

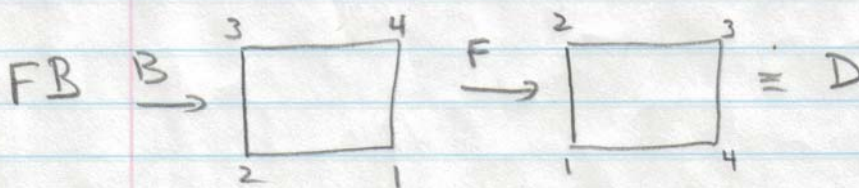
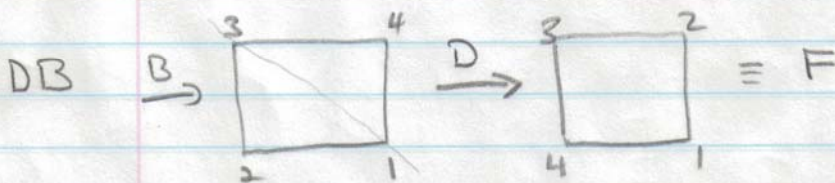
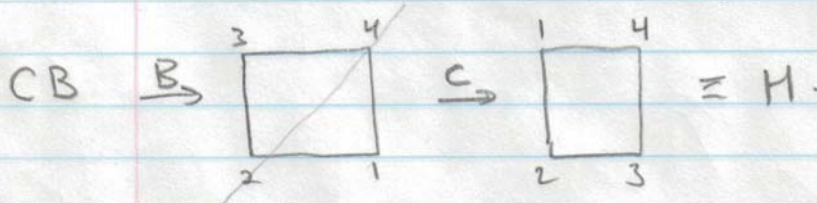
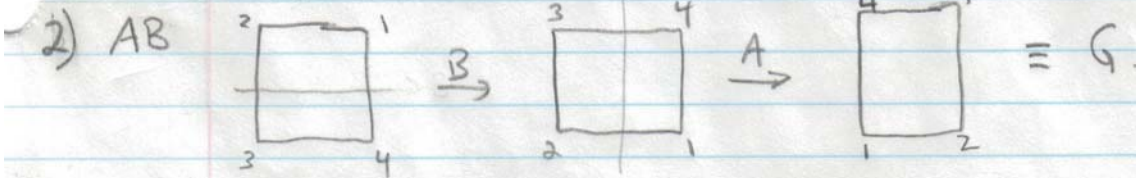
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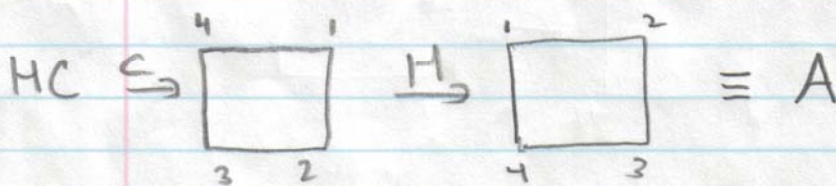
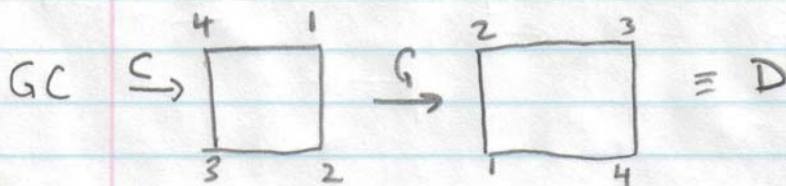
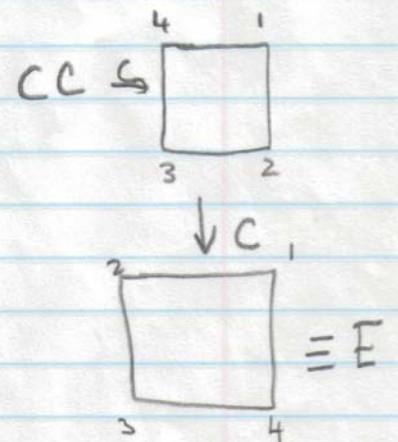
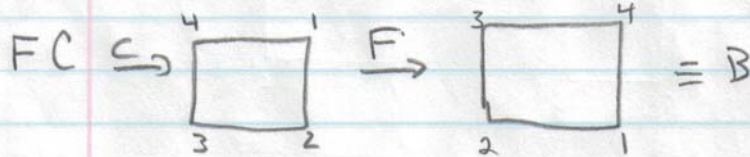
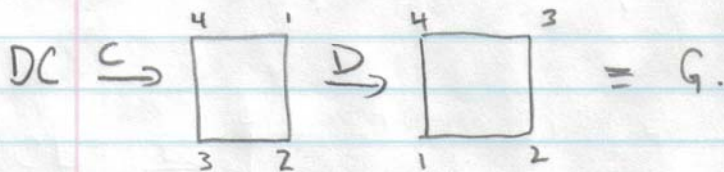
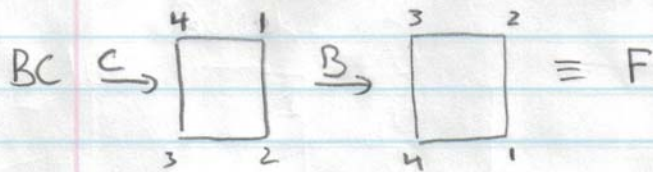
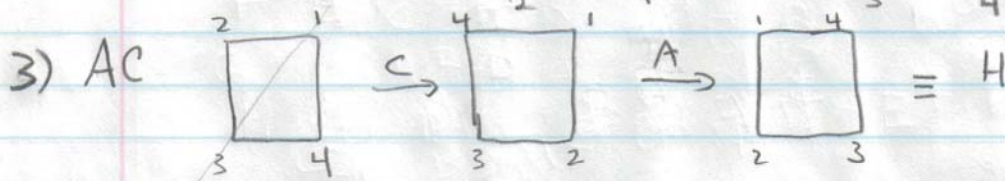
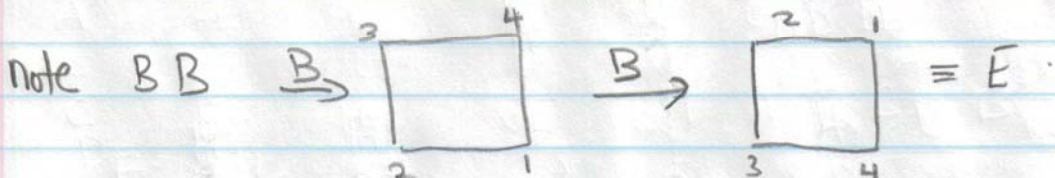
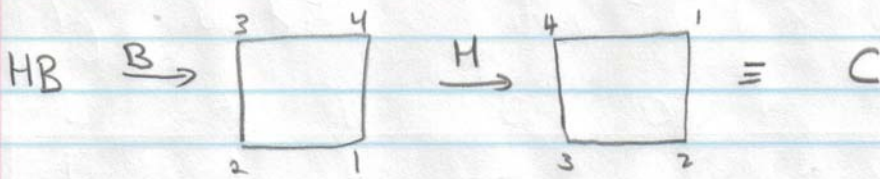
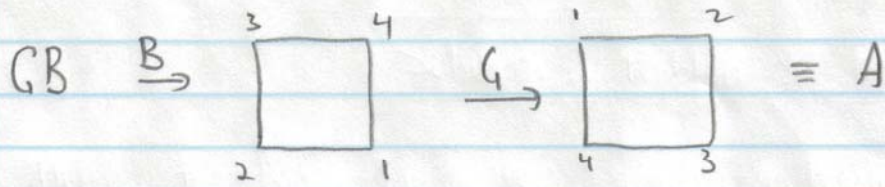
Solutions Problem Set #1.

① a) Here to begin is the group multiplication table followed by work. Convention \equiv col. \times row = product.

	E	A	B	C	D	F	G	H
E	E	A	B	C	D	F	G	H
A	A	E	G	F	H	C	B	D
B	B	G	E	H	F	D	A	C
C	C	H	F	E	G	B	D	A
D	D	F	H	G	E	A	C	B
F	F	D	C	A	B	G	H	E
G	G	B	A	D	C	H	E	F
H	H	C	D	B	A	E	F	G

Work: 1) $EX = XE = X$ for all X .





4) AA $\begin{matrix} 2 & 1 \\ \square & \\ 3 & 4 \end{matrix} \xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{A} \begin{matrix} 2 & 1 \\ \square & \\ 3 & 4 \end{matrix} \equiv E$

BA $\xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{B} \begin{matrix} 4 & 3 \\ \square & \\ 1 & 2 \end{matrix} \equiv G$

CA $\xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{C} \begin{matrix} 3 & 2 \\ \square & \\ 4 & 1 \end{matrix} \equiv F$

DA $\xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{D} \begin{matrix} 1 & 4 \\ \square & \\ 2 & 3 \end{matrix} \equiv H$

FA $\xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{F} \begin{matrix} 4 & 1 \\ \square & \\ 3 & 2 \end{matrix} \equiv C$

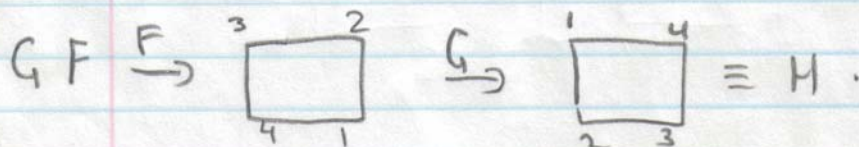
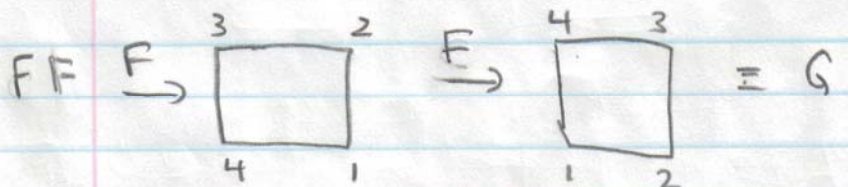
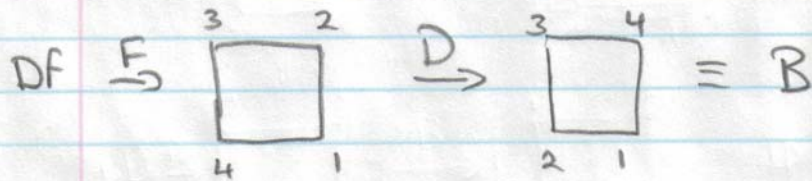
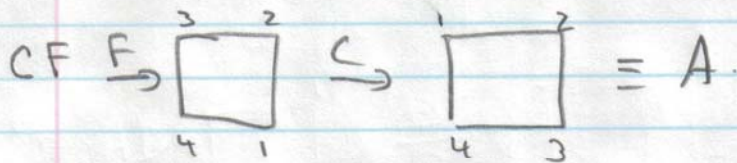
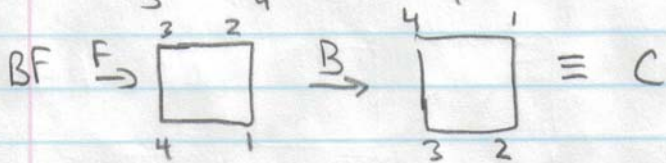
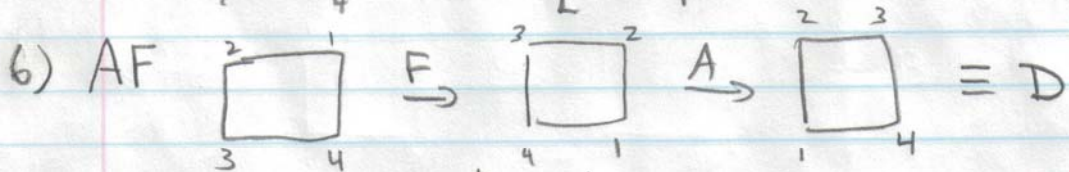
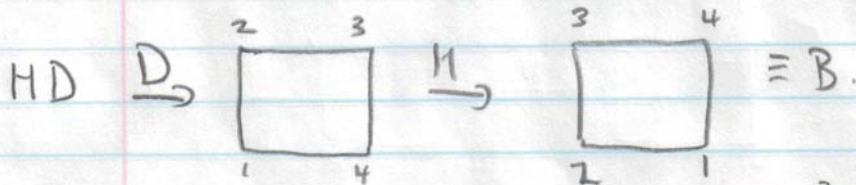
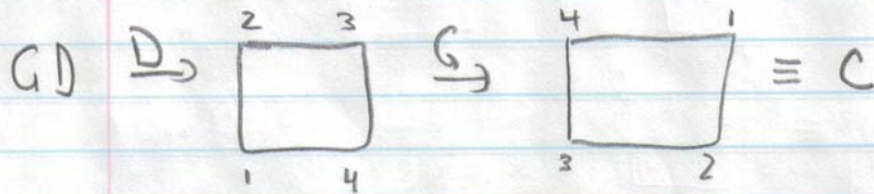
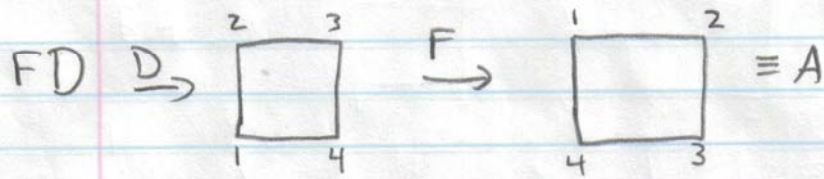
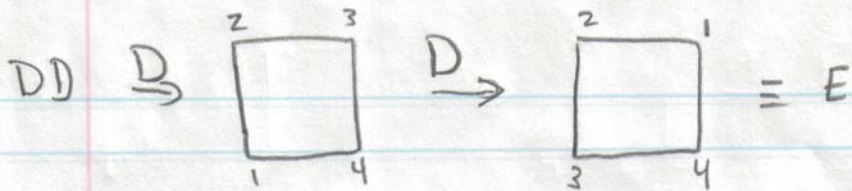
GA $\xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{G} \begin{matrix} 3 & 4 \\ \square & \\ 2 & 1 \end{matrix} \equiv B$

HA $\xrightarrow{A} \begin{matrix} 1 & 2 \\ \square & \\ 4 & 3 \end{matrix} \xrightarrow{G} \begin{matrix} 2 & 3 \\ \square & \\ 1 & 4 \end{matrix} \equiv D$

5) AD $\begin{matrix} 2 & 1 \\ \square & \\ 3 & 4 \end{matrix} \xrightarrow{D} \begin{matrix} 2 & 3 \\ \square & \\ 1 & 4 \end{matrix} \xrightarrow{A} \begin{matrix} 3 & 2 \\ \square & \\ 4 & 1 \end{matrix} \equiv F$

BD $\xrightarrow{D} \begin{matrix} 2 & 3 \\ \square & \\ 1 & 4 \end{matrix} \xrightarrow{B} \begin{matrix} 1 & 4 \\ \square & \\ 2 & 3 \end{matrix} \equiv H$

CD $\xrightarrow{D} \begin{matrix} 2 & 3 \\ \square & \\ 1 & 4 \end{matrix} \xrightarrow{C} \begin{matrix} 4 & 3 \\ \square & \\ 1 & 2 \end{matrix} \equiv G$



$$HF \xrightarrow{F} \begin{array}{|c|c|} \hline 3 & 2 \\ \hline 4 & 1 \\ \hline \end{array} \xrightarrow{H} \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} \equiv E$$

$$7) AG \xrightarrow{G} \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{A} \begin{array}{|c|c|} \hline 3 & 4 \\ \hline 2 & 1 \\ \hline \end{array} \equiv B$$

$$BG \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{B} \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 4 & 3 \\ \hline \end{array} \equiv A ; CG \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{C} \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 1 & 4 \\ \hline \end{array} \equiv D$$

$$DG \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{D} \begin{array}{|c|c|} \hline 4 & 1 \\ \hline 3 & 2 \\ \hline \end{array} \equiv C ; FG \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{F} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \equiv H$$

$$GG \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{G} \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} \equiv E ; HG \xrightarrow{G} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \xrightarrow{H} \begin{array}{|c|c|} \hline 3 & 2 \\ \hline 4 & 1 \\ \hline \end{array} \equiv F$$

$$8) AH \xrightarrow{H} \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{A} \begin{array}{|c|c|} \hline 4 & 1 \\ \hline 3 & 2 \\ \hline \end{array} \equiv C$$

$$BH \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{B} \begin{array}{|c|c|} \hline 2 & 3 \\ \hline 1 & 4 \\ \hline \end{array} \equiv D ; CH \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{C} \begin{array}{|c|c|} \hline 3 & 4 \\ \hline 2 & 1 \\ \hline \end{array} \equiv B$$

$$DH \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{D} \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 4 & 3 \\ \hline \end{array} \equiv A ; FH \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{F} \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} \equiv E$$

$$GH \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{G} \begin{array}{|c|c|} \hline 3 & 2 \\ \hline 4 & 1 \\ \hline \end{array} \equiv F ; HH \xrightarrow{H} \begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \xrightarrow{H} \begin{array}{|c|c|} \hline 4 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \equiv G$$

b) Classes: First use derived multiplication table to deduce inverses. Find

$$A^{-1} = A$$

$$B^{-1} = B$$

$$C^{-1} = C$$

$$D^{-1} = D$$

$$F^{-1} = H \quad ; \quad H^{-1} = F \quad (\text{as required}).$$

$$G^{-1} = G$$

$$E^{-1} = E$$

Consider element A:

$$A^{-1}AA = AAA = AE = A$$

$$B^{-1}AB = BAB = BG = A$$

$$C^{-1}AC = CAC = CH = B$$

$$D^{-1}AD = DAD = DF = B$$

$$E^{-1}AE = EAE = A$$

$$F^{-1}AF = HAF = HD = B$$

$$G^{-1}AG = GAG = GB = A$$

$$H^{-1}AH = FAH = FC = B$$

Next consider element B:

$$A^{-1}BA = ABA = AG = B$$

$$B^{-1}BB = BBB = EB = B$$

$$C^{-1}BC = CBC = CF = A$$

$$D^{-1}BD = DBD = DH = A$$

$$E^{-1}BE = EBE = EB = B$$

$$F^{-1}BF = HBF = HC = A$$

$$G^{-1}BG = GBG = GA = B$$

$$H^{-1}BH = FBH = FD = A$$

$\therefore A, B$ form a class.

element C: $A^{-1}CA = ACA = AF = D$; $B^{-1}CB = BCB = BH = D$

$C^{-1}CC = EC = C$; $D^{-1}CD = DC = DG = C$

$E^{-1}CE = ECE = C$; $F^{-1}CF = HCF = HA = D$

$G^{-1}CG = GCG = GD = C$; $H^{-1}CH = FCH = FB = D$

$\therefore C, D$ form a class.

⑥

element E is in its own class. (always).

element F : $A^{-1}FA = AFA = AC = H$

$B^{-1}FB = BFB = BD = H$; $C^{-1}FC = CFC = CB = H$

$D^{-1}FD = DFD = DA = H$; $E^{-1}FE = EFE = F$

$F^{-1}FF = HFF = HG = F$; $G^{-1}FG = GFG = GH = F$

$H^{-1}FH = FFH = FE = F$

$\therefore F, H$ form a class.

element G : $A^{-1}GA = AGA = AB = G$; $B^{-1}GB = BGB = BA = G$

$C^{-1}GC = CGC = CD = G$; $D^{-1}GD = DGD = DC = G$

$E^{-1}GE = EGE = G$; $F^{-1}GF = HGF = HH = G$

$G^{-1}GG = GGG = GE = G$

$\therefore G$ is in its own class.

② a) the identity operator under multiplication leaves the other group elements (and itself) unchanged.
Inspection of the table $\Rightarrow G \equiv E \equiv$ identity element.

b) $J \cdot L \cdot M$ is associative if $(J \cdot L) \cdot M = J \cdot (L \cdot M)$

$(J \cdot L) \cdot M = HM = P \Rightarrow$ multiplication is associative
 $J \cdot (L \cdot M) = J \cdot K = P$

- next $K \cdot H \cdot P$.

$(K \cdot H) \cdot P = OP = L \Rightarrow$ multiplication is associative.

$K \cdot (H \cdot P) = K \cdot M = L$

c) $O^{-1}O = G$ ($\because G \equiv$ identity element)
 $\therefore O^{-1} = O$

$K^{-1}K = G \quad \therefore K^{-1} = L$

d) $H \cdot K \cdot H^{-1} = H \cdot K \cdot H = HO = L$

$L \cdot O \cdot L^{-1} = L \cdot O \cdot K = LP = J$

$P^{-1}PP = P \cdot P \cdot P = PG = P$

e) consider element H :

$$G^{-1}HG = GHG = GH = H$$

$$K^{-1}HK = LHK = LJ = P$$

$$H^{-1}HH = HHH = HG = H$$

$$J^{-1}HJ = JHJ = JK = P$$

$$L^{-1}HL = KHL = KO = P$$

$$M^{-1}HM = MHM = MP = H$$

$$O^{-1}HO = OHO = OL = P$$

$$P^{-1}HP = PHP = PM = H$$

$\therefore H, P$ are in the same class.

note you could have considered element P and achieved the same result!