Chapter 6 - Response To Gusting Wind

6.1 General

Assume that the structure is smaller than the typical dimension of the atmospheric vortex. Then the quasi-steady approach applies and the time variable drag force is

$$F_D(t) = \frac{1}{2}\rho C_D A V^2(t) = \frac{1}{2}\rho C_D A\left[\overline{V} + v(t)\right]^2 \qquad (6\text{-}1)$$

where ρ = air density = 0.0024 slugs/ft ^{3} = 1.23 kg/m^3, C_D = drag coefficient, A = area exposed to wind, $V(t) = \overline{V} + v(t)$ = wind speed with $\overline{V} > v(t)$. Taking the mean wind speed $\overline{V}$ in front of the bracket

$$V^2(t) = \overline{V}^2\left[1 + \frac{v(t)}{\overline{V}}\right]^2 = \overline{V}^2\left[1 + 2\frac{v(t)}{\overline{V}} + \left(\frac{v(t)}{\overline{V}}\right)^2\right]$$

Because $v(t)/\overline{V} < 1$ and thus $\left(v(t)/\overline{V}\right)^2 << 1$, the square of the ratio $v(t)/\overline{V}$ can be omitted and the total drag force split into two parts:

1. the mean (static drag) $\qquad \overline{F}_D = \frac{1}{2}\rho C_D A\overline{V}^2 \qquad (6\text{-}2)$

   and

2. the fluctuating (dynamic) drag $\qquad F_D(t) = \rho C_D A\overline{V}v(t) \qquad (6\text{-}3)$

in which *v(t)* = fluctuating component of the wind speed. This fluctuating wind speed has a spectrum $S_V(f)$ and, therefore, the spectrum of the drag is, according to equation 5.21a,

$$S_F = \left(\rho C_D A\overline{V}\right)^2 S_v(f) \qquad (6\text{-}4)$$

or

$$S_F = \frac{4\overline{F}_D^2}{\overline{V}^2} S_v(f) \qquad (6\text{-}5)$$

The relation between the size of the structure and the size of the vortex (disturbance) can be introduced through an "aerodynamic admittance function" analogous to the mechanical admittance,

$$\chi^2 = \left|\chi\left(\frac{fL}{\overline{V}}\right)\right|^2 \qquad (6\text{-}6)$$

where L = characteristic dimension for buildings or structures equal to $L = \sqrt{A}$. The ratio $fL/\overline{V}$ is a dimensionless frequency. Only with "point" (small) structures does the Aerodynamic Admittance, $\chi \equiv 1$. Otherwise the values of χ range between these limits:

$$\left(fL/\overline{V}\right) \to 0\ ,\quad \chi^2 \to 1$$
$$\left(fL/\overline{V}\right) \to \infty\ ,\quad \chi^2 \to 0$$

χ^2 is a drooping function indicated in Fig. 6.1.

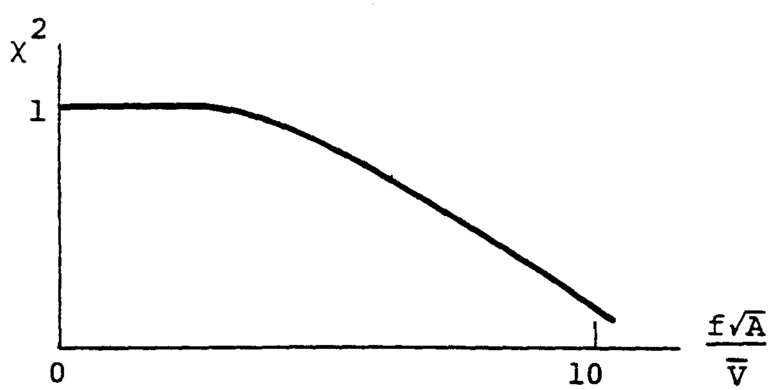


Fig. 6.1 Aerodynamic admittance function

The aerodynamic admittance function is usually established analytically using experimental data. With the aerodynamic admittance introduced into equation (6-5), the fluctuating force spectrum is

$$S_F = \frac{4\overline{F}_D^2}{\overline{V}}\left|\chi\left(\frac{fL}{\overline{V}}\right)\right|^2 S_v(f) \qquad (6\text{-}7)$$

With the spectrum given by equation (6-7), the response of a single degree of freedom structure can be readily predicted. However, for large structures, the approach must be somewhat extended to include the variation of wind speed with height and the shape of the vibration mode.

These aspects were included in an approach adopted in the National Building Code of Canada. This Code assumes that the first vibration mode is linear and sufficient for the analysis and that the width of the face of the building is constant.

The approach is called the Gust Factor Approach. The gust factor C_g is defined as the ratio of the total peak displacement $\hat{U}$ (or load) in a period T to the mean displacement $\bar{u}$ (or load). Thus,

$$C_g = \frac{\hat{U}}{\bar{u}} \quad \text{or} \quad \hat{U} = C_g \bar{u}$$

and the peak load $\hat{p} = C_g \bar{p}$. The peak value is obtained from σ_u by means of the peak factor g; adding the static (average) displacement $\bar{u}$, the total peak displacement is (Fig. 6.2)

$$\hat{U} = \bar{u} + g\sigma_u = \bar{u}\left(1 + g\frac{\sigma_u}{\bar{u}}\right) \tag{6-8}$$

Hence, the gust factor actually becomes

$$C_g = 1 + g\frac{\sigma_u}{\bar{u}} \tag{6-9}$$

The gust factor C_g can be evaluated using the method given for the evaluation of wind loading in the NBC of Canada. This method incorporates all the ingredients of the random vibration approach.

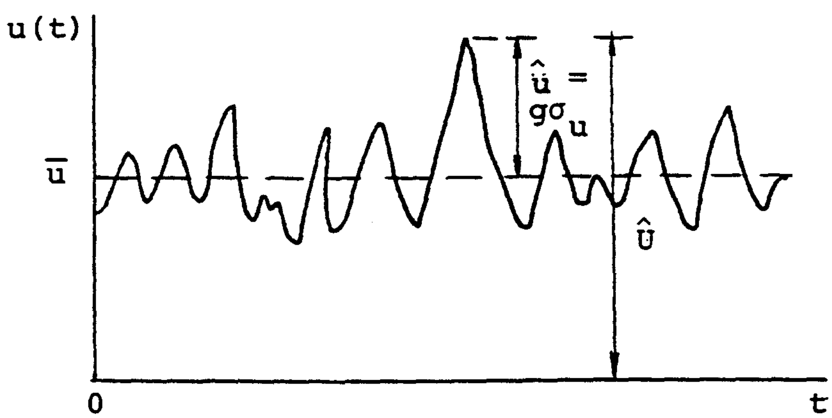


Fig. 6.2 Total structural response to gusting wind

6.2 Wind Pressure By NBC

Detailed Procedure by the National Building Code of Canada, 1995

This procedure is facilitated by the charts given in Fig. 6.3.

The *design external pressure* $p = p(Z)$ varies with height Z on the windward wall, is constant for other walls and is calculated as

$$p = q\, C_e\, C_g\, C_p \qquad (6\text{-}10)$$

in which:

$q = q_{10} =$ *reference mean velocity pressure* in kN/m$^{2} = 1/2\,\rho \overline{V}^2$. It is given in Chapter 1 of the Supplement to the NBC for different localities and return periods of 10, 30 and 100 years; it corresponds to open terrain and the height of 10m, hence $\overline{V} = \overline{V}_{10}$

C_e = *exposure factor.* It accounts for the variation of the wind pressure (wind speed) with height and therefore, varies with height and exposure (surface roughness). It can be read from Fig. 6.3b or calculated for three exposures:

A - open terrain: $C_e = (Z/10)^{0.28}$, $C_e > 1.0$ (6-11a)
B - suburban areas: $C_e = 0.5(Z/12.7)^{0.50}$, $C_e > 0.5$ (6-11b)
C - centres of large cities: $C_e = 0.4(Z/30)^{0.72}$ $C_e > 0.4$ (6-11c)

In equation (6-10), C_e varies with height for the windward wall, i. e. $C = C_e(Z)$, for the roof $C_{eH} = C_e(H)$ and for the leeward wall $C_e = C_e\,(H/2)$.

C_g = *gust effect factor.* It accounts for dynamic effect of gusting wind and is defined as the ratio: total peak response (load)/mean response(load). C_g has one value for the whole building and is evaluated using the mean wind velocity at the top of the building, V_H. This velocity is obtained from the reference velocity $\overline{V} = \overline{V}_{10}$ as:

$$V_H = \overline{V}\sqrt{C_{eH}} \text{ (m/s)} \qquad (6\text{-}12)$$

where $C_{eH} = C_e(H)$ and $\overline{V} = \overline{V}_{10}$ is:

$$\overline{V} = \sqrt{q/C} \text{ (m/s)} \qquad (6\text{-}13)$$

with $C = \frac{1}{2}\rho = 650 \times 10^{-6}$ for $\overline{V}$ in m/s and q in kN/m^2.

The gust effect factor follows from the formula:

$$C_g = 1 + g_p(\sigma / \mu) \qquad (6\text{-}14)$$

in which the coefficient of variation:

$$\sigma / \mu = \sqrt{\frac{K}{C_e}\left(B + \frac{sF}{\beta}\right)} \qquad (6\text{-}15)$$

Here, K = surface roughness factor equal to:
0.08 for exposure A,
0.10 for exposure B,
0.14 for exposure C;

$C_{eH} = C_e(H)$ = the exposure factor for height H;

B = background turbulence factor from Fig. 6.3c;

s = size reduction factor from Fig. 6.3d in which n_o = natural frequency of the building in Hz;

F = gust energy ratio from Fig. 6.3e; and

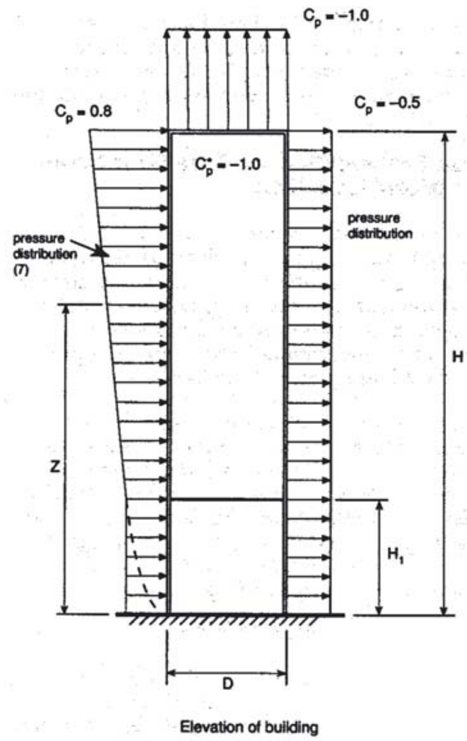
β = damping ratio of the building.

In equation (6-14),
g_p = peak factor = $\bar{\hat{u}} / \sigma_u$ follows from Fig. 6.3f depending on the average fluctuation rate (equation 5.34)

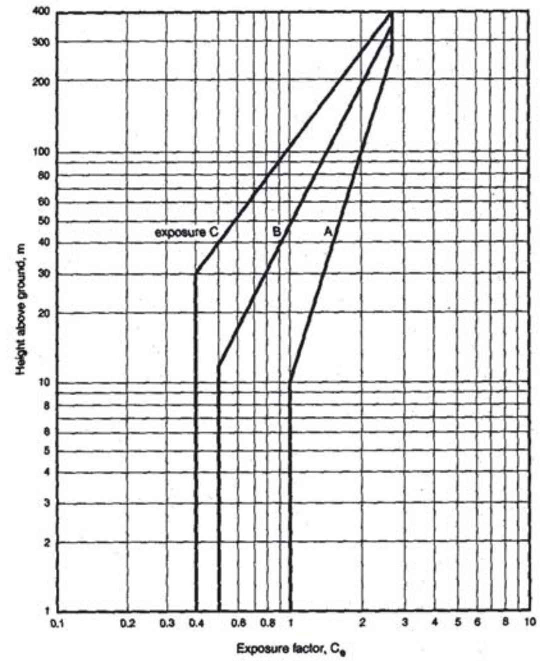
$$\nu = n_o \sqrt{\frac{sF}{sF + \beta B}}$$

With these factors C_g is obtained from equation (6-14). Finally in equation (6-10),

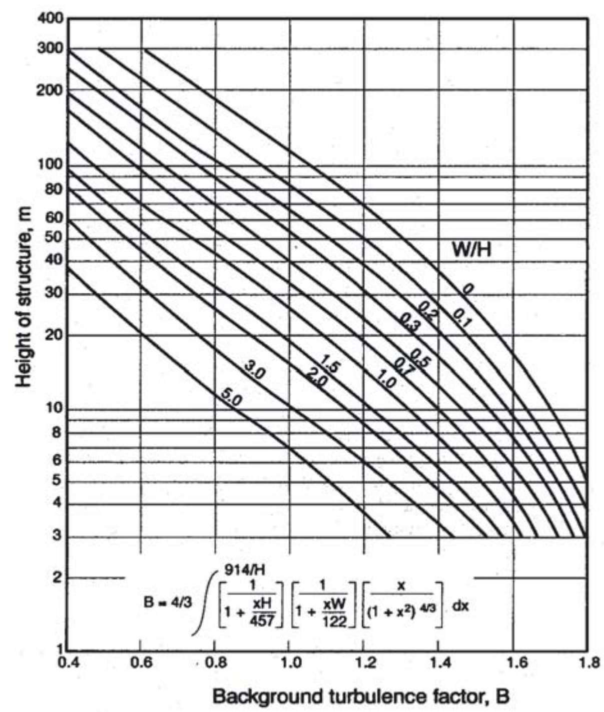
C_p = *pressure coefficient* given in Fig. 6.3a.



a) Pressure Coefficients

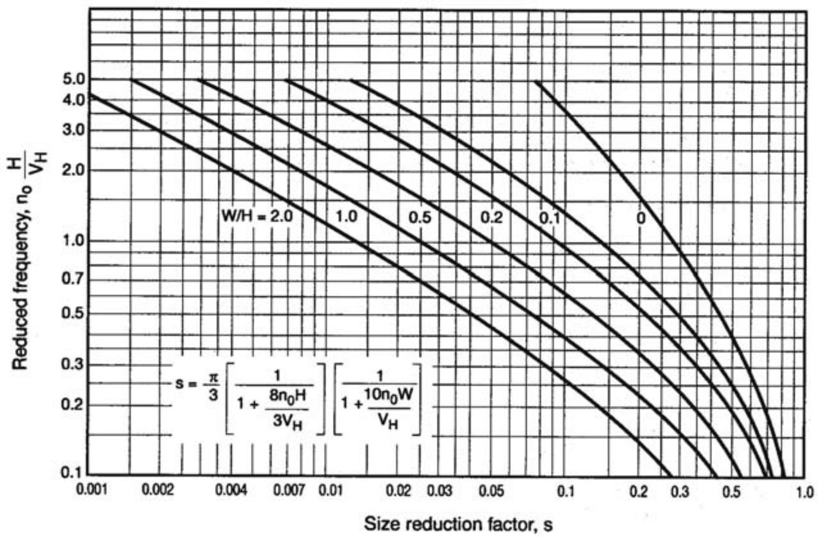


b) Exposure Factor, C_e



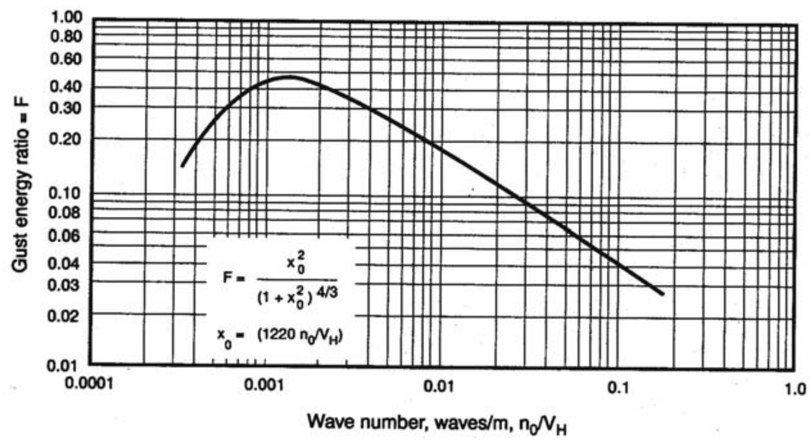
c) Background Turbulence Factor, B

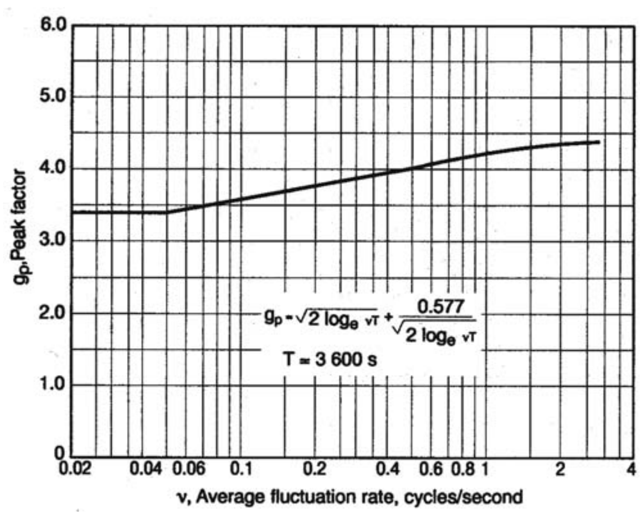
Fig. 6.3a,b,c Charts for evaluation of gust effect factor by NBC, 1995



d) Size Reduction Factor, *S*

e) Gust Energy Ratio, *F*





f) Peak Factor, g_P

Fig. 6.3d,e,f Charts for evaluation of gust effect factor by NBC, 1995

Acceleration - The gust factor can also be used to estimate the peak acceleration at the top of the building, which is an important design criterion of human comfort. Assume that the peak acceleration, $\hat{a}$, derives only from the peak fluctuating displacement $\hat{u}$ (Fig. 6.2) and that it can be approximately evaluated as in harmonic motion; assume further that background turbulence does not contribute to it because the associated frequencies are very low and thus the background turbulence factor B = 0. Then, the peak acceleration is equal to:

$$\hat{a} = \omega_o^2 \hat{u} = 4\pi^2 n_o^2 g_p \frac{\sigma_u}{\mu} \mu$$

With σ / μ from equation (6-15) and $\mu = \overline{u} = \hat{U} / C_g$, where $\hat{U}$ is the total peak displacement due to loading described by equation (6-10), the peak acceleration becomes:

$$\hat{a} = 4\pi^2 n_o^2 \frac{g_p}{C_g} \sqrt{\frac{KsF}{C_e \beta}} \cdot \hat{U} \qquad (6\text{-}17)$$

PROBLEM 6.1:

Evaluate the gust factor, design pressure and acceleration for a tall building to be built on the waterfront in Toronto's downtown area. Consider wind blowing from the south (lake) and from the north (downtown), i.e. exposures A,C, respectively.

Height	H = 100 m
Width	W = 24 m
Depth	D = 20 m
Natural Frequency	n_o = 0.2 cps
Damping ratio	β = 0.01
Exposure	A,C

For a 30 year return period, the reference hourly wind pressure in Toronto is from Chapter 1 of the Supplement, q = 0.48 kN/m^2. Evaluate the maximum acceleration assuming that $\hat{U}$ was found to be 0.02 m for both exposures. Explain the difference between the two exposures.

SOLUTION TO PROBLEM 6.1:

Given:

Height	H = 100 m
Width	W = 24 m
Depth	D = 20 m
Natural Frequency	n_o = 0.2 cps
Damping ratio	β = 0.01
Exposure	A,C
Reference Wind Pressure	q = 0.48 kN/m^2 for a Return Period of 30 years
Peak Displacement	$\hat{U}$ = 0.02 m

a) Evaluate response under NBCC Exposure “A” (Open Terrain Exposure)

- The gust factor is determined through the evaluation of:

$$C_g = 1 + g_p\left(\frac{\sigma}{u}\right) \qquad (6\text{-}14)$$

$$\frac{\sigma}{u} = \sqrt{\frac{K}{C_e}\left(B + \frac{sF}{\beta}\right)} \qquad (6\text{-}15)$$

- $K = 0.08$ for Exposure A

- The exposure factor is evaluated for a height of 100 m,
$C_e = C_e(100) = (Z/10)^{0.28} = (100/10)^{0.28} = 1.91$, $C_e \geq 1.0$

- The Background Turbulence Factor is found from Figure B-2 as
B = 0.78 since W/H =24/100 = 0.24 and H = 100 m

- The reference wind speed is determined from
$\bar{V} = \sqrt{0.48 / 650x10^{-6}} = 27.2$ m/s and the wind speed at the top of the building, $V_H = 27.2\sqrt{1.91} = 37.5$ m/s.

- The Reduced Frequency is: $\frac{n_o H}{V_H} = \frac{0.2x100}{37.5} = 0.53$ and with the width to height ratio, W/H = 0.24, from Fig. B-3, the Size Reduction Factor, s = 0.18

- The Gust Energy Ratio is obtained from Figure B-4, for a Wave Number, $\frac{n_o}{V_H} = \frac{0.2}{37.5} = 0.0053$, then F = 0.28,

- The Coefficient of Variation, for a structural damping ratio, β = 0.01 from equation (5) is now:

$$\frac{\sigma}{u} = \sqrt{\frac{K}{C_e}\left(B + \frac{sF}{\beta}\right)} = \sqrt{\frac{0.08}{1.91}\left(0.78 + \frac{0.18x0.28}{0.01}\right)} = 0.49$$

- The Average Fluctuation Rate ν, is needed to evaluate the Peak Factor, g_p, and is determined from:

$$\nu = n_o\sqrt{\frac{sF}{sF + \beta B}} \qquad (6\text{-}16)$$

$$\nu = 0.2\sqrt{\frac{0.18x0.28}{0.18x0.28 + 0.01x0.78}} = 0.19 \text{ Hz}$$

- The Peak Factor, is found from Figure B-5 as $g_p = 3.8$ for a time duration of 1 hour (T = 3600 seconds), or from the familiar equation:

$$g_p = \sqrt{2\ln\nu T} + \frac{0.5772}{\sqrt{2\ln\nu T}}$$

$$= \sqrt{2\ln(0.19x3600)} + \frac{0.5772}{\sqrt{2\ln(0.19x3600)}}$$

$$= 3.77$$

- The Gust Effect Factor is then, from equation (6-14):

$$C_g = 1 + g_p\left(\frac{\sigma}{u}\right)$$

$$= 1 + 3.77x0.49$$

$$= 2.86$$

- The pressure coefficient at the <u>Windward Face</u> at roof level is $C_p = 0.8$ according to NBCC, so then has a design pressure of:

$$p = qC_eC_gC_p \qquad (6\text{-}10)$$

= 0.48 x 1.91 x 2.86 x 0.8 = 2.10 kPa

- The pressure coefficient on the <u>Leeward Face</u> at roof level is $C_p = -0.5$ according to NBCC, so then has a design pressure of:

p = 0.48 x 1.91 x 2.86 x (-0.5) = -1.31 kPa

- The maximum acceleration is determined from equation (6-17)

$$\hat{a} = 4\pi^2 n_o^2 \frac{g_p}{C_g}\sqrt{\frac{KsF}{C_e\beta}}\cdot\hat{U} \qquad (6\text{-}17)$$

$$\hat{a} = 4\pi^2 0.2^2 \frac{3.77}{2.86}\sqrt{\frac{0.08x0.18x0.28}{1.91x0.01}}\cdot 0.02$$

$$= 0.019\ \text{m/s}^2$$

Or, about 2 milli-g's

b) Evaluate response under NBCC Exposure "C" (City Exposure)

- The Gust Effect Factor is then, from equation (6-14):
  $C_g = 3.71$

- The pressure at the Windward Face at roof level is
  p = 0.48 x 0.95 x 3.71 x 0.8 = 1.35 kPa

- The pressure on the Leeward Face at roof level is:
  p = -0.85 kPa
- The maximum acceleration is determined from equation (6-17)
  $\hat{a} = 0.021\text{m/s}^2$

  Or, again, about 2 milli-g's

  Notice that :
  - In Exposure A, (Open Terrain) we have higher mean wind speeds at the top of the building, but lower gust factors, which results in higher mean pressures, but lower gust pressures
  - In Exposure C, (City Terrain) we have lower mean wind speeds at the top of the building, but higher gust factors, which results in lower mean pressures, but higher gust pressures