Review of Random Loading Concepts - I

- We require new mathematical tools to describe "random" loads and responses of structures
- Random loads vary in:
  - Magnitude
  - Time
  - Spatially
- We use:
  - Probability Distributions
  - Power Spectra and Auto-correlations
  - Cross-Spectra and Cross-correlations

Stationary Random Process:

Strict Stationarity
- All statistical properties are invariant with time

Weak Stationarity
- Mean μ_x and autocorrelation $\rho_x(\tau)$ are invariant with time

Ergodic Process
- Stationary process where the statistical properties of one sample are identical to the ensemble statistics

- There are fully developed mathematical methods available for Ergodic Processes and thus there are enormous advantages if a process can be regarded as stationary. This includes processes that can be described as having *weak stationarity* or *local stationarity*

Auto-correlation

- $R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(x(t) - \bar{x}\right)\left(x(t-\tau) - \bar{x}\right) dt$
- $R_x(\tau) = R_x(-\tau)$ (an "even" function)
- $R_x(\tau = 0) = \sigma_x^2 = \lim_{T\to\infty} \int_0^T (x(t) - \bar{x})^2 dt$
- $\frac{dR(\tau = 0)}{d\tau} = 0$
- $R(\tau = \infty) = 0$
- If $x(t) = y_1(t) + y_2(t) + y_3(t) + ... + y_n(t)$ where the y 's are independent, then $R_x(\tau) = R_{y1}(\tau) + R_{y2}(\tau) + ... + R_{yn}(\tau)$ and since $R_x(\tau = 0) = \sigma_x^2$, then $\sigma^2{}_x(\tau) = \sigma^2{}_{y1} + \sigma^2{}_{y2} + ... + \sigma^2{}_{yn}$

Power Spectrum

- Two-sided spectrum $G(f) = \int_{-\infty}^{\infty} R(\tau) e^{-i2\pi f\tau} d\tau$
- One-sided spectrum *S(f)* defined for $f \geq 0$

  $S(f) = 2G(f)$ and recalling that $R(\tau) = R(-\tau)$

  $$\left.\begin{aligned} S(f) &= 4\int_{0}^{\infty} R(\tau)\cos(2\pi f\tau)d\tau \\ R(\tau) &= \int_{0}^{\infty} S(f)\cos(2\pi f\tau)df \end{aligned}\right\} \text{This is a “Cosine Fourier Transform Pair”}$$

- The area under the power spectrum equals the variance, σ^2
- Dimensions of power spectrum are (dimension of x^2)/frequency, for example if *“x”* is displacement in metres, then the unsits of the power spectrum are: m^2 / Hz
- Power spectra are often normalized by the variance, σ^2, so the area is 1.0
- Examples of Random Processes:

x(t) *p(x)* *$S_x(f)$* *$R_x(\tau)$*