Table of Integrals

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Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

$$\int \frac{1}{x} dx = \ln x$$

$$\int u dv = uv - \int v du$$

Integrals of Rational Functions

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$$

$$\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$$

$$\int (x+a)^n dx = (x+a)^n \left(\frac{a}{1+n} + \frac{x}{1+n} \right), \quad n \neq -1$$

$$\int x(x+a)^n dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}(x/a)$$

$$\int \frac{x dx}{a^2+x^2} = \frac{1}{2} \ln(a^2+x^2)$$

$$\int \frac{x^2 dx}{a^2+x^2} = x - a \tan^{-1}(x/a)$$

$$\int \frac{x^3 dx}{a^2+x^2} = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln(a^2+x^2)$$

$$\int (ax^2+bx+c)^{-1} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right)$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{l \mathrm{n}(a+x) - \ln(b+x)}{b-a}, \; a \neq b \;,$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln(a+x)$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{\ln(ax^2+bx+c)}{2a} - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right)$$

Integrals Involving Roots

$$\int \sqrt{x-a} dx = \frac{2}{3}(x-a)^{3/2}$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a}$$

$$\int \frac{1}{\sqrt{a-x}} dx = 2\sqrt{a-x}$$

$$\int x\sqrt{x-a} dx = \frac{2}{3} a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}$$

$$\int \sqrt{ax+b} dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{b+ax}$$

$$\int (ax+b)^{3/2} dx = \sqrt{b+ax} \left(\frac{2b^2}{5a} + \frac{4bx}{5} + \frac{2ax^2}{5} \right)$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3}(x \pm 2a)\sqrt{x \pm a}$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x}\sqrt{a-x} - a \tan^{-1} \left(\frac{\sqrt{x}\sqrt{a-x}}{x-a} \right)$$

$$\int \sqrt{\frac{x}{x+a}} dx = \sqrt{x}\sqrt{x+a} - a \ln \left[\sqrt{x} + \sqrt{x+a} \right]$$

$$\int x\sqrt{ax+b} dx = \left(-\frac{4b^2}{15a^2} + \frac{2bx}{15a} + \frac{2x^2}{5} \right) \sqrt{b+ax}$$

$$\int \sqrt{x}\sqrt{ax+b} dx = \left(\frac{b\sqrt{x}}{4a} + \frac{x^{3/2}}{2} \right) \sqrt{b+ax} - \frac{b^2 \ln \left(2\sqrt{a}\sqrt{x} + 2\sqrt{b+ax} \right)}{4a^{3/2}}$$

$$\int x^{3/2} \sqrt{ax+b} dx = \left(-\frac{b^2\sqrt{x}}{8a^2} + \frac{bx^{3/2}}{12a} + \frac{x^{5/2}}{3} \right) \sqrt{b+ax} - \frac{b^3 \ln \left(2\sqrt{a}\sqrt{x} + 2\sqrt{b+ax} \right)}{8a^{5/2}}$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x\sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right)$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x\sqrt{a^2 - x^2} - \frac{1}{2} a^2 \tan^{-1} \left(\frac{x\sqrt{a^2-x^2}}{x^2-a^2} \right)$$

$$\int x\sqrt{x^2 \pm a^2} = \frac{1}{3}(x^2 \pm a^2)^{3/2}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left(x + \sqrt{x^2 \pm a^2} \right)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x\sqrt{x^2 \pm a^2} \mp \frac{1}{2} \ln \left(x + \sqrt{x^2 \pm a^2} \right)$$

$$\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{1}{2} x\sqrt{a - x^2} - \frac{1}{2} a^2 \tan^{-1} \left(\frac{x\sqrt{a^2-x^2}}{x^2-a^2} \right)$$

$$\int \sqrt{ax^2+bx+c} \; dx = \left(\frac{b}{4a} + \frac{x}{2} \right) \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left(\frac{2ax+b}{\sqrt{a}} + 2\sqrt{ax^2+bc+c} \right)$$

$$\int x\sqrt{ax^2+bx+c} \; dx = \left(\frac{x^3}{3} + \frac{bx}{12a} + \frac{8ac-3b^2}{24a^2} \right) \sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^{5/2}} \ln \left(\frac{2ax+b}{\sqrt{a}} + 2\sqrt{ax^2+bc+c} \right)$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}} \ln \left[\frac{2ax+b}{\sqrt{a}} + 2\sqrt{ax^2+bx+c} \right]$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \ln \left[\frac{2ax+b}{\sqrt{a}} + 2\sqrt{ax^2+bx+c} \right]$$

Integrals Involving Logarithms

$$\int \ln x dx = x \ln x - x$$

$$\int \frac{\ln(ax)}{x} dx = \frac{1}{2} (\ln(ax))^2$$

$$\int \ln(ax+b) dx = \frac{ax+b}{a} \ln(ax+b) - x$$

$$\int \ln(a^2x^2 \pm b^2) dx = x \ln(a^2x^2 \pm b^2) + \frac{2b}{a} \tan^{-1} \left(\frac{ax}{b} \right) - 2x$$

$$\int \ln(a^2 - b^2x^2) dx = x \ln(a^2 - b^2x^2) + \frac{2a}{b} \tan^{-1} \left(\frac{bx}{a} \right) - 2x$$

$$\int \ln(ax^2+bx+c) dx = \frac{1}{a}\sqrt{4ac-b^2} \tan^{-1} \left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right) - 2x + \left(\frac{b}{2a} + x \right) \ln \left(ax^2+bx+c \right)$$

$$\int x \ln(ax+b) dx = \frac{b}{2a} x - \frac{1}{4} x^2 + \frac{1}{2} \left(x^2 - \frac{b^2}{a^2} \right) \ln(ax+b)$$

$$\int x \ln(a^2 - b^2x^2) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left(x^2 - \frac{a^2}{b^2} \right) \ln(a^2 - bx^2)$$

Integrals Involving Exponentials

$$\int e^{ax}dx = \frac{1}{a}e^{ax}$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}\left(i\sqrt{ax}\right), \text{ where}$$

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

$$\int xe^x dx = (x-1)e^x$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$$

$$\int x^2 e^x dx = e^x(x^2 - 2x + 2)$$

$$\int x^2 e^{ax}dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$$

$$\int x^3 e^x dx = e^x(x^3 - 3x^2 + 6x - 6)$$

$$\int x^n e^{ax}dx = (-1)^n \frac{1}{a}\Gamma[1+n, -ax] \text{ where}$$

$$\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$$

$$\int e^{ax^2}dx = -i\frac{\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}\left(ix\sqrt{a}\right)$$

Integrals Involving Trigonometric Functions

$$\int \sin x dx = -\cos x$$

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$\int \sin^3 x dx = -\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$$

$$\int \cos x dx = \sin x$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$

$$\int \cos^3 x dx = \frac{3}{4}\sin x + \frac{1}{12}\sin 3x$$

$$\int \sin x \cos x dx = -\frac{1}{2}\cos^2 x$$

$$\int \sin^2 x \cos x dx = \frac{1}{4}\sin x - \frac{1}{12}\sin 3x$$

$$\int \sin x \cos^2 x dx = -\frac{1}{4}\cos x - \frac{1}{12}\cos 3x$$

$$\int \sin^2 x \cos^2 x dx = \frac{x}{8} - \frac{1}{32}\sin 4x$$

$$\int \tan x dx = -\ln\cos x$$

$$\int \tan^2 x dx = -x + \tan x$$

$$\int \tan^3 x dx = \ln[\cos x] + \frac{1}{2}\sec^2 x$$

$$\int \sec x dx = \ln|\sec x + \tan x|$$

$$\int \sec^2 x dx = \tan x$$

$$\int \sec^3 x dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x \tan x|$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \sec^2 x \tan x dx = \frac{1}{2}\sec^2 x$$

$$\int \sec^n x \tan x dx = \frac{1}{n}\sec^n x, \quad n \neq 0$$

$$\int \csc x dx = \ln|\csc x - \cot x|$$

$$\int \csc^2 x dx = -\cot x$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x|$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x, \quad n \neq 0$$

$$\int \sec x \csc x dx = \ln\tan x$$

Integrals Involving Trigonometric Functions and x^n

$$\int x\cos x dx = \cos x + x\sin x$$

$$\int x\cos(ax)dx = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax$$

$$\int x^2\cos x dx = 2x\cos x + (x^2-2)\sin x$$

$$\int x^2\cos axdx = \frac{2}{a^2}x\cos ax + \frac{a^2x^2-2}{a^3}\sin ax$$

$$\int x^n \cos x dx =$$

$$-\frac{1}{2}(i)^{1+n}\left[\Gamma(1+n,-ix) + (-1)^n\Gamma(1+n,ix)\right]$$

$$\int x^n \cos axdx =$$

$$\frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(1+n,-iax) - \Gamma(1+n,iax)\right]$$

$$\int x\sin x dx = -x\cos x + \sin x$$

$$\int x\sin(ax)dx = -\frac{x}{a}\cos ax + \frac{1}{a^2}\sin ax$$

$$\int x^2\sin x dx = (2-x^2)\cos x + 2x\sin x$$

$$\int x^2\sin axdx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2}{a^3}x\sin ax$$

$$\int x^n \sin x dx = -\frac{(i)^n}{2}\left[\Gamma(n+1,-ix) - (-1)^n\Gamma(n+1,-ix)\right]$$

Integrals Involving both Trigonometric and Exponential Functions

$$\int e^x \sin x dx = \frac{1}{2}e^x\left[\sin x - \cos x\right]$$

$$\int e^{bx}\sin(ax)dx = \frac{1}{b^2+a^2}e^{bx}\left[b\sin ax - a\cos ax\right]$$

$$\int e^x\cos x dx = \frac{1}{2}e^x\left[\sin x + \cos x\right]$$

$$\int e^{bx}\cos(ax)dx = \frac{1}{b^2+a^2}e^{bx}\left[a\sin ax + b\cos ax\right]$$

$$\int xe^x\sin x dx = \frac{1}{2}e^x\left[\cos x - x\cos x + x\sin x\right]$$

$$\int xe^x\cos x dx = \frac{1}{2}e^x\left[x\cos x - \sin x + x\sin x\right]$$

Integrals Involving Hyperbolic Functions

$$\int \cosh x dx = \sinh x$$

$$\int e^{ax}\cosh bxdx = \frac{e^{ax}}{a^2-b^2}\left[a\cosh bx - b\sinh bx\right]$$

$$\int \sinh x dx = \cosh x$$

$$\int e^{ax}\sinh bxdx = \frac{e^{ax}}{a^2-b^2}\left[-b\cosh bx + a\sinh bx\right]$$

$$\int e^x\tanh x dx = e^x - 2\tan^{-1}(e^x)$$

$$\int \tanh axdx = \frac{1}{a}\ln\cosh ax$$

$$\int \cos ax\cosh bxdx =$$

$$\frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right]$$

$$\int \cos ax\sinh bxdx =$$

$$\frac{1}{a^2+b^2}\left[b\cos ax\cosh bx + a\sin ax\sinh bx\right]$$

$$\int \sin ax\cosh bxdx =$$

$$\frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx + b\sin ax\sinh bx\right]$$

$$\int \sin ax\sinh bxdx =$$

$$\frac{1}{a^2+b^2}\left[b\cosh bx\sin ax - a\cos ax\sinh bx\right]$$

$$\int \sinh ax\cosh axdx = \frac{1}{4a}\left[-2ax + \sinh(2ax)\right]$$

$$\int \sinh ax\cosh bxdx =$$

$$\frac{1}{b^2-a^2}\left[b\cosh bx\sinh ax - a\cosh ax\sinh bx\right]$$