Using dice to demonstrate how selection makes the impossible inevitable

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- Try this yourself with real dice.
- Chance does not lead to the accumulation of favourable elements of a complex outcome.
- Selection is a property of imperfectly self-replicating entities (e.g., life) that allows favourable elements of a complex outcome to accumulate gradually.
- Note that there are large numbers of complex outcomes. Once a complex outcome has occurred, its probability is 1 (it is no longer unlikely). In other words, life as we know it was not the only possible outcome.

Selection makes
the improbable
inevitable

How many throws of 6 dice are needed,
on average, to obtain 6 ones?

6?

36?

46,656

$$46{,}656 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6$$

At one throw per second, it would take on average 13 hours, nonstop, with no guarantee that 6 ones would appear!

Number of dice (n)	Expected number (E) of throws to obtain all ones
2	36
4	1,296
6	46,656
8	1,679,616
10	60,466,176
50	8.1×10^{38}
100	6.5×10^{77}
200	4.3×10^{155}
300	2.8×10^{233}
n	n^n

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All humans, 1 throw per second, a trillion times the age of the universe

Greater than the number of atoms in the universe

Number of dice (n)	Expected number (E) of throws to obtain all ones
2	36
4	1,296
6	46,656
300	2.8×10^{233}
n	n^n

In other words, impossible!

Selection makes
the *impossible*
inevitable

How many throws of 6 dice are needed,
on average, to obtain all ones,

gradually, with selection?

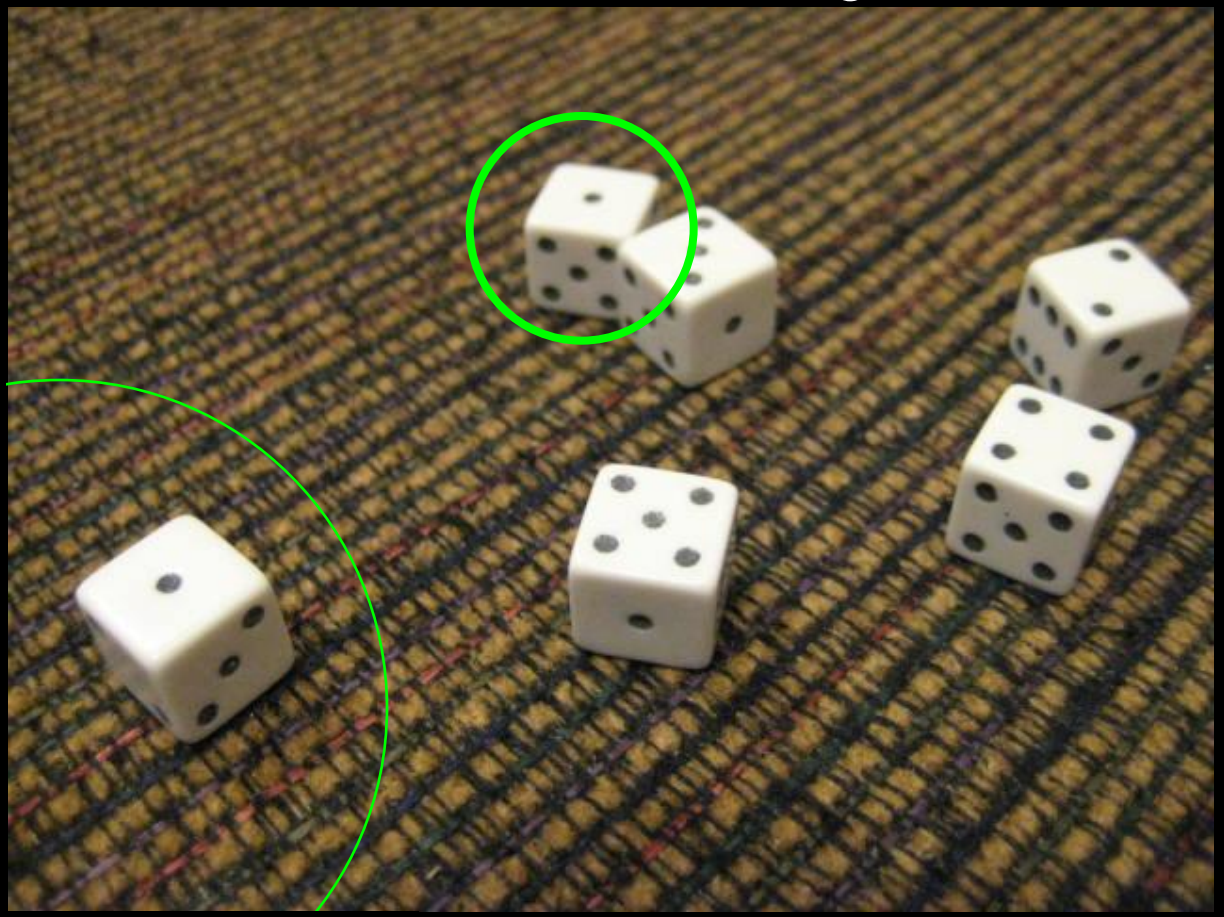
Throw 1 ◆ *E*(at least 1 one in 6) = 1.50 ◆ no luck

Throw 2 ◆ *E*(at least 1 one in 6) = 1.50 ◆ success

One down, five to go

Throw 3 ◆ *E*(at least 1 one in 5) = 1.67 ◆ success

Two down, four to go



Throw 4 ◆ *E*(at least 1 one in 4) = 1.93 ◆ success

Three down, three to go

Throw 5 ◆ *E*(at least 1 one in 3) = 2.37 ◆ no luck

Still three to go

Throw 6 ◆ *E*(at least 1 one in 3) = 2.37 ◆ no luck

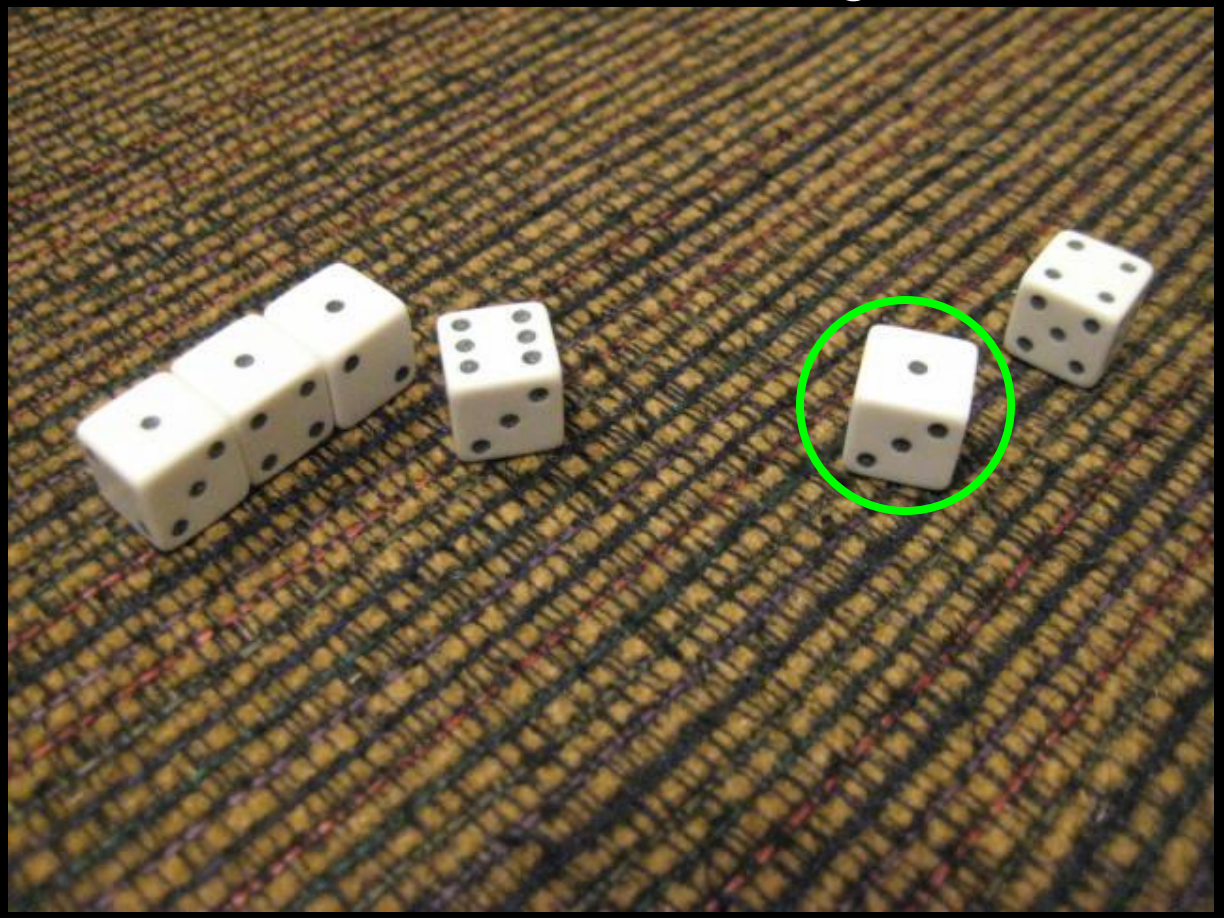
Still three to go

Throw 7 ◆ *E*(at least 1 one in 3) = 2.37 ◆ no luck

Still three to go

Throw 8 ◆ *E*(at least 1 one in 3) = 2.37 ◆ success

Four down, two to go



Throw 9 ◆ *E*(at least 1 one in 2) = 3.27 ◆ no luck

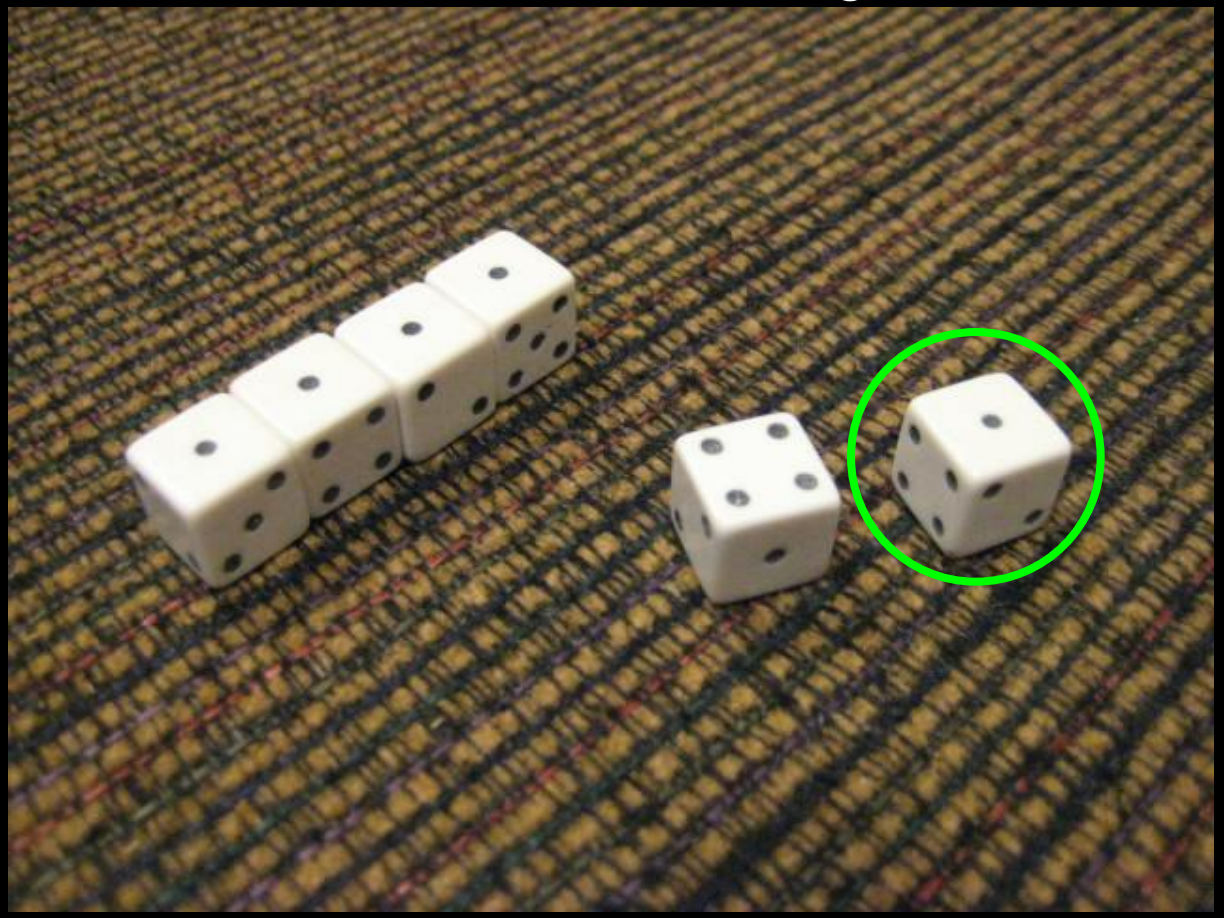
Still two to go

Throw 10 ◆ *E*(at least 1 one in 2) = 3.27 ◆ no luck

Still two to go

Throw 11 ◆ *E*(at least 1 one in 2) = 3.27 ◆ success

Five down, one to go



Throw 12 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 13 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 14 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 15 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 16 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 17 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 18 ◆ *E*(one) = 6 ◆ no luck

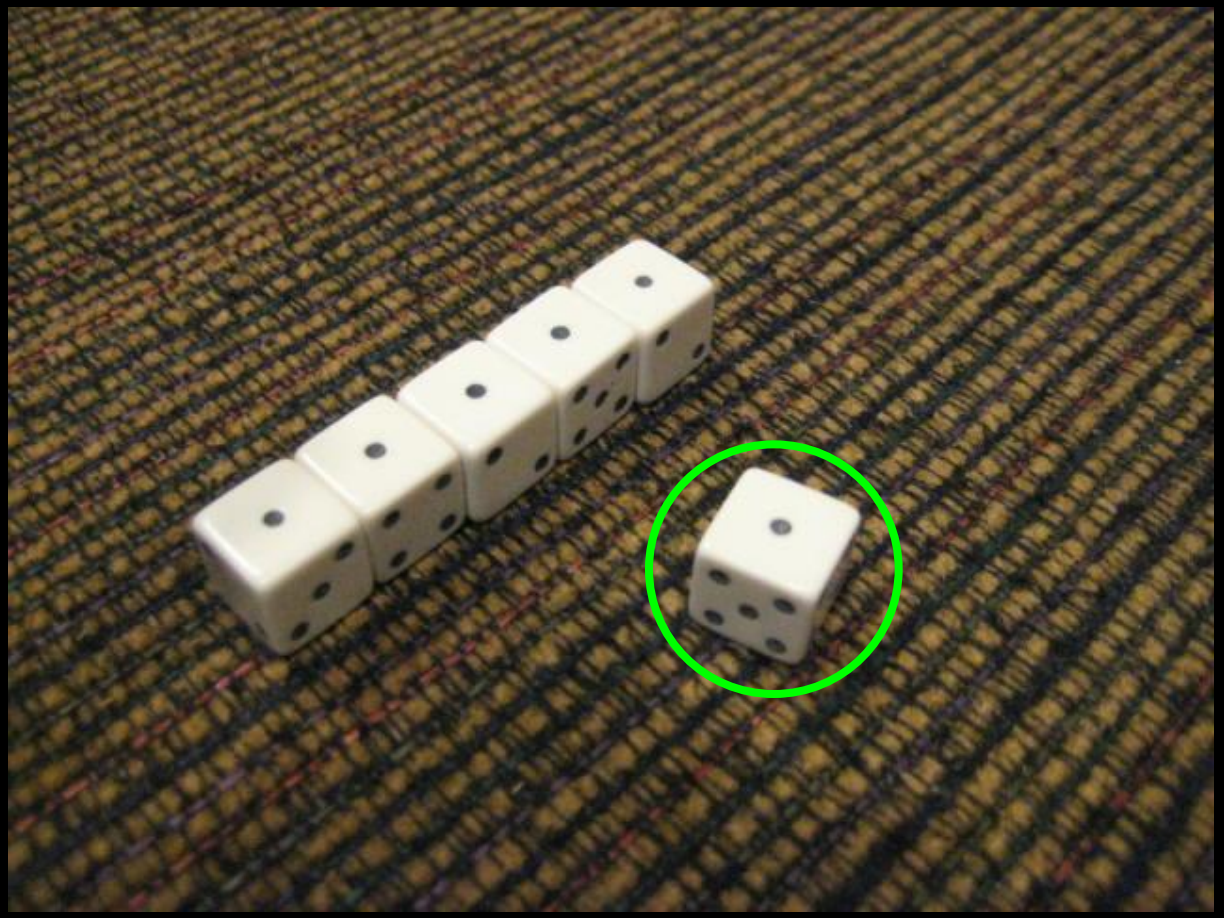
Still one to go

Throw 19 ◆ *E*(one) = 6 ◆ no luck

Still one to go

Throw 20 ◆ *E*(one) = 6 ◆ success

Done!



Selection did in **20** throws what
chance might have done in 46,656 throws

Selection builds on past successes, step by step

$$E(\text{all ones without selection}) = 6^n$$

$$P = \frac{1}{E}; \quad P(\text{all ones}) = \left(\frac{1}{6}\right)^n$$

$$P(\text{none one}) = \left(\frac{5}{6}\right)^n; \quad P(\text{at least one}) = 1 - \left(\frac{5}{6}\right)^n$$

$$E(\text{at least one}) = \frac{1}{1 - \left(\frac{5}{6}\right)^n}$$

$$E(\text{all ones with selection}) \leq \sum_{i=1}^{n} E_i(\text{at least one})$$

Dice	E(all ones, chance)	E(all ones, selection)
2	36	9
4	1,296	14
6	46,656	17
8	1,679,616	19
10	60,466,176	22
50	8.1×10^{38}	63
100	6.5×10^{77}	113
200	4.3×10^{155}	213
300	2.8×10^{233}	313

Dice	*E*(all ones, chance)	*E*(all ones, selection)
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50	Impossible	Inevitable
100		
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