

Proportions of recessive alleles in the various genotypes when q is small

Let q be the proportion of allele a .

Overall proportions of 2N genotypes

The overall proportion of individuals carrying allele a in the homozygous state is q^2 and the overall proportion of individuals carrying allele a in the heterozygous state is $2pq$.

What proportion of a alleles are found where?

Each of the $2pq$ heterozygotes has one a and each of the p^2 homozygotes aa has two.

Therefore, the proportion of a alleles that are found in the homozygote is:

$$\frac{2q^2}{2q^2 + 2pq} = \frac{q}{q + p} = q$$

and the proportion of a alleles that are found in the heterozygote is:

$$\frac{2pq}{2pq + 2q^2} = \frac{p}{p + q} = p$$

Among genotypes carrying a alleles, what proportion consists of homozygotes and what proportion consists of heterozygotes?

The proportion of homozygotes carrying a compared to heterozygotes is:

$$\frac{q^2}{2pq + q^2} = \frac{q}{2p + q} = \frac{q}{2 - q} = \frac{1 - p}{1 + p}$$

and the proportion of heterozygotes carrying a compared to homozygotes is:

$$\frac{2pq}{2pq + q^2} = \frac{2p}{2p + q} = \frac{2p}{1 + p}$$

When q is very small, the first expression approaches 0 and the second approaches 1.