6.3 Distributed Random Loading Due to Wind

Multiple Discrete Loads

Consider a randomly varying quantity *z(t)* made up of two components, *x(t)* and *y(t)*,

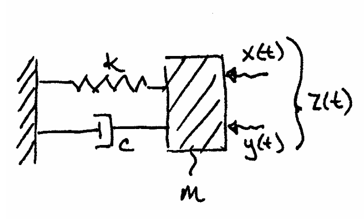

Fig. 6.4 Multiple loads

$$z(t)=x(t)+y(t)$$

The autocorrelation function of the combined input, $R_z(\tau)$, is:

$$\begin{aligned} R_z(\tau) &= < z(t)z(t+\tau) > \\ &= < [x(t)+y(t)][x(t+\tau)+y(t+\tau)] > \\ &= < x(t)x(t+\tau)+x(t)y(t+\tau)+y(t)x(t+\tau)+y(t)y(t+\tau) > \end{aligned}$$

Thus, $R_z(\tau)=R_{xx}(\tau)+R_{xy}(\tau)+R_{yx}(\tau)+R_{yy}(\tau)$

In which $R_{xx}(\tau)$ and $R_{yy}(\tau)$ are the autocorrelation functions of signals *x* and *y* and:

$$\begin{aligned} R_{xy}(\tau) &= < x(t)y(t+\tau) > \\ R_{yx}(\tau) &= < y(t)x(t+\tau) > \end{aligned}$$

the *cross-correlation or cross-covariance* functions.

The correlation function of a combined signal is determined by the autocorrelations of the components and by their cross-correlations. The cross-correlation will be zero only if the components *x(t)* and *y(t)* are completely uncorrelated or unrelated.

When a signal consists of more components, so that:

$$z(t)=x_1(t)+x_2(t)+x_3(t)+...+x_n(t)$$

The autocorrelation $R_z(\tau)$ is:

$$\begin{aligned} R_z(\tau) = {} & R_{11}(\tau)+R_{12}(\tau)+R_{13}(\tau)+...+R_{1n}(\tau)+ \\ & R_{21}(\tau)+R_{22}(\tau)+R_{23}(\tau)+...+R_{2n}(\tau)+ \\ & R_{31}(\tau)+R_{32}(\tau)+R_{13}(\tau)+...+R_{1n}(\tau) \end{aligned}$$

which can be written as a double sum: $R_z(\tau)=\sum_r \sum_s R_{rs}(\tau)$

or as a correlation matrix: $[R_z] = [R_{ij}]$

$$[R_z] = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix}$$

Properties of cross-correlations of stationary processes:

$$R_{xy}(-\tau) = < x(t)y(t-\tau) > = < y(t)x(t+\tau) > = R_{yx}(\tau)$$
$$R_{yx}(-\tau) = < y(t)x(t-\tau) > = < x(t)y(t+\tau) > = R_{xy}(\tau)$$
$$R_{xx}(-\tau) = < x(t)x(t-\tau) > = < x(t)x(t+\tau) > = R_{xx}(\tau)$$

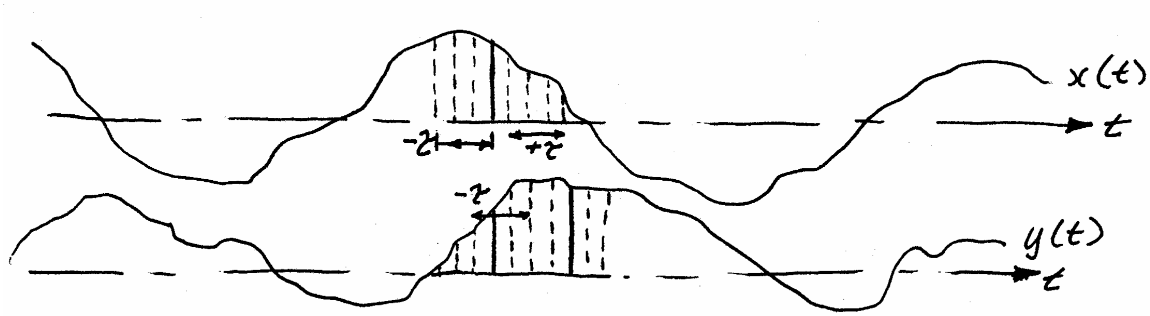

Fig. 6.5 Time history of two signals, *x(t)* and *y(t)*

There is no relation between $R_{xy}(\tau)$ and $R_{xy}(-\tau)$; $R_{xy}(\tau)$ and $R_{yx}(\tau)$ are in general, unrelated.

$R_{xy}(\tau)$ and $R_{yx}(\tau)$ do not necessarily have their maximum values at τ =0.

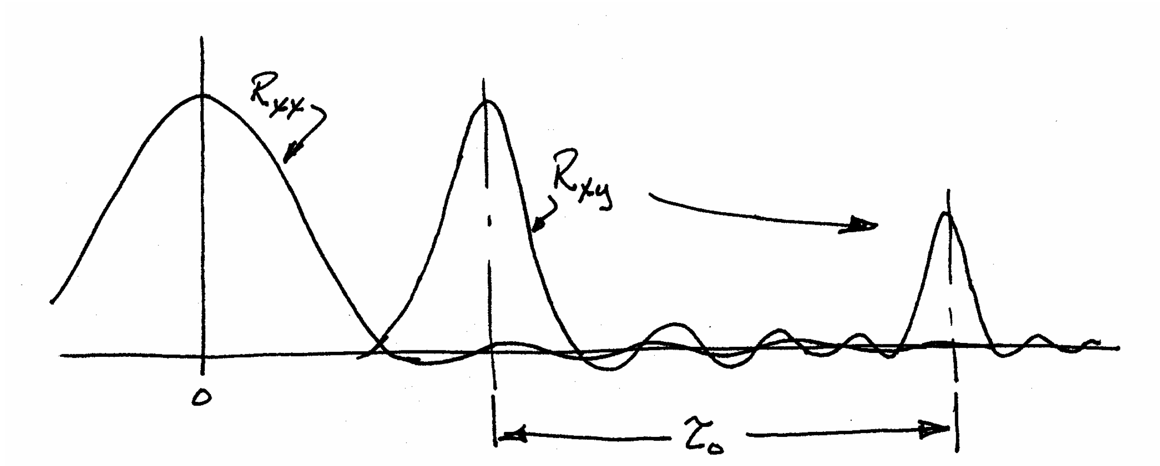

Fig. 6.6 Auto and Cross Correlation Functions

The distance between two peaks determines the average time delay between two processes, e.g. wind speed and wind pressure.

Covariance

When τ =0, $R_{xy}(0) = <x(t)y(t)> = \sigma_{xy}^2$, which is covariance. This is the measure of the extent to which two random variables, *x,y* are correlated. In they are completely independent, $\sigma_{xy}^2 = 0$ and also $R_{xy} = R_{yx} = 0$.

Correlation Coefficient

A dimensionless form of covariance

$$R = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} \leq 1.0 \qquad ,$$

or, when $\sigma_x = \sigma_y = \sigma$, $R = \sigma_{xy}^2 / \sigma^2$

When R = 1, the process is fully correlated and if R = 0 , the process is completely uncorrelated.

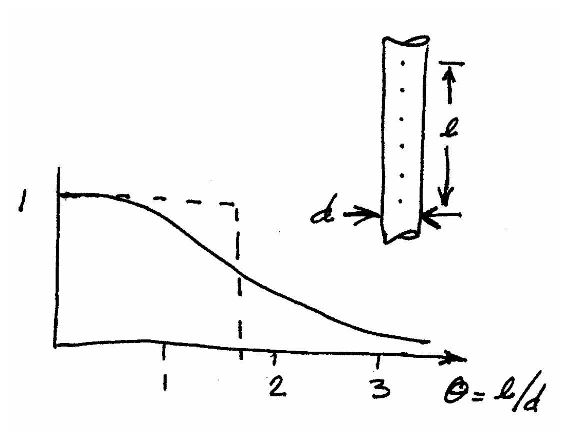

Fig. 6.7 Correlation Coefficient

The correlation length, L, can measure the span-wise correlation and is defined as:

$$\frac{L}{d} = \int_0^\infty R(\theta) d\theta$$ in numbers of diameters, d.

Further useful relationships are:

$$\left|R_{xy}(\tau)\right|^2 \le R_x(0)R_y(0)$$
$$R_{xy}(\tau) \le \tfrac{1}{2}\left[R_x(0) + R_y(0)\right]$$

The *correlation function coefficient*, (normalized cross-covariance or cross-correlation function), is, with zero means,

$$\rho_{xy}(\tau) = \frac{R_{xy}(\tau)}{\sqrt{R_x(0)R_y(0)}} = \frac{R_{xy}(\tau)}{\sigma_x \sigma_y}$$

The area beneath the curve is a measure of the time over which the processes are correlated,

$T = \int_0^\infty \rho_{xy}(\tau)d\tau$ and is called the *time scale*. In the case of wind , T is the time scale of turbulence, determined from ρ_{xx}, ρ_{yy} respectively.

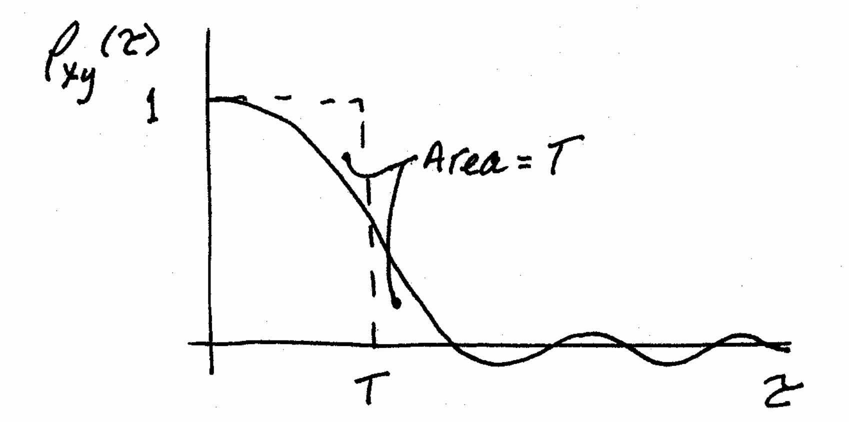

Fig. 6.8 Effective time scale

A more complete picture of the process with respect to its frequency content can be obtained from "cross-correlations" and "cross-spectral densities". In most cases, we can describe the process in a simplified way through the spectral density and the correlation coefficient, which can be replaced with the correlation length.

i.e. Local Spectra x Correlation Length