Chemistry 474b 2007

Problem Set 6, Due Thursday, March 15, 2007

1.) Consider 3 electrons with spin ½ and $\ell = 2$.

a) What are the possible values of total orbital angular momentum L?
b) What are the possible values of total spin S?
c) What are the possible values for the total angular momentum J?
d) What are the possible values of total angular momentum, j, for each of the electrons?
e) Find all possible values J from the set $\{j_1, j_2, j_3\}$ determined in part d) and compare with results of part c).

2.) Consider two spinless particles but each with angular momentum $\ell = 1$. The total angular momentum $J = \ell_1 + \ell_2$ can take on values J = 0, 1, and 2.
a) Derive the Clebsch-Gordon coefficients for J = 2.
b) Derive the Clebsch-Gordon coefficients for J = 1.
c) Derive the Clebsch-Gordon coefficients for J = 0.
Note: For part b) first that the coupled wave function |1,1> be a linear combination like the uncoupled wave function derived for |2,1>, and then require |1,1> to be orthogonal to |2,1> and normalized. For part c) require |0,0> to be orthogonal to both |2,0> and |1,0> from parts a) and b) respectively, and normalized.

3.) A particle of spin ½ is in a D-state of orbital angular momentum.
a) What are it possible states of total angular momentum, J?
b) Suppose the single particle Hamiltonian is:

$$\hat{H} = A + B\hat{L}\cdot\hat{S} + C\hat{L}\cdot\hat{L}$$

What are the values of the energy for each of the different states of total angular momentum in terms of the constants A, B, and C?