C474b Problem Set #4 (Due Thursday Feb. 15, 2007)

1.) In matrix form a Hamiltonian is the sum: $\widetilde{H} = \widetilde{H}^{(0)} + \widetilde{H}^{(1)}$, where

$$\widetilde{H}^{(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \; ; \; \widetilde{H}^{(1)} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

and the basis functions used are orthonormal.

a) Use first-order degenerate perturbation theory to calculate the first order corrections to the energies for the first three levels and the "correct" zeroth order eigenstates under the perturbation. Show that the correct eigenstates are orthonormal.

b) Calculate the energy of the fourth level to second order.

2.) Consider a one-dimensional harmonic oscillator with an angular frequency ω_0 and electric charge q. At t = 0 the oscillator is in its ground state. An electric field of strength, E, is applied for a time τ, so the perturbation is

$\hat{H}^{(1)} = -qEx$; $0 \le t \le \tau$. At all other times: $\hat{H}^{(1)} = 0$. Here x is the position operator.

a) Using first-order time-dependent perturbation theory, calculate the probability of a transition from $|n = 0\rangle$ to $|n = 1\rangle$.

b) Using the results of first-order perturbation time dependent theory, show that a transition from $|n = 0\rangle$ to $|n = 2\rangle$ is forbidden.

Note: This is a harmonic oscillator problem so it will be easiest to use the raising and lowering operator method to evaluate integrals. See previous problem sets and notes.