7.2 Fatigue Analysis

The most common method used to examine the effect of cyclic stress on a structural component is by examining the "S-N Curve". This is a plot of the stress level required to cause a fatigue failure of a component for a given number of repetitive cycles.

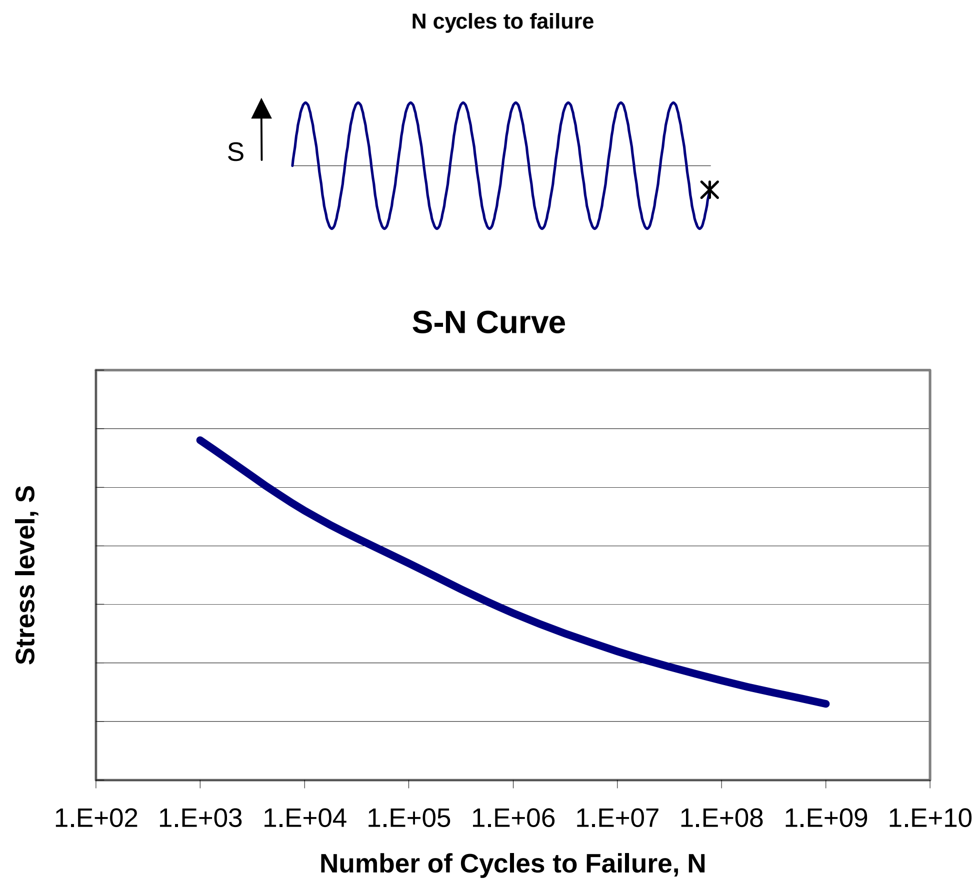


Figure 7.9: Typical S-N Curve

However, most loading of structures does not consist of a single, cyclic stress level, but a number of stress conditions, all contributing to the fatigue damage to the structure. An empirical method was developed to evaluate the cumulative damage done to a structural component due to different cyclic stress levels, each with their own contribution to fatigue. This is the so-called "Miner's Damage Law".

Miner's Damage Law

Miner postulated that fatigue failure would occur when:

$$\sum \frac{n(y)}{N(y)} = 1$$

where:

$N(y)$ is the number of cycles required to produce failure at amplitude, y
$n(y)$ is the number of cycles realized at amplitude, y

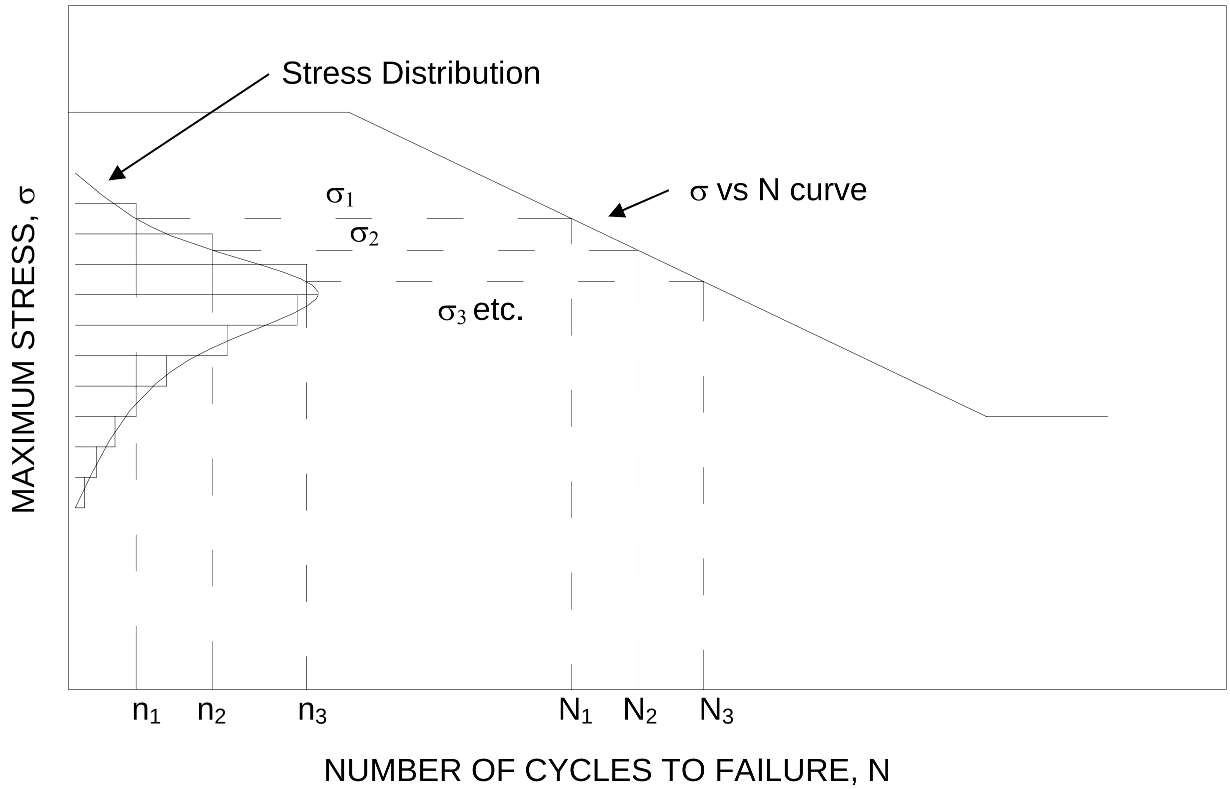


Figure 7.10 Miner's Damage Law

If we have a normally distributed process, the number of cycles in a given time *T* becomes:

$$n(y) = \nu T \frac{(y_o - \bar{y})}{\sigma^2} e^{-(y_o - \bar{y})/2\sigma^2}$$

Hence: $\int \frac{n(y)}{N(y)} dy = 1$ is the damage criteria.

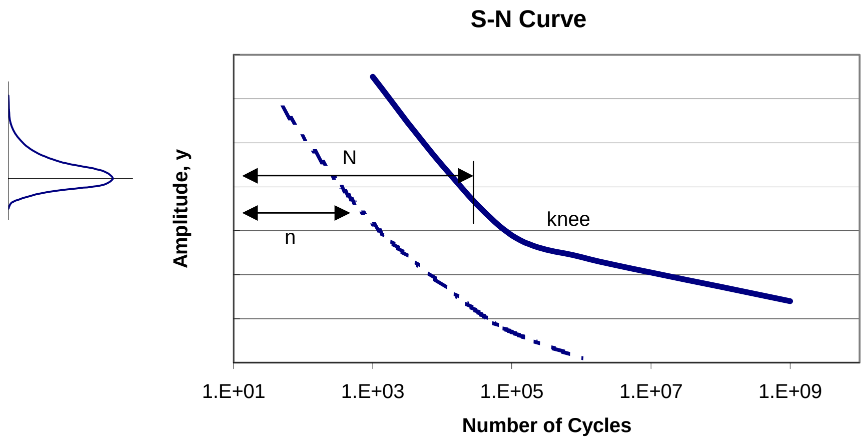
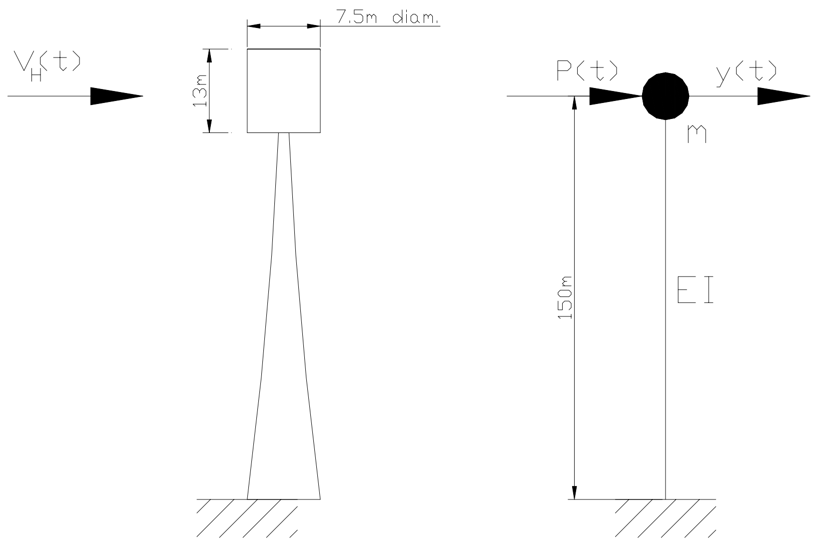


Figure 7.11 Damage Criteria

Problem 7.1

Check the possible fatigue damage at the base of the tower shown here due to resonant vortex shedding from the observation gondola.



Data:

Effective mass,	m = 68,000 kg
Young's Modulus,	$E = 2 \times 10^5$ Mpa
Height,	h = 152.4 m
Equivalent uniform inertia of the cross section, I_{eq} = 1.116 m^4	
Inertia at Base	$I_{base} = 3 \times I_{eq}$
Diameter at base,	d_{base} = 15.24 m
Damping Ratio,	D = 0.4%
Lift Coefficient,	C_L = 0.2
Strouhal Number,	S = 0.2

The lateral force due to vortex shedding at resonance is $P(t) = P_o \cos \omega_o t$, where $P_o = \frac{1}{2} \rho V_{crit}^2 C_L \ell d$ and ℓ and d are the height and diameter of the observation gondola, respectively. The critical velocity for vortex shedding is $V_{crit} = f_o d / S$

Assume that:

a) Resonant Vortex Shedding occurs when $0.85 \leq V / V_{crit} \leq 1.15$, where V is the hourly mean wind speed.

b) The hourly mean wind speed has a "Weibull" probability distribution of the form:

$$P(>V) = e^{-\left(V/C\right)^K} \quad \text{where: } C = 8 \text{ m/s and } K = 1.8$$

The natural frequency of the tower is equal to:

$$
\begin{aligned}
f_o &= \frac{1}{2\pi}\sqrt{k/m} \\
&= \frac{1}{2\pi}\sqrt{\frac{3EI/h^3}{m}} \\
&= 0.1592\sqrt{\frac{3\cdot 2x10^{11}\cdot 1.116}{152.4^3\cdot 68000}} \\
&= 0.265\ \text{Hz}
\end{aligned}
$$

The critical wind speed for vortex shedding is equal to:

$$
\begin{aligned}
V_{crit} &= f_o d/S \\
&= 0.265\cdot 7.5/0.2 \\
&= 9.94\,\text{m/s}
\end{aligned}
$$

The stress at the base due to the vortex shedding forces at resonance with the natural frequency of the tower is determined from:

$$P(t) = P_o \cos\omega_o t$$

$$
\begin{aligned}
P_o &= \tfrac{1}{2}\rho V_{crit}^2 C_L \ell d \\
&= \tfrac{1}{2}\cdot 1.23\cdot 9.94^2\cdot 0.2\cdot 7.5\cdot 13.0 \\
&= 1.185\ \text{kN}
\end{aligned}
$$

at resonance, the maximum dynamic force is $P_o \cdot \dfrac{1}{2D}$

$$
\begin{aligned}
\sigma_{base} &= \frac{P_o\cdot \frac{1}{2D}\cdot h\cdot d_{base}/2}{I_{base}} \\
&= \frac{1185\cdot 152.4\cdot 15.24/2}{3\cdot 1.116} \\
&= 51.1\text{x}10^6\,\text{N/m}^2 = 51.1\text{MPa}
\end{aligned}
$$

The dynamic stress range is:

$$F_{sr} = \sigma_{\max} - \sigma_{\min} = 51.1-(-51.1) = 102.2\,\text{MPa}$$

From Figure K1 or Table 6A of CAN3-S16.1-M8, $F_{sr} <$ Endurance Limit or Fatigue strength for Categories A and B, therefore no further action is needed. However, $F_{sr} >$ Endurance Limit for Categories C to F, therefore it would be necessary to estimate the number of cycles that the tower would likely be subjected to over its lifetime.

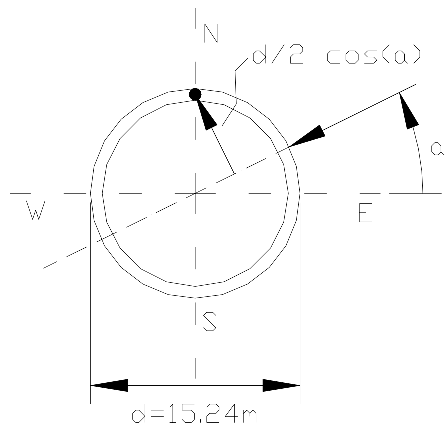
To address this problem fully, one should know the likelihood of obtaining this 9.94 m/s wind speed for different directions. Then one could assess the fatigue at a specific location around the circumference of the tower base, since:

$$(F_{sr})_A = \left(\frac{M_{base} \cdot {}^{d}\!/_{2} |\cos\alpha|}{I_{base}} \right)$$

$$= 102.2 \cdot |\cos\alpha| \text{ MPa}$$

$$(F_{sr})_A = 102.2 \text{ MPa when } \alpha = 0$$

$$= 0 \text{ when } \alpha = 90$$



A)

The most conservative assumption that can be made is to assume that the wind blows from the same direction all the time, so $(F_{sr})_A$ is always 102.2 Mpa, whenever the wind speed is in the critical range.

The number of cycles in the lifetime is equal to $n = (0.265 \text{ x } 60 \text{ x } 60) \text{ x M}$ or 0.265 Hz x 60 seconds x 60 minutes x the number of hours during the lifetime*, L* when *V* is within +/- 15% of V_{cr} or 8.4 m/s $\leq V_{cr} \leq 11.4$ m/s.

Assuming a lifetime of 50 years (from NBCC, for example), then

$$M = 365 \text{ x } 24 \text{ x } 50 \text{ x } (\text{P}(\text{V} > 8.4) - \text{P}(\text{V} > 11.4))$$

$$= 438{,}000 \text{ x } (e^{-(8.4/8)^{1.8}} - e^{-(11.4/8)^{1.8}})$$

$$= 438{,}000 \text{ x } 0.1848$$

$$= 80{,}946 \text{ hours per 50 years}$$

$$\text{n} = 0.265 \text{ x } 60 \text{ x } 60 \text{ x } 80{,}946$$

$$= 7.72 \text{ x } 10^7 \text{ cycles per 50 years}$$

From Figure K1, it can be seen that there would be fatigue damage unless the connection used was designed to fall into Allowable Stress Category B.

B) Assuming that the wind is equally likely to come from any direction,

α	$(F_{sr})_A = 102.2 \text{ x } \cos\alpha$	$n_1 = n/7$	Cycle Life "N" Category C, Fig K1
0°	102.2	1.1×10^7	1.2×10^6
15°	98.7	1.1×10^7	1.6×10^6
30°	88.5	1.1×10^7	2.2×10^6
45°	72.3	1.1×10^7	3.8×10^6
60°	51.1	1.1×10^7	∞
75°	26.5	1.1×10^7	∞
90°	0	1.1×10^7	∞

F_{sr} far below endurance limit

Miners Damage Law assumes that:

$$\sum_j \frac{n_j}{N_j} = \text{Cumulative Damage} \qquad j \text{ is the stress level}$$

There would be fatigue damage over a broad range of wind directions, $0° < \alpha < 45°$, should the detailing of the connection fall into Category C.