0 + rt \quad \Rightarrow \quad N_t = N_0 e^{rt}$.