Note on Random Loading Concepts Due to Wind

The general form of the response of structures due to random loads was found to be of the form:

$$\begin{aligned}\sigma_y^2 = \overline{y^2} &\cong \frac{1}{k^2}\int_o^{f_o} S_p(f)df + \frac{1}{k^2}\int_0^{\infty} S_p(f_o)|H(f)|^2 df \\ &\cong \frac{\sigma_p^2}{k^2}\left[1+\frac{\pi}{4D}\frac{f_o S_p(f_o)}{\sigma_p^2}\right]\end{aligned} \qquad (1)$$

The form of this relationship was observed in Fig. 8.12

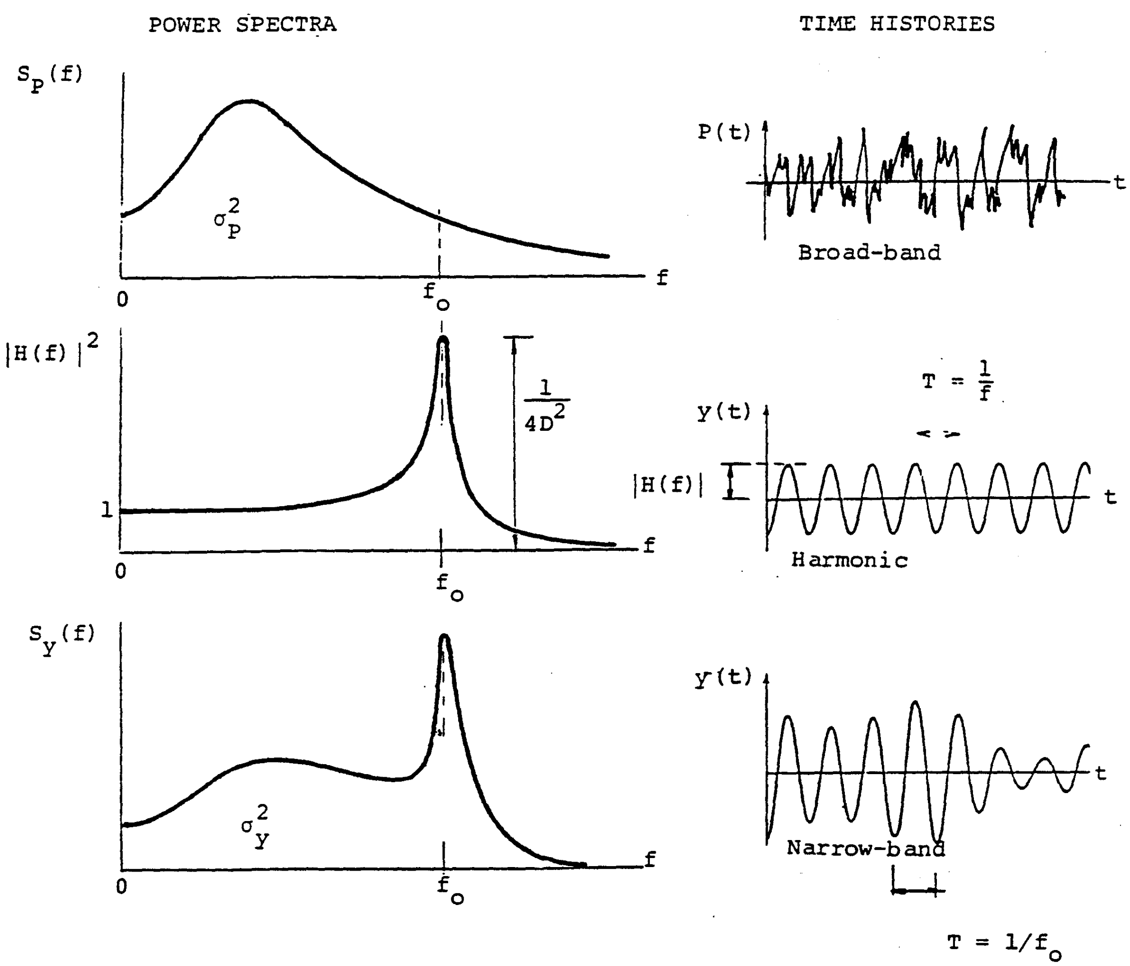

Fig. 8.12 The relationship between spectrum of input and spectrum of output

However when the random excitation come from turbulent wind, the description of the wind load has additional components that must be taken into account in addition to the Spectra of the oncoming wind.

If the area of the structure is sufficiently small so that when the bulk of the energy of the turbulence of wind gusts is at wavelengths much greater than a typical dimension of the structure, then the structure can be considered point-like. The flow past the structure is quasi-steady and the wind load in the drag direction can be written:

$$F_D(t) = \frac{1}{2}\rho V^2(t) C_D A \qquad (2)$$

where $V(t) = \overline{V} + v'(t)$. Ignoring terms of the order $(v'/\overline{V})^2$, the mean drag force and fluctuating drag force are, respectively:

$$F_D(t) = \frac{1}{2}\rho \overline{V}^2 C_D A \text{ and } F'_D(t) = \frac{1}{2}\rho \overline{V} v'(t) C_D A \qquad (3)$$

where the “prime” denotes the fluctuating component of either velocity or force.

The power spectrum of the fluctuating drag force is then:

$$S_F(f) = (C_D A \rho \overline{V})^2 S_u(f) = \frac{4\overline{F}^2}{\overline{V}^2} S_u(f) \qquad (4)$$

If, on the other hand, the size of the structure ceases to become small in comparison to the size of the wind gust, some adjustment must be made in the above formulation to account for the reduced spatial correlation of the forces. The Influence of the size of the gust in relation to the size of the structure is introduced through an “Aerodynamic Admittance Function”,

$$\left|\chi\left(\frac{fL}{\overline{V}}\right)\right|^2 \qquad (5)$$

where L is a characteristic dimension of the structure, often taken as $\sqrt{A}$. So, the spectrum of the wind force becomes:

$$S_F(f) = \frac{4\overline{F}^2}{\overline{V}^2} \left|\chi\left(\frac{fL}{\overline{V}}\right)\right|^2 S_u(f) \qquad (6)$$

As $f\sqrt{A/V} \rightarrow 0$, $|\chi|^2 \rightarrow 1.0$, (i.e. the response of a point-like structure) and as $f\sqrt{A/V} \rightarrow \infty$, $|\chi|^2 \rightarrow 0$

Theoretical estimates of the function $\left|\chi(\frac{f\sqrt{A}}{\overline{V}})\right|^2$ have been made with simplified flow past a bluff object. These estimates seem to be in satisfactory agreement with experimental observations as shown in Fig. 1.

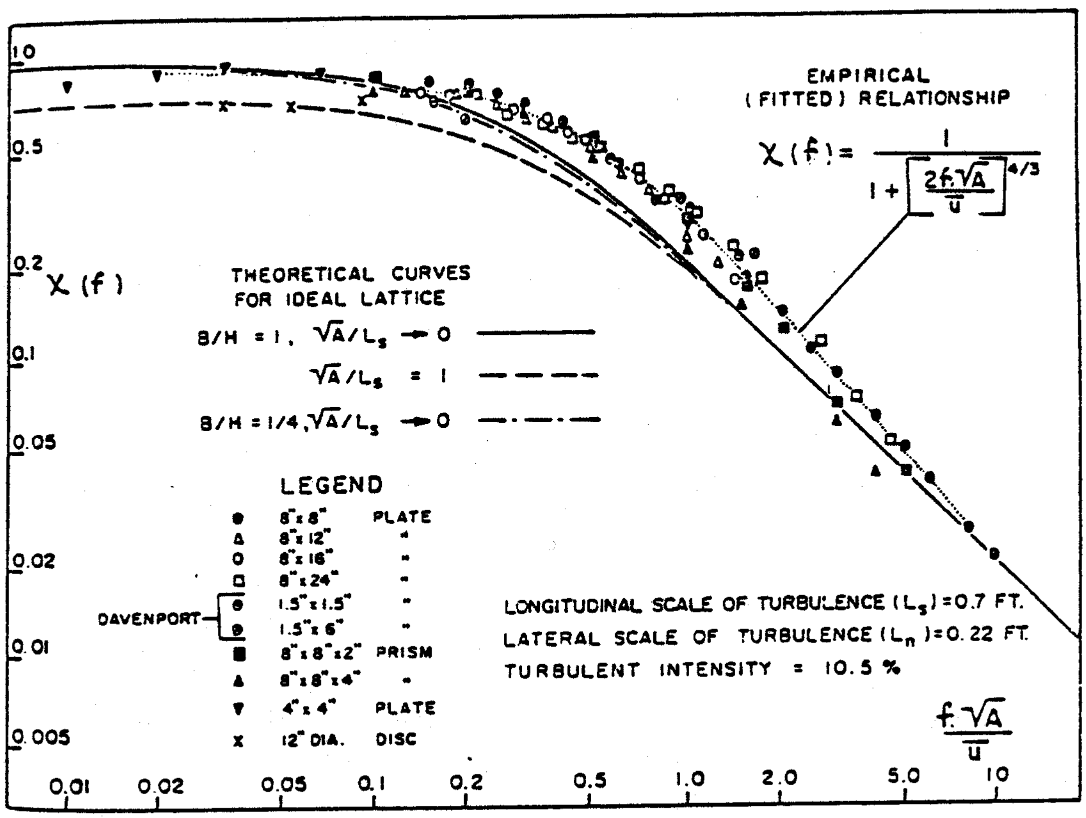

Fig. 1 Experimental and Theoretical Values of Aerodynamic Admittance for Flat Plates and Prisms Normal to the Flow ($R_e \approx 2x10^4$) (after Vickery and Davenport, 1967)

The resulting input – output diagram of loading due to random wind gusting is modified as shown in Fig. 2.

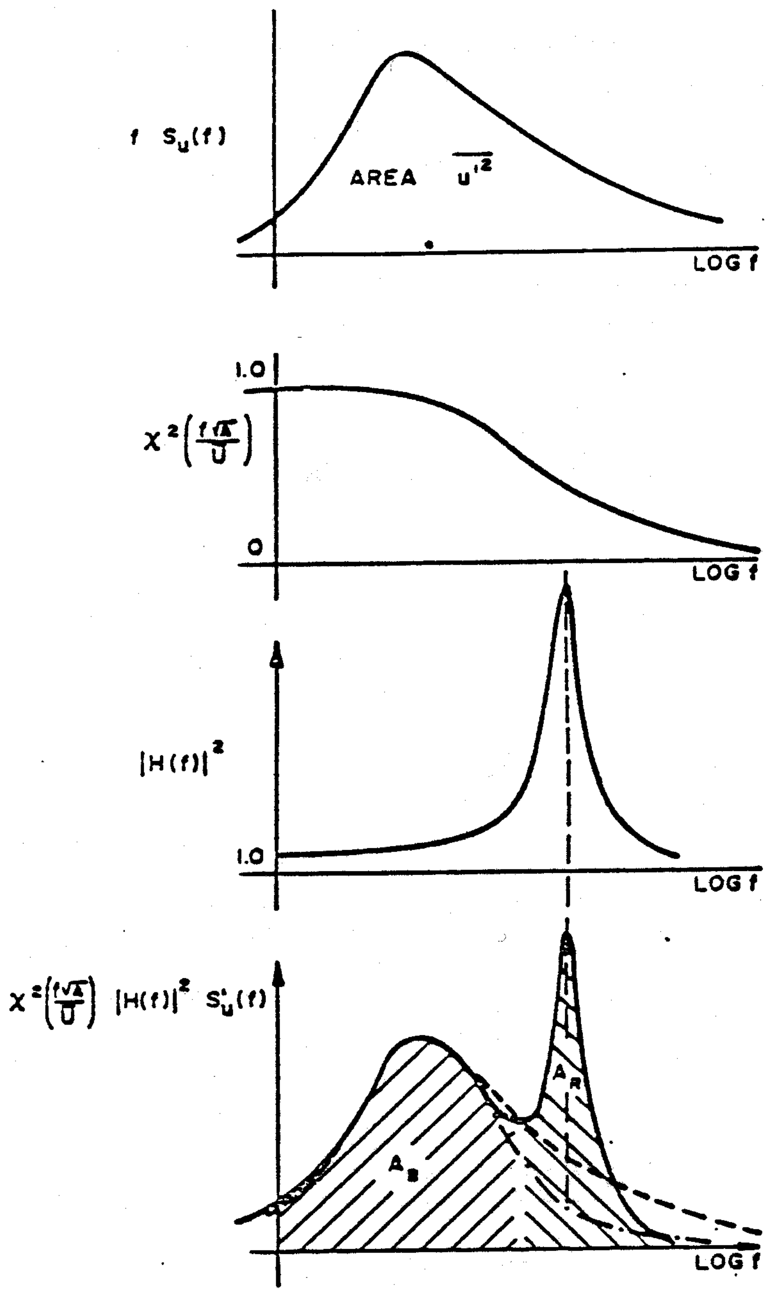

Fig. 2 Diagrammatic Representation of Mean-Square Response