6.4 The Along-Wind Response of Line-Like Structures (after Davenport)

We have examined the response of small, point-like structures to wind. These structures are those that are sufficiently small, so that the bulk of the energy of the turbulence of gusting is at wavelengths much greater than the typical dimension of the structure. A line-like structure, on the other hand is one where that has significant dimension transverse to the wind, however remains small in the sense that the smallest wavelengths likely to be of significance must be large compared to the breadth of the structure. The approach is applicable to slender towers, transmission lines and some long span bridges.

The load per unit length on a slender structure of width *B* and length *L* may be expressed as:

$$\begin{aligned} F(t) &= \overline{F} + F'(t) \\ &= \tfrac{1}{2}\rho\overline{U}^2 C_D B + \rho\overline{U} C_D B u(t) \end{aligned} \qquad (6\text{-}18)$$

The response of the structure to the fluctuating component of the force, $\rho\overline{U}C_D Bu(t)$, may be computed mode-by-mode using modal analysis. If the mode shape for the *i*th mode is $\mu_i(z)$, then the modal force, $F_i(t)$, is as follows;

$$F_i(t) = \int_0^L C_D B\rho\overline{U} u(z,t)\mu_i(z)dz \qquad (6\text{-}19)$$

The spectral density function $S_F(f)$ then becomes;

$$S_F(f) = \left(C_D B\rho\overline{U}\right)^2 S_v(f)\int_0^L\int_0^L R(z,z',f)\mu_i(z)\mu_i(z')dzdz' \qquad (6\text{-}20)$$

in which;

$S_v(f)$	is the spectrum of turbulence
$R(z,z',f)$	is the narrow band correlation function or normalized co-spectrum of turbulence

The mean square value of the modal coefficient $\overline{a_i^2}$ is then obtained from the relationship;

$$\overline{a_i^2} = \frac{\displaystyle\int_0^\infty S_F(f)\cdot\frac{1}{(1-(f/f_i)^2)^2 + 4(f/f_i)^2\zeta^2}df}{(2\pi f_i)^4 M_i^2} \qquad (6\text{-}21)$$

in which;

ζ is the damping as a fraction of critical
f_i is the natural frequency of the *i*th mode
M_i is the Generalized Mass of the *i*th mode

$$M_i = \int_0^L m(z)\mu_i(z)dz$$

The displacement of the structure can now be computed by the superposition of modes as follows;

$$y(z,t) = \sum_i a_i(t)\mu_i(z) \qquad (6\text{-}22)$$

or, in terms of the mean-square response;

$$\overline{y^2(z)} = \sum_i \overline{a_i^2}\mu_i^2(z) \qquad (6\text{-}23)$$

The mean-square bending moment at position, *z* is given by:

$$\overline{BM^2(z)} = \sum_i a_i^2 \cdot BM_i^2(z) \qquad (6\text{-}24)$$

in which $BM_i(z)$ is the bending moment at position *z,* when the deformed shape is $\mu_i(z)$. If the spanwise distribution of force varies due to spanwise changes of diameter, the drag coefficient, the mean wind velocity and / or velocity spectrum, Equation 6-20 can be written:

$$S_F(f) = \left(C_D B\rho\overline{U}\right)^2 S_v(f) \cdot \left|J(f)\right|^2 \qquad (6\text{-}25)$$

where

$$\left|J(f)\right|^2 = \int_0^L\int_0^L R(\xi,\xi';f)\,\gamma(\xi)\gamma(\xi')\,\mu_i(\xi)\mu_i(\xi')\,d\xi d\xi' \qquad (6\text{-}26)$$

ξ is a normalized coordinate, $\xi = z/L$ and $\gamma(\xi)$ is a function incorporating the span -wise changes in diameter, the drag coefficient, the mean wind velocity and velocity spectrum. The function $\left|J(f)\right|^2$ is called the “Joint Acceptance Function” and defines the sensitivity between the turbulence and the structural modes of vibration, which is critical in defining the response of the structure.

The Joint Acceptance Function

Equations (6-25) and (6-21) define the link between the gust fluctuations (which are described by the velocity spectrum $S_v(f)$) and the modal force fluctuations and displacements (provided by the Joint Acceptance Function). This function depends on the mode shape and the velocity field, which can vary from structure to structure as indicated in Figure 6.9.

The lamp standard in (a) can oscillate in both a fore and aft as well as in a twisting mode, however, the wind excitation is concentrated at essentially one elevation, namely that of the lamps. In the vertical structures in (b-d), the wind speed varies with height; the fundamental mode of the building in (b) is nearly a straight line. The diameter of the chimney in (c) varies with height and has a fundamental bending type of mode shape. The guyed mast in (d) will have mode shapes consisting of several half-waves. The horizontal bridge structures in (e) and (f) will likely have near constant mean wind velocity along the span. The suspension bridge will have near-sinusoidal mode shapes, while the cantilever bridge will have a twisting mode about the axis of the pier as well as one in the fore and aft direction.

For a slender structure it is common practice to assume that the correlation of the forces is the same as that for the transverse correlation of the longitudinal component of the wind. That is;

$$R(z,z',f) \approx \exp\left[-c\frac{f|z-z'|}{\bar{V}}\right] \tag{6-27}$$

or

$$R(\xi,\xi',f) = \exp\left[-\phi\frac{f|z-z'|}{L}\right] \tag{6-28}$$

where $\phi = \dfrac{cfL}{\bar{V}}$, a dimensionless frequency, and c is the correlation coefficient, commonly found to be between 8-10.

The JAF can now be written:

$$|J(f)|^2 = \int_0^1\int_0^1 e^{-\phi|\xi-\xi'|}\ \gamma(\xi)\gamma(\xi')\ \mu_i(\xi)\mu_i(\xi')\,d\xi d\xi' \tag{6-29}$$

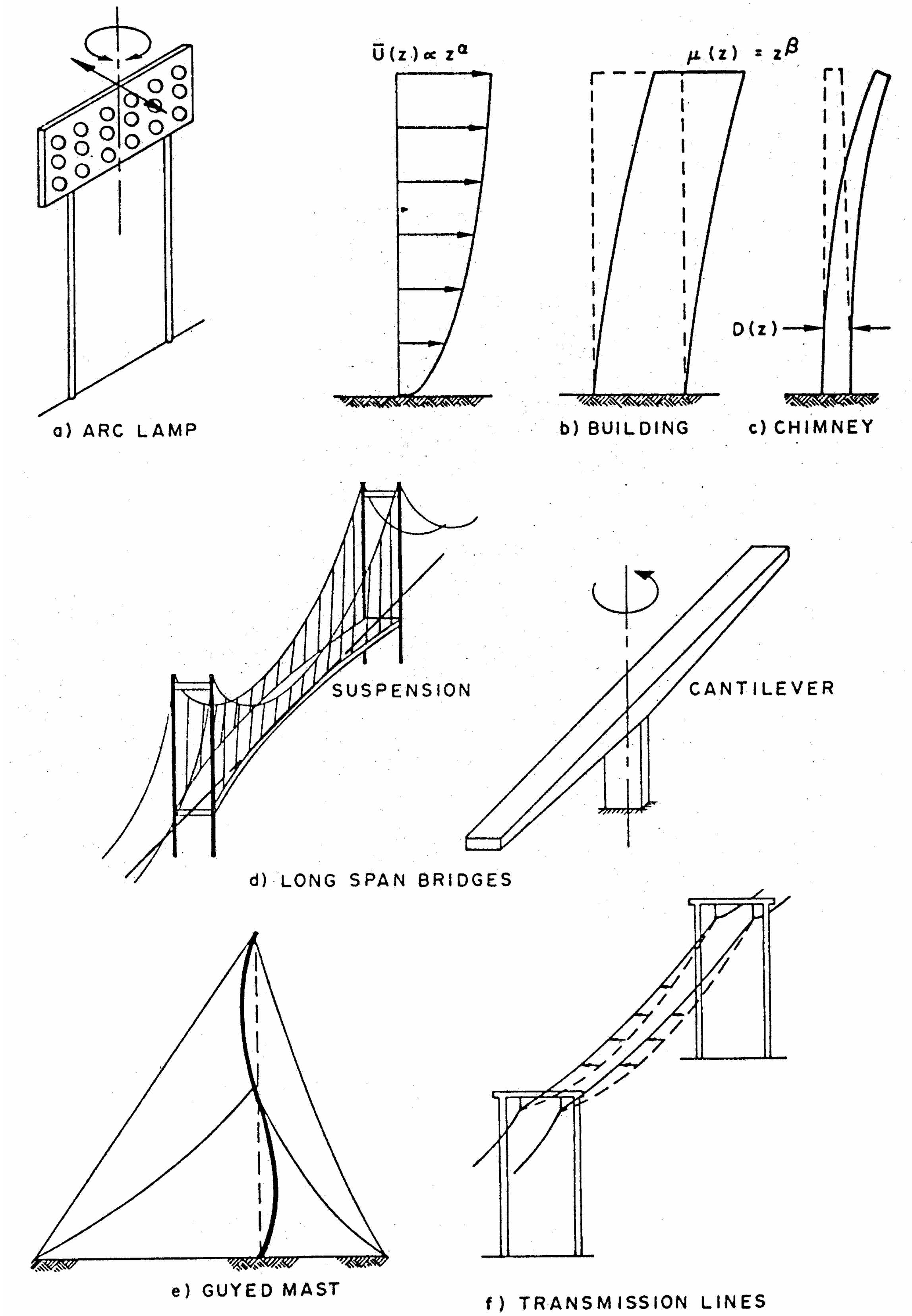


Fig. 6.9 Oscillations of structures in turbulent wind (after Davenport)

Table 6.1 indicates the form of the Joint Acceptance Function for a number of common mode shapes, assuming that $\gamma(\xi) = 1.0$, or there is no variation in the force per unit length. The functions are also plotted in Figures 6.10 and 6.11.

Several features of these functions are worth noting:

a) The mode shapes composed of deflections of the same sign decrease monotonically for higher frequencies and can be well represented by approximations of the form

$$|J(f)|^2 = \frac{1}{A + B\phi}$$

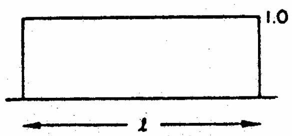
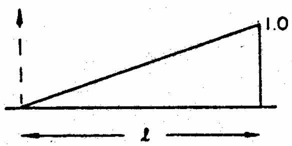
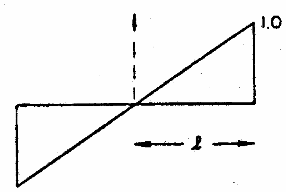
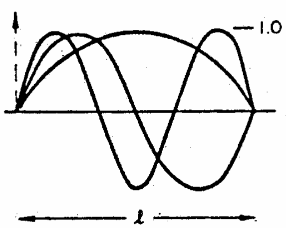
b) The antisymmetric shapes have no response for small values of ϕ (i.e. for large gust wavelengths). The gust then envelops the entire structure and the antisymmetric mode shape neutralizes its effect. These mode shapes have a peaked JAF, indicating that there will be a maximum response at a specific wavelength.

c) Mode shapes having deflections of opposite sign, but not necessarily antisymmetric have a finite lower asymptote as $\phi \to 0$, reach a peak for intermediate values of ϕ and fall off again as $1/\phi$ at larger values of ϕ. It is important to note that for sinusoidal mode shapes, the peak of the JAF occurs at $\phi \approx 3n - 2$, where n is the number of half waves.

The qualification of "line-like" in the use of the Joint Acceptance Function implies that all significant wavelengths influencing the structural response are greater than the diameter of the structure (i.e. $fD/\overline{V} << 1$). The flow is then "quasi-steady" For structures such as transmission lines this assumptions is accurate, however as the structure becomes larger and the slenderness decreases, (from chimneys to bridge decks to buildings, for example) this assumption will exaggerate the response at higher frequencies and provide overly conservative estimates of the wind loads. Introducing the Aerodynamic Admittance Function, $|A(f)|^2$, allows for the reduction in force for the less slender structures.

In most cases in the estimation of the response of a structure to wind it is rarely necessary to extend the summation of equation (6-20) beyond more than a few modes, and in many cases the first term (especially in the case of buildings of moderate height) is quite adequate. This is the basis for the wind load provisions in the National Building Code of Canada. Long span bridges may require several modes to adequately represent the loading with sufficient detail in order to adequately predict the force effects. To date, satisfactory wind codes for long span bridges have not been developed and wind tunnel testing must be undertaken to define loads for design.

Table 6.1 Joint Acceptance Functions for line-like structures

MODE SHAPE AND REFERENCE LENGTH		JOINT ACCEPTANCE FUNCTION $\mid J(\phi)\mid^2$	ASYMPTOTES $\phi \to 0$	ASYMPTOTES $\phi \to \infty$	APPROXIMATION	MEAN SQUARE AMPLITUDE $\overline{\mu^2(\zeta)}$	MEAN AMPLITUDE $\overline{\mu(\zeta)}$
	$(\zeta = x/l)$	$(\phi = C\,fl/\overline{V})$	$\lvert J\rvert^2 \to$	$\lvert J\rvert^2 \to$	$\lvert J(\phi)\rvert^2 =$		
	$\mu(\zeta)=1.0$	$\frac{2}{\phi^2}[\phi-1+e^{-\phi}]$	1.0	$\frac{2}{\phi}$	$\frac{1}{1+.5\phi}$	1.0	1.0
	$\mu(\zeta)=\zeta$	$\frac{2}{\phi^4}[\frac{\phi^3}{3}-\frac{\phi^2}{2}+1-e^{-\phi}(1+\phi)]$	.25	$\frac{2}{3\phi}$	$\frac{.25}{1+.375\phi}$	$\frac{1}{3}$	$\frac{1}{2}$
	$\mu(\zeta)=\zeta$	$\frac{2}{\phi^4}[\frac{2\phi^3}{3}-\phi^2+1-e^{-2\phi}(1+\phi)^2]$	$\frac{8}{15}\phi$	$\frac{4}{3\phi}$		$\frac{1}{3}$	0
	$\mu(\zeta)=\sin n\pi\zeta$ n = number of $\frac{1}{2}$ waves	$\frac{\phi}{\phi^2+(n\pi)^2}+2[\frac{n\pi}{\phi^2+(n\pi)^2}]^2(1+e^{-\phi}\cos n\pi)$	n even $\frac{4}{n^2\pi^2}$ n odd $\frac{3\phi}{(n\pi)^2}$	$\frac{1}{\phi}$	$n = 1$ $= \frac{.404}{1+.404\phi}$	$\frac{1}{2}$	$\frac{1-\cos n\pi}{n\pi}$

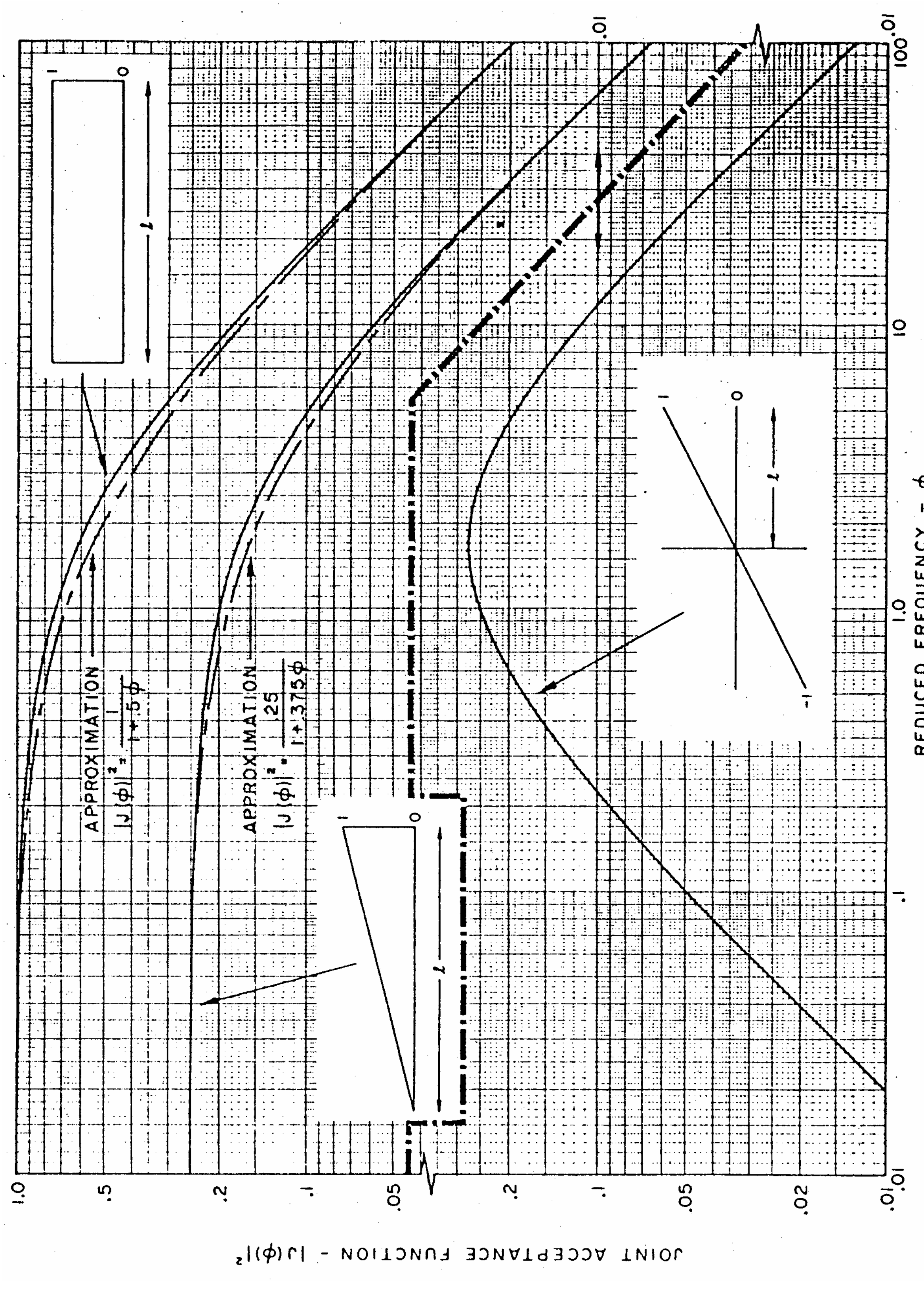


Fig. 6.10 Joint Acceptance Functions for line-like mode shapes – linear modes (after Davenport)

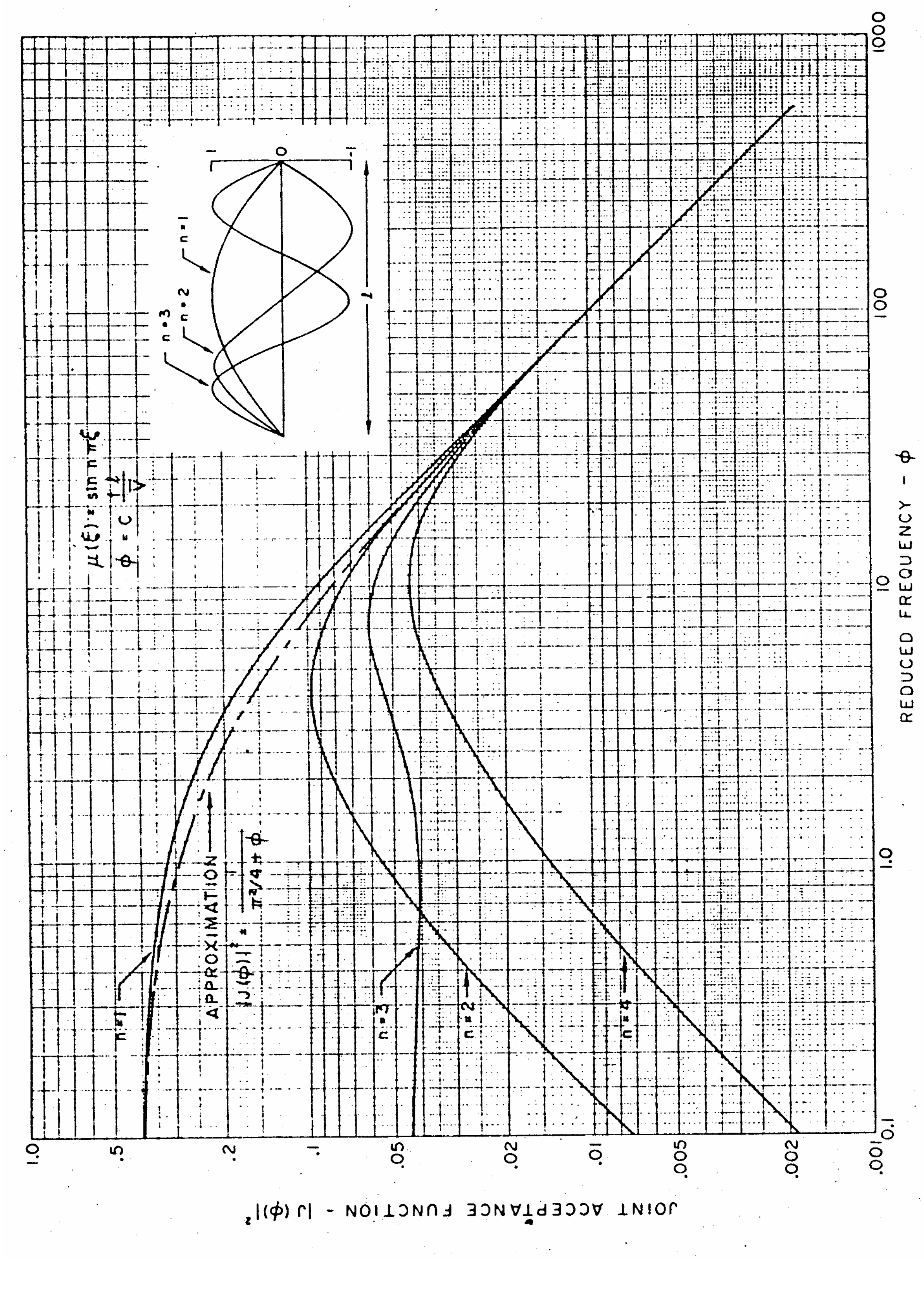


Fig. 6.11 Joint Acceptance Functions for line-like mode shapes – Isinusoidal modes (after Davenport)