Marginal Value Theorem

1. Used when there are repeated investments (as opposed to a single investment); e.g., honey bees foraging for nectar to bring back to their colony.
2. Must be diminishing returns to the investment.
3. Assume that the optimal behaviour maximizes the *rate* of return (e.g., the rate that a bee delivers nectar to the colony).

Marginal Value Theorem

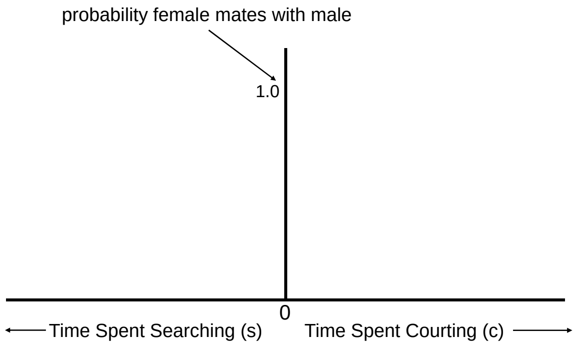
Guppies: A male's dilemma – how to mate with as many females as possible

Marginal Value Theorem

1. Guppies are a small tropic fish found in freshwater streams in Trinidad.
2. Males court females using a sigmoid display and female chooses whether or not to mate with male.
3. The longer a male courts a female, the greater his chance is of mating with her.
4. Males are polygynous (they may mate with many females) but they must spend time searching for each female.
5. Males provide no care for their offspring.

Marginal Value Theorem

How long should a male guppy court a female?



Marginal Value Theorem

Optimal investment maximizes:

rate of return → $F(c)$; probability of mating → $R(c)$; time spent searching + time spent courting → $s_1 + c$

$$F(c) = \frac{R(c)}{s_1 + c}$$

Marginal Value Theorem

Maximize F(c):

$$F'(c^*) = 0$$

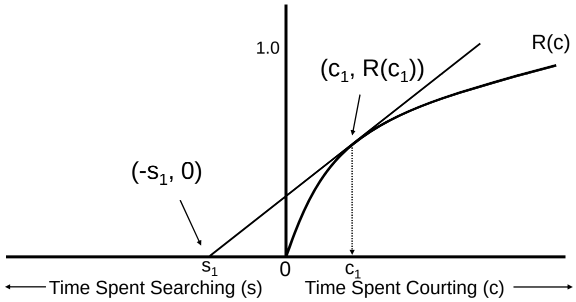
$$F'(c^*) = \frac{R'(c^*) \cdot (s_1 + c^*) - R(c^*)}{(s_1 + c^*)^2} = 0$$

Optimal Investment:

$$c^* = \frac{R(c^*)}{R'(c^*)} - s_1$$

Marginal Value Theorem

Show that point of intersection between R(c) and the line passing through point $(-s_1, 0)$ and tangent to R(c) maximizes the rate of return, F(c).



Marginal Value Theorem

Recall: $$c^* = \frac{R(c^*)}{R'(c^*)} - s_1$$ maximizes F(c)

Eq. of Line: $y = m \cdot x + b$

Two Points: $(-s_1, 0)$ & $(c_1, R(c_1))$

$$m = \frac{\text{rise}}{\text{run}} = \frac{R(c_1)}{c_1 + s_1}$$

Marginal Value Theorem

$$m = \frac{\text{rise}}{\text{run}} = \frac{R(c_1)}{c_1 + s_1}$$

$$y = \frac{R(c_1)}{c_1 + s_1} \cdot c + b$$ (could also solve for b)

We know that line must also be tangent to R(c) at c_1
Thus slope of the line must also equal $R'(c_1)$:

$$\frac{R(c_1)}{c_1 + s_1} = R'(c_1)$$

Marginal Value Theorem

$$\frac{R(c_1)}{c_1 + s_1} = R'(c_1)$$

Solving for c_1:

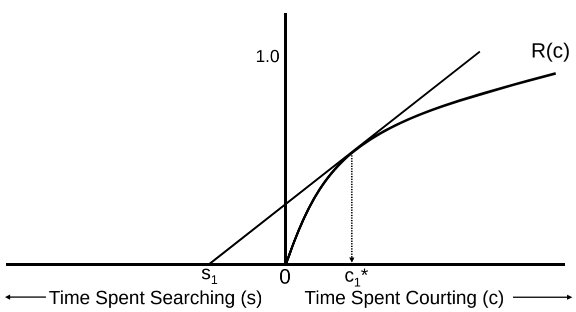
$$c_1 = \frac{R(c_1)}{R'(c_1)} - s_1$$

$$c_1 = c^*$$

Thus, the point of intersection between R(c) and the line passing through point ($-s_1$, 0) and tangent to R(c) maximizes the rate of return F(c).

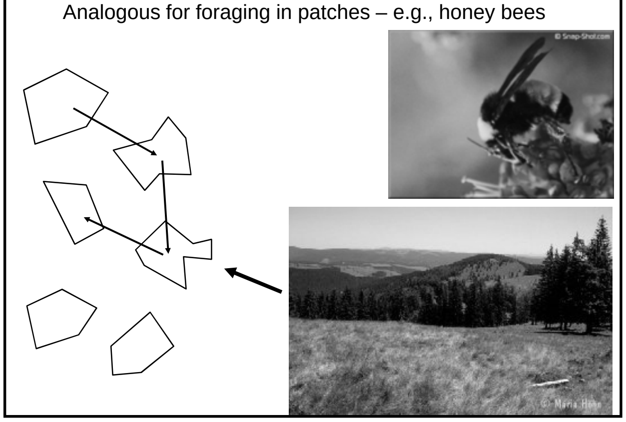
Marginal Value Theorem

What would happen to c* if search time increased (e.g., fewer females in stream)?



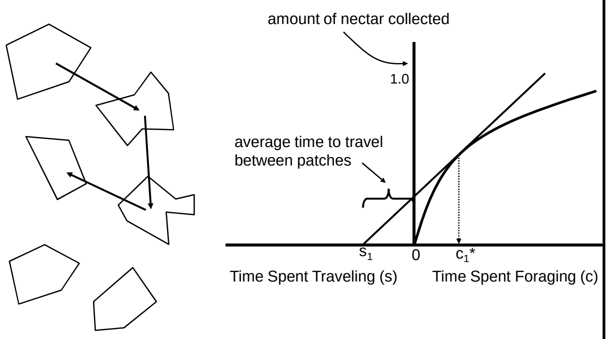
Marginal Value Theorem

Analogous for foraging in patches – e.g., honey bees



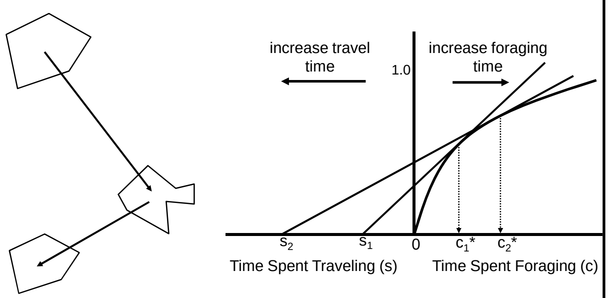
Marginal Value Theorem

Analogous for foraging in patches – e.g., honey bees

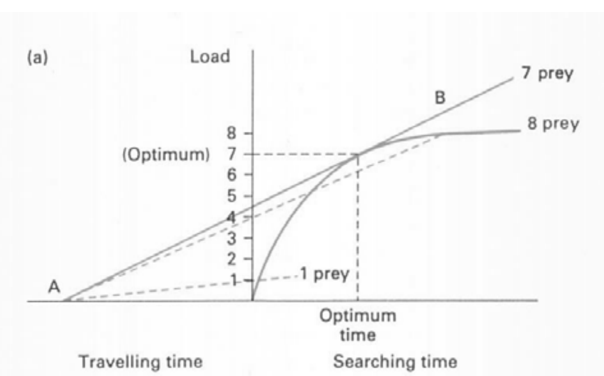


Marginal Value Theorem

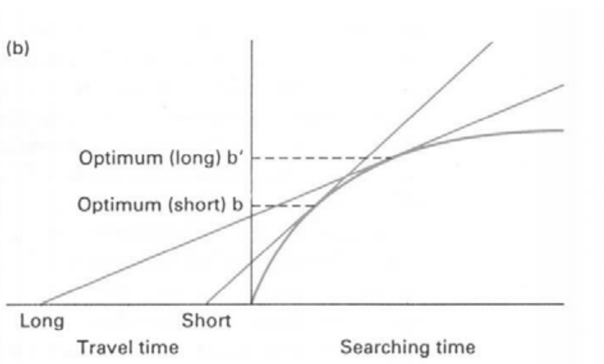
Optimal foraging time increases as travel time between patches increases

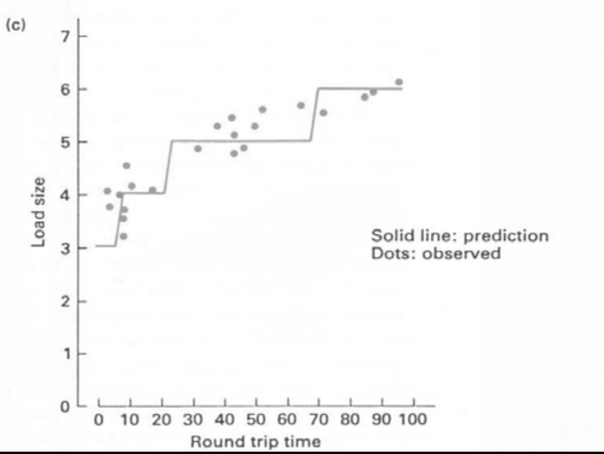


MVT: Starlings and Leatherjackets



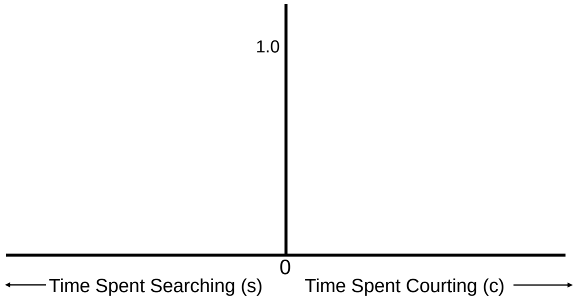
MVT: Starlings and Leatherjackets





Marginal Value Theorem

What would be the optimal investment for a low quality and high quality male guppy (in the same river)?



MVT: Courting and copulating in dung flies

How long should a male dung fly copulate with a female given the following information:

1. The proportion of eggs fertilized by a male follows the following distribution:

Time Spent Copulating (min)	Eggs Fertilized (%)
10	30
20	50
30	70
40	80
75	90
100	95

MVT: Courting and copulating in dung flies

2. A male must guard the female after copulating with her until she lays her eggs to ensure that no other male removes his sperm. Females typically lay their eggs about 106 minutes after copulating with a male.

3. It takes about 50 minutes to find the next female to copulate with.

Summary

1. Used when there are repeated investments.
2. Must be diminishing returns to the investment.
3. Assume that the optimal behaviour maximizes the *rate of return*.
4. Can be used to solve for the optimal time an individual should forage in a patch, or the optimal time an individual should court a female.