THE UNIVERSITY OF WESTERN ONTARIO
FACULTY OF ENGINEERING SCIENCE

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 490 TERM B
DYNAMICS OF STRUCTURES

QUIZ

DATE: Wednesday, February 14, 2001

TIME: 4:00 p.m. to 5:00 p.m.

ROOM: EC 2116

INSTRUCTOR: Prof. N. Isyumov

INSTRUCTIONS: Closed Book (no notes or textbooks allowed).
Civil and Environmental Engineering Department approved calculators which include calculator models HP48G and HP48S

ANSWER ALL THREE (3) QUESTIONS *Total Marks = 50*

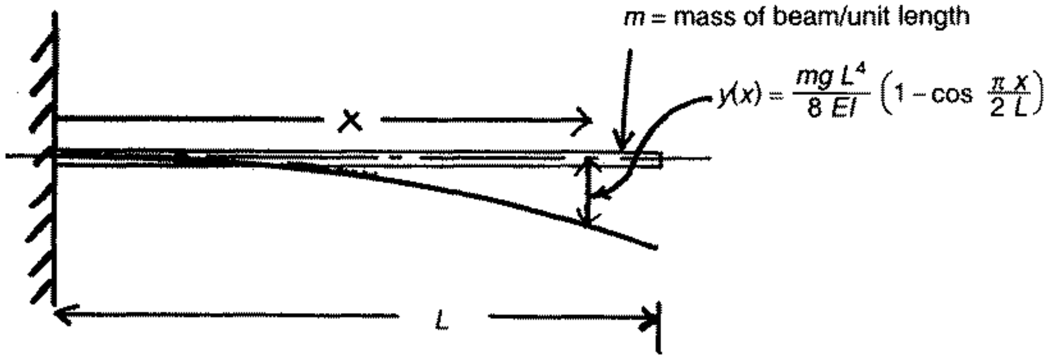
Bonus Question *Bonus Marks = 10*

Fact Sheet Provided

Marks

(14) **Question 1**

Estimate the frequency of the fundamental mode of vertical vibration of the cantilever beam, shown below



L = 10m

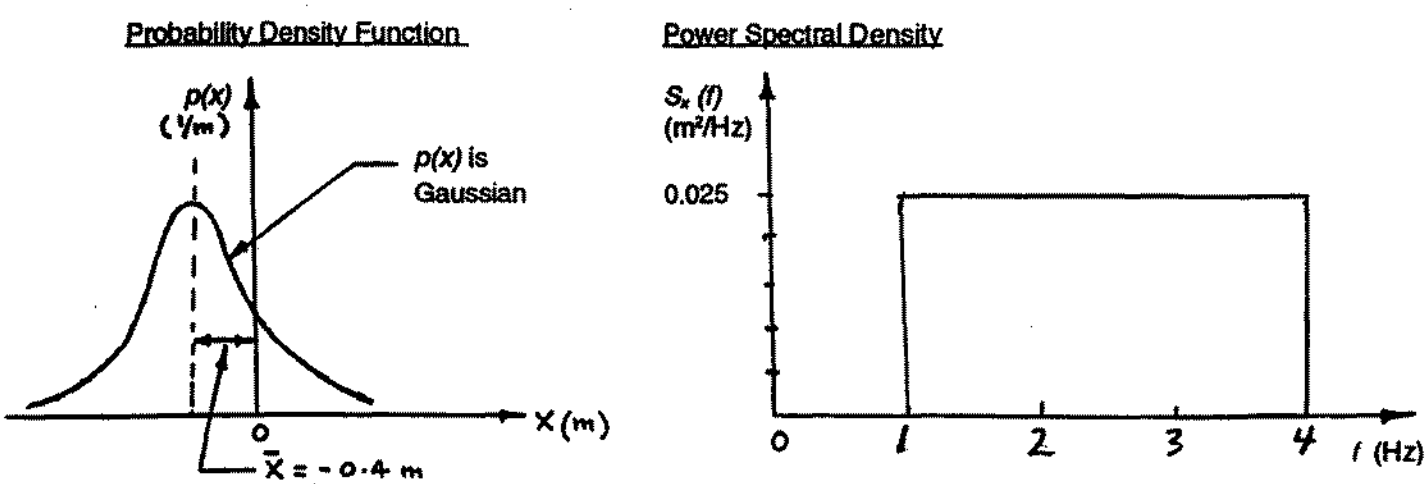
EI = 200 MN-m^2

m = 400 Kg/m

g = 9.81 m/sec^2

(16) **Question 2**

$x(t)$ is a stationary random variable in time with the following properties



(8) a) Estimate the maximum (largest) value of $x(t)$ expected within a time interval of T = 1 hour. (Hint: use the spectrum to determine σ_x and the cycling rate).

[redacted]

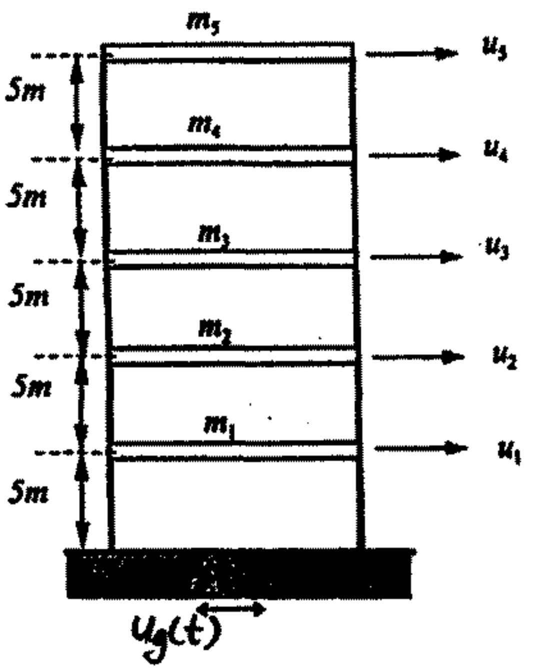
Question 2 (continued)

(8) **b)** $S_F(f)$ is the power spectral density of a stationary random excitation acting on a S.D.F. system with stiffness K, undamped natural frequency f_o and a damping ratio D. Write an expression for the variance of the deflection of this structure.

What approach is taken to get an approximate estimate of the variance if the damping ratio is small and the response is highly resonant?

(20) **Question 3**

A five storey structure has been approximated by a 5 D.O.F. system. The damping ratio can be taken as $D = 0.01$ for all modes of vibration. All masses are equal, $m_i = 500{,}000$ (kg), $i = 1,2,\ldots5$.



An eigenvalue analysis has provided the following information:

$$\text{Eigenvalues:} \quad \{\omega_i\} = \left\{ \begin{array}{l} \omega_1 = 5.70 \\ \omega_2 = 16.62 \\ \omega_3 = 26.20 \\ \omega_4 = 33.64 \\ \omega_5 = 38.38 \end{array} \right\} \text{(rad/s)}$$

Marks

Question 3 (continued)

Floor level

$$\text{Eigenvectors:}\quad [\phi_{ij}] = \begin{bmatrix} 0.334 & -0.895 & 1.173 & -1.078 & 0.641 \\ 0.641 & -1.173 & 0.334 & 0.895 & -1.078 \\ 0.895 & -0.641 & -1.078 & 0.334 & 1.173 \\ 1.078 & 0.334 & -0.641 & -1.173 & -0.895 \\ 1.173 & 1.078 & 0.895 & 0.641 & 0.334 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$

Mode of Vibration 1 2 3 4 5

Heavy machinery operating nearby causes a harmonic horizontal ground motion, of the form,

$$u_g(t) = u_{g_o} \sin \omega t$$

where $u_{g_o} = 0.1$ (mm) and $0 \leq \omega \leq 29$ (rad/s). The machine can continue operating harmonically at any frequency within this range.

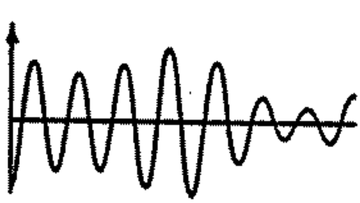
For steady state vibrations due to this machine, estimate the maximum horizontal displacement at level 5, namely maximum value of u_5. State your assumptions.

(10) **BONUS QUESTION**

i) What information does the power spectral density (P.S.D.) provide?

Ii) The time history of a random variable $x(t)$ is shown below. Sketch its expected power spectral density $S_x(f)$ versus frequency f and explain its form.

$x(t)$



$t(s)$

$S_x(f)$

f (Hz)

FACT SHEET

- $\eta_j = \dfrac{u_{g_o}\, \omega^2}{K_j}\ L_j \left| H_j\,(\omega) \right|$

- $M_j = \sum_{i=1}^{n} \phi_{ij}^2\, m_i$

- $L_j = \sum_{i=1}^{n} \phi_{ij}\, m_i$

- $p(x) = \dfrac{1}{\sqrt{2\pi}}\ \dfrac{1}{\sigma_x}\ e^{-\dfrac{(x-\overline{x})^2}{2\,\sigma_x^2}}$

- $\nu = \left[\dfrac{\int_o^\infty f^2\, S(f)\, df}{\int_o^\infty S(f)\, df} \right]^{1/2}$

- $g = \sqrt{2\ \ln\ \nu T} + \dfrac{0.5772}{\sqrt{2\ \ln\ \nu T}}$

- If $y(t) = a\, x(t)$

  $S_y(f) = a^2\ S_x(f)$

- $\sigma_y^2 \simeq \dfrac{\sigma_F^2}{K^2} \left[1 + \dfrac{\pi}{4}\, f_o\, \dfrac{S_F\,(f_o)}{\sigma_F^2}\ \dfrac{1}{D} \right]$

- $|H(\omega)|^2 = \varepsilon^2(\omega) = \dfrac{1}{\left(\left(1 - \dfrac{\omega^2}{\omega_o^2} \right)^2 + 4\ D^2\ \dfrac{\omega^2}{\omega_o^2} \right)}$

- $D = \zeta$

- $\cos^2 x = \dfrac{1}{2}\,(1 + \cos\ 2\,x)$

END