C474b 2007 Solutions Problem Set #5

1. From class, force on beam $\equiv F = -\mu_B (m_l + 2m_s)(\partial B/\partial z)$

$\mu_B = e\hbar/2m_e > 0$ and $\partial B/\partial z > 0$

a)

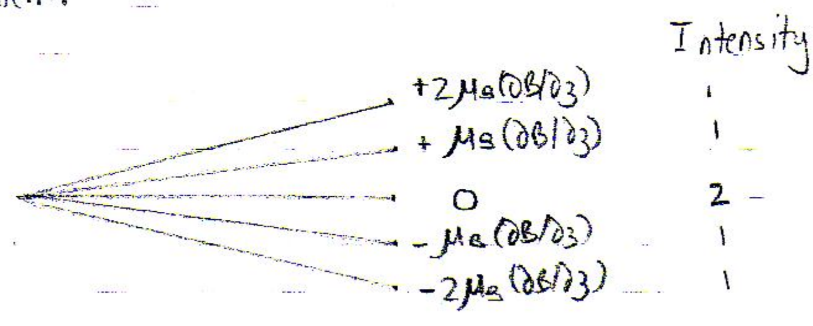
n	l	m_l	m_s	deflection
4	0	0	$+\frac{1}{2}$	$-\mu_B(\partial B/\partial z)$
			$-\frac{1}{2}$	$+\mu_B(\partial B/\partial z)$

Splitting pattern:

b)

n	l	m_l	m_s	deflection
4	1	-1	$+\frac{1}{2}$	0
		-1	$-\frac{1}{2}$	$-2\mu_B(\partial B/\partial z)$
		0	$+\frac{1}{2}$	$+\mu_B(\partial B/\partial z)$
		0	$-\frac{1}{2}$	$-\mu_B(\partial B/\partial z)$
		$+1$	$+\frac{1}{2}$	$+2\mu_B(\partial B/\partial z)$
		$+1$	$-\frac{1}{2}$	0

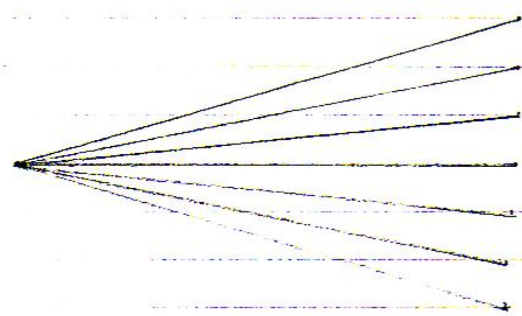
Splitting pattern:



c)

n	ℓ	m_ℓ	m_s	deflection
4	2	-2	$+\frac{1}{2}$	$\mu_B(\partial B/\partial z)$
		-2	$-\frac{1}{2}$	$3\mu(\partial B/\partial z)$
		-1	$+\frac{1}{2}$	0
		-1	$-\frac{1}{2}$	$2\mu_B(\partial B/\partial z)$
		0	$+\frac{1}{2}$	$-\mu_B(\partial B/\partial z)$
		0	$-\frac{1}{2}$	$\mu_B(\partial B/\partial z)$
		$+1$	$+\frac{1}{2}$	$-2\mu_B(\partial B/\partial z)$
		$+1$	$-\frac{1}{2}$	0
		$+2$	$+\frac{1}{2}$	$-3\mu_B(\partial B/\partial z)$
		$+2$	$-\frac{1}{2}$	$-\mu_B(\partial B/\partial z)$

splitting pattern:

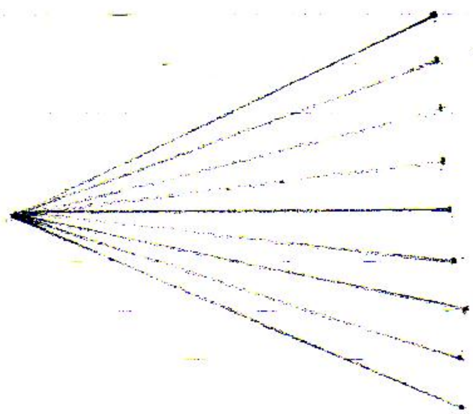


	Intensity
$+3\mu_B(\partial B/\partial z)$	1
$+2\mu_B(\partial B/\partial z)$	1
$+\mu_B(\partial B/\partial z)$	2
0	2
$-\mu_B(\partial B/\partial z)$	2
$-2\mu_B(\partial B/\partial z)$	1
$-3\mu_B(\partial B/\partial z)$	1

d)

n	ℓ	m_ℓ	m_s	deflection
4	3	-3	$+\frac{1}{2}$	$2\mu_B(\partial B/\partial z)$
		-3	$-\frac{1}{2}$	$4\mu_B(\partial B/\partial z)$
		-2	$+\frac{1}{2}$	$\mu_B(\partial B/\partial z)$
		-2	$-\frac{1}{2}$	$3\mu_B(\partial B/\partial z)$
		-1	$+\frac{1}{2}$	0
		-1	$-\frac{1}{2}$	$2\mu_B(\partial B/\partial z)$
		0	$+\frac{1}{2}$	$-\mu_B(\partial B/\partial z)$
		0	$-\frac{1}{2}$	$\mu_B(\partial B/\partial z)$
		$+1$	$+\frac{1}{2}$	$-2\mu_B(\partial B/\partial z)$
		$+1$	$-\frac{1}{2}$	0
		$+2$	$+\frac{1}{2}$	$-3\mu_B(\partial B/\partial z)$
		$+2$	$-\frac{1}{2}$	$-\mu_B(\partial B/\partial z)$
		$+3$	$+\frac{1}{2}$	$-4\mu_B(\partial B/\partial z)$
		$+3$	$-\frac{1}{2}$	$-2\mu_B(\partial B/\partial z)$

Splitting pattern:



	Intensity
$+4\mu_B(\partial B/\partial z)$	1
$+3\mu_B(\partial B/\partial z)$	1
$+2\mu_B(\partial B/\partial z)$	2
$+\mu_B(\partial B/\partial z)$	2
0	2
$-\mu_B(\partial B/\partial z)$	2
$-2\mu_B(\partial B/\partial z)$	2
$-3\mu_B(\partial B/\partial z)$	1
$-4\mu_B(\partial B/\partial z)$	1

2) a) Can relate kinetic energy to the thermal energy

$\Rightarrow \frac{1}{2} m v_x^2 = 2 k_B T$

$$\therefore \; v_x = \sqrt{\frac{4 k_B T}{m}} = \sqrt{\frac{(4)(1.38\times10^{-23}\,\mathrm{J\,K^{-1}})(400\,\mathrm{K})}{(1.0079\times10^{-3}\,\mathrm{kg}) / N_{Avogadro} = 6.022\times10^{23}\,\mathrm{mol^{-1}}}}$$

$$= 3633.012\ \mathrm{m s^{-1}}.$$

b) The time, t, the atom experiences the transverse force in traveling through the magnet of length x is

$$t = x / v_x = (1\,\mathrm{m}) / (3633.02\,\mathrm{m s^{-1}}) = 2.752\times10^{-4}\,\mathrm{s} = 0.27\,\mathrm{ms}.$$

c) From class, the transverse force due to the inhomogeneous $\vec{B}$ field $\equiv \vec{F}_z = -\mu_B(m_\ell + 2m_s)(\partial B/\partial z)$

$$m_\ell = 0, \quad m_s = \pm\tfrac{1}{2} \quad \Rightarrow \vec{F}_z = \mp \mu_B(\partial B/\partial z) = m a_z$$

$$\therefore \; a_z = \mp \frac{\mu_B(\partial B/\partial z)}{m} = \mp \frac{(9.27408\times10^{-24}\,\mathrm{J T^{-1}})(10\,\mathrm{T/m})}{\dfrac{(1.0079\times10^{-3}\,\mathrm{kg})}{(6.02205\times10^{23}\,\mathrm{mol^{-1}})}}$$

$$= \mp\ 5.541\times10^{4}\ \mathrm{m s^{-2}}.$$

d) The transverse deflection $z = \frac{1}{2} a_z t^2$

$= \pm \frac{1}{2} (5.541 \times 10^4 \, ms^{-2})(2.752 \times 10^{-4} s)^2$

$= \pm 2.1 \times 10^{-3} \, m \quad = \pm 2.1 mm$

e) easily fits onto a postcard !

(3.) From class $E_{n, m_\ell, m_s} = E_n^{(0)} + E^{(1)}_{zeeman}$

$= -e^2/2n^2 a_0 + \mu_B B \, (m_\ell + 2m_s)$.

n	ℓ	m_ℓ	m_s	$E^{(1)}_{zeeman}$
1	0	0	$\pm \frac{1}{2}$	$\pm \mu_B B$
3	0	0	$\pm \frac{1}{2}$	$\pm \mu_B B$
	1	-1	$+\frac{1}{2}, -\frac{1}{2}$	$0, -2\mu_B B$
		0	$+\frac{1}{2}, -\frac{1}{2}$	$\mu_B B, -\mu_B B$
		1	$+\frac{1}{2}, -\frac{1}{2}$	$2\mu_B B, 0$
	2	-2	$+\frac{1}{2}, -\frac{1}{2}$	$-\mu_B B, -3\mu_B B$
		-1	$+\frac{1}{2}, -\frac{1}{2}$	$0, -2\mu_B B$
		0	$+\frac{1}{2}, -\frac{1}{2}$	$\mu_B B, -\mu_B B$
		1	$+\frac{1}{2}, -\frac{1}{2}$	$2\mu_B B, 0$
		2	$+\frac{1}{2}, -\frac{1}{2}$	$3\mu_B B, \mu_B B$

recall: $\mu_B = \frac{e\hbar}{2m_e} > 0$

b) c) Cont'd

$n=1 \longrightarrow n=3$