7.3 The Estimation of Fatigue Life

The analysis of cracking in steel bridges can be accomplished through using a linear-elastic fracture mechanics. With this method, the stresses very close to the crack front, which cause crack extension, are treated as proportional to the stress intensity factor, *K*. The size, shape and orientation of the crack play a major role in determining the applicable *K* value.

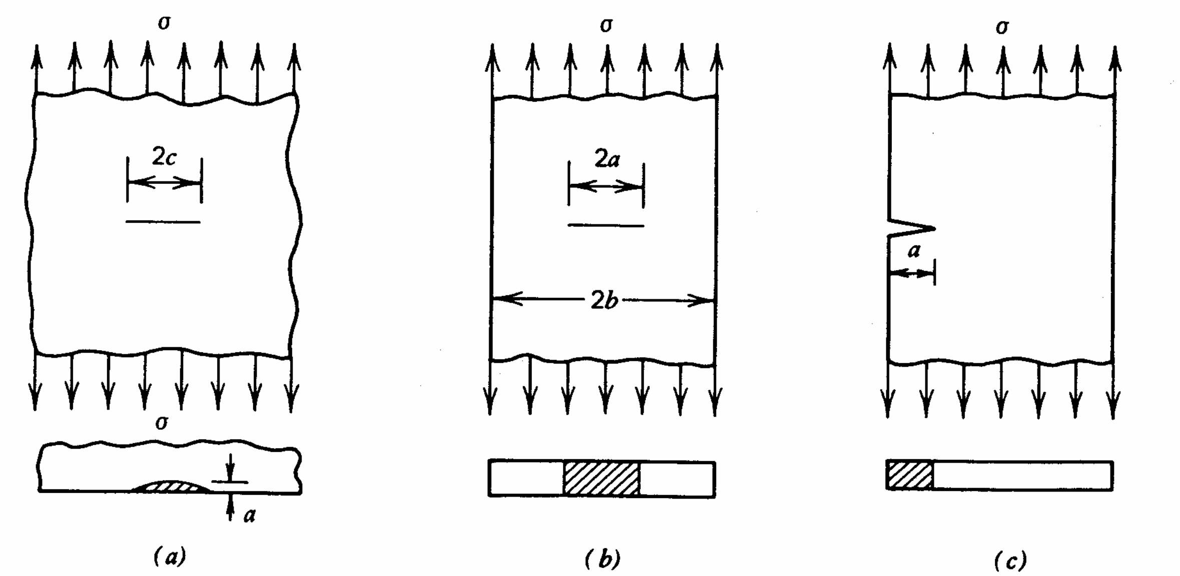

Figure 7.12: Idealized crack conditions. *(a)*, Surface crack; *(b)*, through crack; and *(c)*, edge crack.

The stress intensity factor, *K*, for a surface crack depth, *a*, shown in Figure 7.12 for three types of cracks can be related as follows:

$$K = F_e \cdot F_s \cdot F_w \cdot F_g \cdot \sigma\sqrt{\pi a}$$

These correction factors modify $\sigma\sqrt{\pi a}$, which is for the idealized case, to account for the following:

- F_s - the free surface
- F_w - the finite width
- F_g - the non-uniform stresses acting on the crack
- F_e - the crack shape

Numerous solutions for these correction factors can be found in the literature. A few of the more important types are:

$F_S = 1.211 - 0.186\sqrt{\dfrac{a}{c}}$ for a semicircular crack in a semi-infinite plate subjected to uniform stress

$F_W = \sqrt{\sec\dfrac{\pi a}{2b}}$ for a central crack in a plate of uniform width

$F_e = \dfrac{1}{E(k)}$ for a three-dimensional elliptical crack shape, where $E(k)$ is an elliptical integral:

$$E(k) = \int_0^{\pi/2}\left[1 - k^2\sin^2\phi\right]^{1/2} d\phi$$

and $k^2 = \dfrac{c^2 - a^2}{c^2}$, where a/c is the minor to major axis ratio

There are a number of approximate expressions for the gradient correction factor, depending on the structural detail – gussets, stiffeners, cover plates, toe welds etc. [1].

Fatigue Crack Growth Model

In order to assess the fatigue behaviour, the crack propagation relationship between the crack growth rate and the range of the stress intensity factor, $\Delta K = K_{\max} - K_{\min}$. Since the crack size at the upper and lower limits of the load cycle are the same, the stress intensity range is a function of the stress range. The “Paris Power Law” has the form:

$$\frac{da}{dN} = C\Delta K^n$$

Figure 7.13 is a schematic representation of the crack growth relationship. A crack growth exponent of $n = 3$ has been observed to be applicable to basic crack growth rate data for structural steels as well as test data on welded members. The corresponding average crack growth constant, C, was found to be between 1.2 to 2.18 x 10^{-13} if one uses mm for crack size and MPa $\sqrt{m}$ for ΔK. In summary an upper bound for the crack growth relationship with the number of cycles can be taken as:

$$\frac{da}{dN} = 2.18 \times 10^{-13} \Delta K^3$$

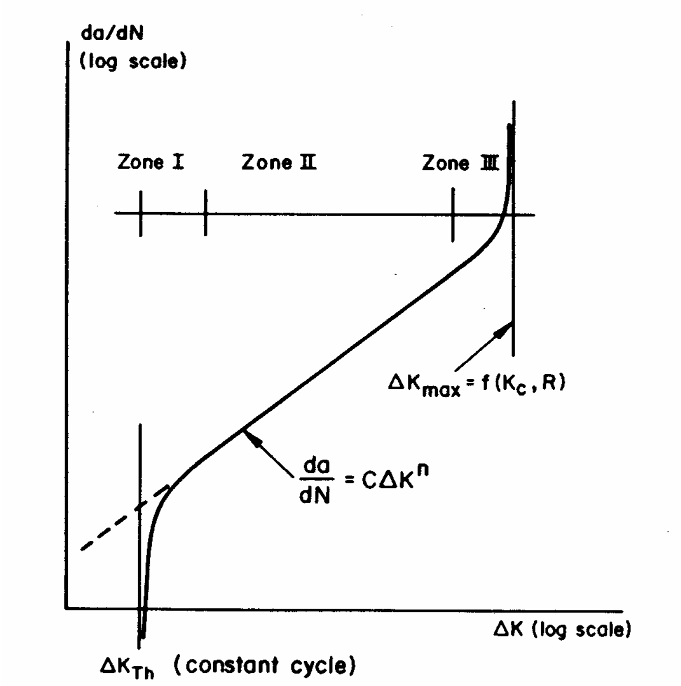

Figure 7.13 General crack propagation relationship.

Since randomly variable loading is usually involved in every case of fatigue crack propagation, an effective stress intensity range can be used, based on Miner's Law and a corresponding Miner's Effective Stress Range, S_{rMiner}. Hence, ΔK can now be defined as:

$$\Delta K_e = S_{rMiner} F_i \sqrt{\pi a}$$

where

$$S_{rMiner} = \left[\sum \alpha_i S_{ri}^3\right]^{1/3}$$

This is the same relationship between cumulative damage and stress range, namely, damage from variable loading is given by:

$$\sum \frac{n_i}{N_i} = 1$$

where the ratio, n_i / N_i is the incremental damage done from a block of stress range cycles S_{ri} that occurs n_i times. Failure is defined when the sum of the increments equals unity.

Fatigue Failure Estimates of the Aguasabon River Bridge

This bridge, located on the Trans-Canada Highway near Terrace Bay Ontario, is a three span continuous plate girder bridge, constructed in 1948. The bridge has a composite steel beam – reinforced concrete slab construction, with spans of 18.3-24.4-18.3 m as shown in Figure 7.14. The longitudinal structural members consist of four WF33 x 141 (84 cm deep) girders, haunched over the piers and abutments to an overall depth of 1.3m. The haunch was fabricated by cutting the bottom flange from the web fillet and welding a 16mm parabolic insert plate at the desired locations. The main girders were field spliced at two points in the center span 6.7 m from each pier as shown in Figure 7.15. The splice plates were located at the point of dead load inflection.

In 1963, cracks were discovered at the vertical butt weld detail in three of the six haunch inserts of the north interior main girder as shown in Figure 7.16. One of these cracks extended 1.12 m into the girder web along a diagonal line starting from this weld detail. These cracks were repaired by welding cover plates on each side of the web and welding an insert in the bottom flange.

In 1973, the structure was subjected to a detailed investigation and dynamic testing. As a result, four additional cracks were found. During the repairs, it was found that the cracks stemmed from initial weld imperfections or inclusions in the short transverse groove welds at the ends of the parabolic haunches. All cracks were discovered before the flanges fractured because the details were located near the point of inflection where the dead load stresses were small. Hence, large fatigue cracks were able to develop from repetitive live loads without brittle fracture of the remaining section.

During the 1973 tests, a test vehicle loaded to a gross weight of 405 kN traversed the bridge and dynamic strains were recorded. Based on the dynamic strain measurements and traffic conditions, a representative stress range histogram (S_r) was determined as shown in Figure 7.17. The stress range histogram was used to estimate the effective RMS stress range S_{rRMS} and S_{rMiner}. If all stress range conditions above 6 Mpa are considered, the S_{rRMS} = 13.2 MPa. The effective stress range using Miner's Law was equal to S_{rMiner} = 13.2 Mpa for all stress cycles above 3.1 Mpa.

A sample of vehicles crossing the bridge indicated that 6 to 15 stress cycles above 43.1 Mpa would result on each passage. The smoothed histogram shown in Figure 7.17, indicated 25 million stress cycles exceeded 3.1 Mpa between 1948 and 1963 and an additional 16 million cycles between 1963 and 1973.

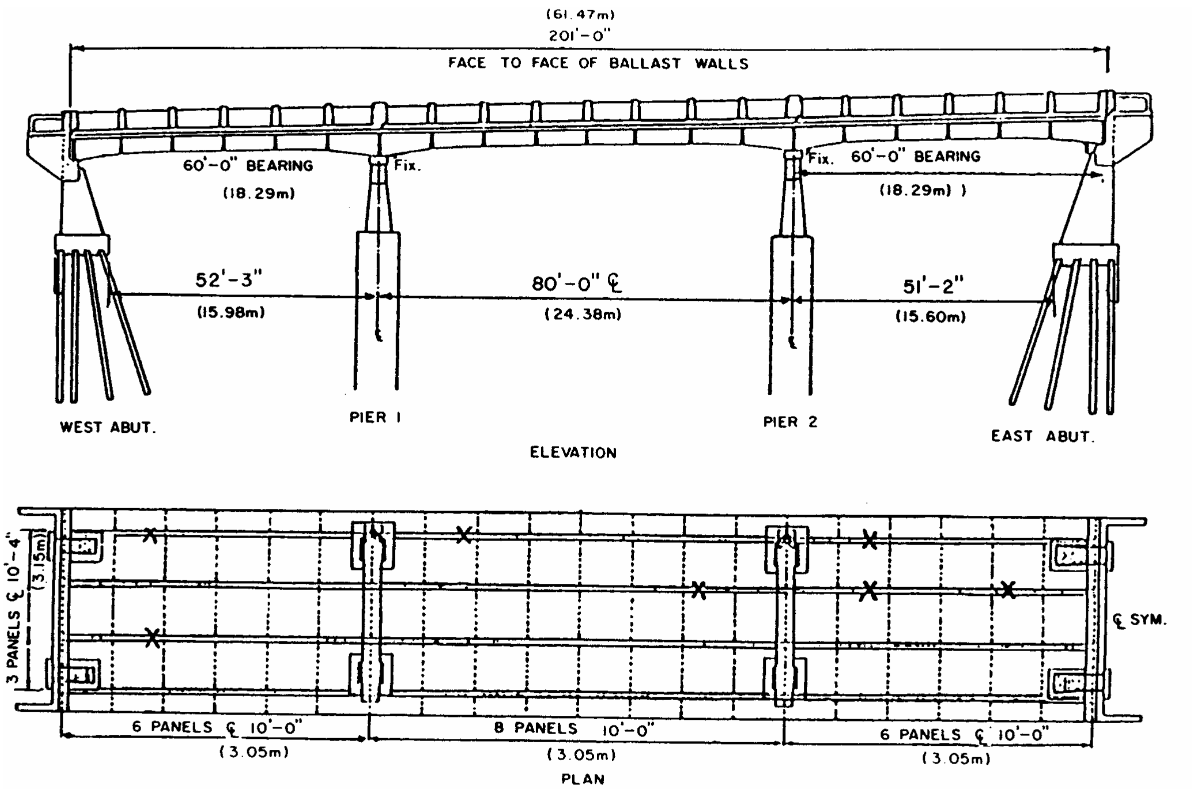

Figure 7.14 Plan and Elevation of Aguasabon Bridge (crack locations are indicated with the mark X).

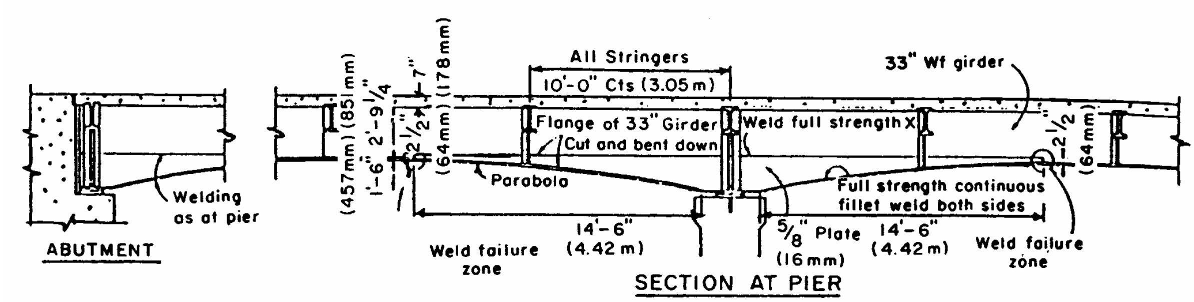

Figure 7.15 Main girder haunch detail, field shear splice and connector detail of Aguasabon Bridge.

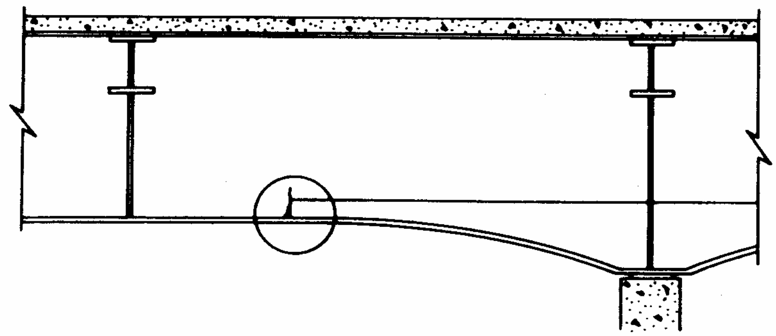

Figure 7.16 Location of weld crack in main girder (see Figure 5.3b for a photograph of circled area).

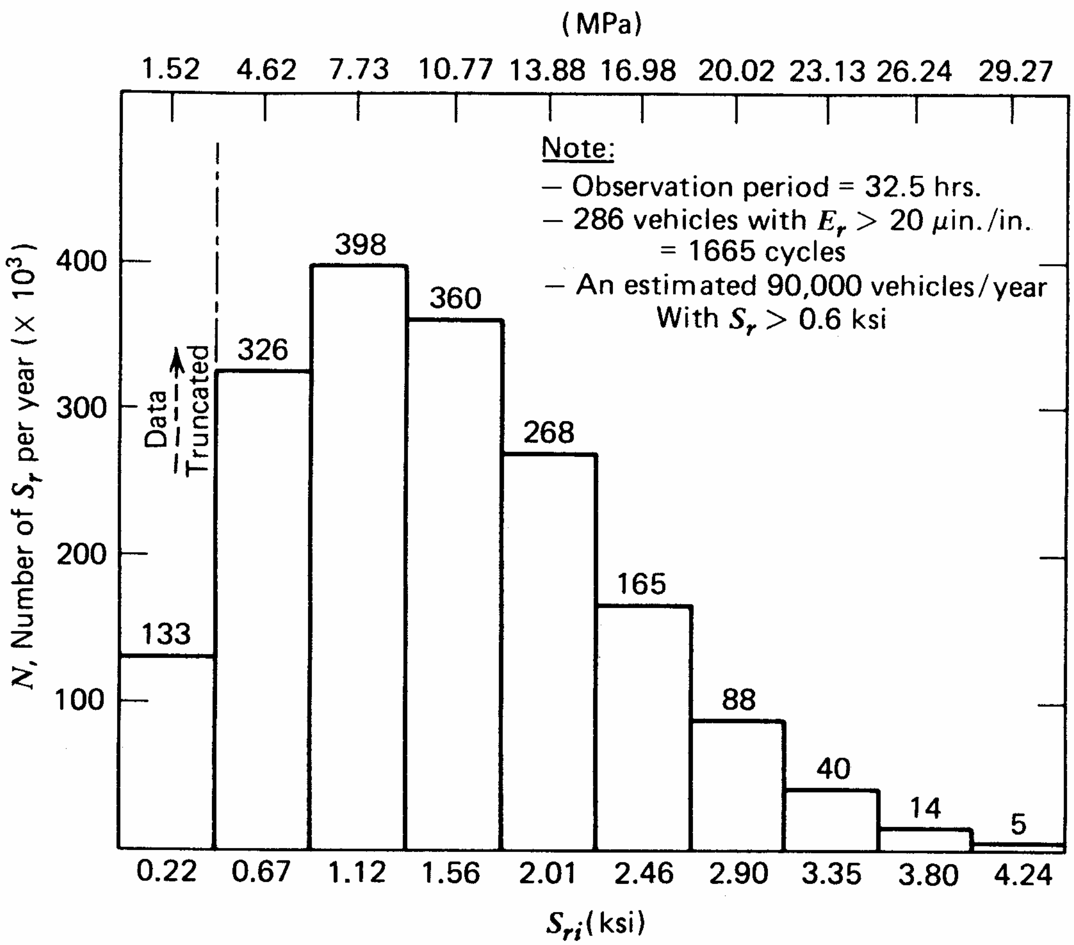

Figure 7.17 Stress range histogram (smoothed) of Aguasabon Bridge.

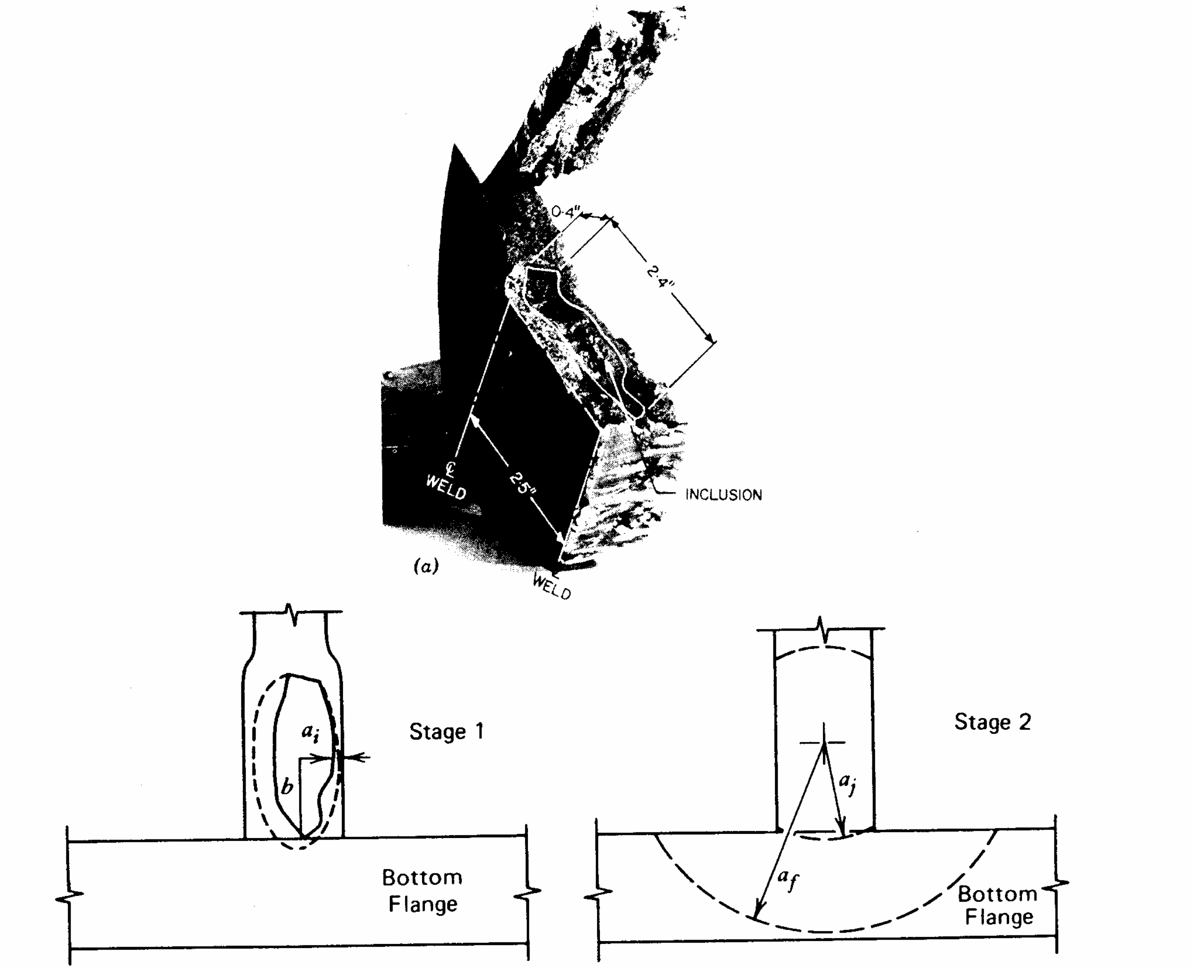

Figure 7.18 Assumed stages of crack growth (outer edge of weld inclusion is shown as dark line).

The fatigue propagation stages were modeled by approximating the weld defect by an ellipse as shown in Figure 7.18. The stress intensity factor for Stage 1 was modeled as:

$$K = F_e F_W \sigma \sqrt{\pi a}$$

where:

$$F_e = \frac{1}{E(k)} \quad \text{and} \quad E(k) = \int_0^{\pi/2} \left[1 - k^2 \sin^2 \phi\right]^{1/2} d\phi \quad \text{and} \quad k^2 = \frac{c^2 - a^2}{c^2}$$

$$F_W = \sqrt{\sec \frac{\pi a}{2b}}$$

The minor to major diameters of the ellipse $a/c = 0.11$, $b = 61$ mm

The "penny" shape of the crack assumed in the second stage of growth resulted in:

$$K = \frac{\pi}{2}\sqrt{\sec\frac{\pi a}{2b}}\ \sigma\sqrt{\pi a}$$

where a is the crack radius and $a_{initial} = c_{final}$ for Stage 1.

The crack growth propagation equation used to estimate the fatigue life for each stage of crack propagation was:

$$N = \int_{a-initial}^{61mm} \frac{da}{2.18\times10^{-13}\,\Delta K^3}$$

Several different crack sizes were assumed to assess the sensitivity of the initial inclusion to the fatigue life. The results are shown in Table 1.

For the crack shown in Figure 7.19, it was estimated that the effective stress range was 13.2 Mpa, occurring at a rate of 1.5 million cycles per year. At this rate approximately, two to three years would be required before the crack penetrated into the bottom flange during the first stage of crack growth. An additional 15 or 20 years would be required before the crack penetrated the full depth of the bottom flange. These estimates are in good agreement with that observed for the Aguasabon River Bridge.

Table 1 Estimated Fatigue Life: S_{rMiner} = 1.92 ksi (13.2 MPa)

Stage 1: Crack Growth through Weld			Stage 2: Crack Growth through Flange		
Initial Crack/Size a_i (in.)	Cycles of Stress	Years to Achieve	Initial Crack/Radius (in.)	Cycles of Stress	Years to Achieve
0.20 (5 mm)	14,050,000	9.4	1.40 (35.6 mm)	25,500,000	17
0.25 (6.4 mm)	2,664,000	1.8	1.42 (36 mm)	24,150,000	16
0.30 (7.6 mm)	40,500	0.3	1.45 (36.8 mm)	21,960,000	14.6
			1.50 (38 mm)	18,600,000	12.4

References

1. Fisher, J.W., "Fatigue and Fracture in Steel Bridges", John Wiley & Sons, New York, 1984.