Chemistry 474ba 2007
Problem Set 3 Due Monday, February 5, 2007

1.) Given:

$$\tilde{A} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{pmatrix} \qquad \tilde{B} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

Find $\tilde{A}\tilde{B}$ and $\tilde{B}\tilde{A}$. Do the two matrices commute?

2.) Evaluate the following determinant:

$$\begin{vmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

3.) Given the matrix:

$$\tilde{A} = \begin{pmatrix} 1 & i & 1 \\ -i & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

a) Is $\tilde{A}$ Hermitian?
b) Find the eigenvalues of $\tilde{A}$

4.) Consider a three-dimensional problem. In a given orthonormal basis the Hamiltonian is represented by the matrix:

$$\tilde{H} = \tilde{H}^{(0)} + \tilde{H}^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix} + \begin{pmatrix} 0 & c & 0 \\ c & 0 & 0 \\ 0 & 0 & c \end{pmatrix}$$

c is a constant << 1.

a) Find the exact eigenvalues of $\tilde{H}$

b) Determine the eigenvalues by non-degenerate perturbation to second order.

c) Compare the results of part a) and b)

Note: you will find the binomial expansion useful for part c): $(1+x)^n \sim 1+nx$; $x < 1$.