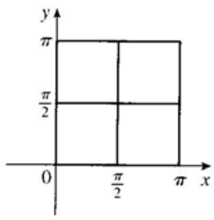
Solutions to Assignment #1

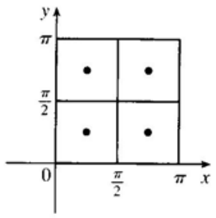
1)

3. (a) The subrectangles are shown in the figure. Since $\Delta A = \pi^2/4$, we estimate

$$\iint_R \sin(x+y)\,dA \approx \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$
$$= f(0,0)\,\Delta A + f\left(0,\tfrac{\pi}{2}\right)\Delta A + f\left(\tfrac{\pi}{2},0\right)\Delta A + f\left(\tfrac{\pi}{2},\tfrac{\pi}{2}\right)\Delta A$$
$$= 0\left(\tfrac{\pi^2}{4}\right) + 1\left(\tfrac{\pi^2}{4}\right) + 1\left(\tfrac{\pi^2}{4}\right) + 0\left(\tfrac{\pi^2}{4}\right) = \tfrac{\pi^2}{2} \approx 4.935$$



(b) $$\iint_R \sin(x+y)\,dA \approx \sum_{i=1}^{2}\sum_{j=1}^{2} f(\overline{x}_i, \overline{y}_j)\,\Delta A$$
$$= f\left(\tfrac{\pi}{4},\tfrac{\pi}{4}\right)\Delta A + f\left(\tfrac{\pi}{4},\tfrac{3\pi}{4}\right)\Delta A + f\left(\tfrac{3\pi}{4},\tfrac{\pi}{4}\right)\Delta A + f\left(\tfrac{3\pi}{4},\tfrac{3\pi}{4}\right)\Delta A$$
$$= 1\left(\tfrac{\pi^2}{4}\right) + 0\left(\tfrac{\pi^2}{4}\right) + 0\left(\tfrac{\pi^2}{4}\right) + (-1)\left(\tfrac{\pi^2}{4}\right) = 0$$



2)

11. $z = 3 > 0$, so we can interpret the integral as the volume of the solid S that lies below the plane $z = 3$ and above the rectangle $[-2,2] \times [1,6]$. S is a rectangular solid, thus $\iint_R 3\,dA = 4 \cdot 5 \cdot 3 = 60$.

3)

13. $z = f(x,y) = 4 - 2y \geq 0$ for $0 \leq y \leq 1$. Thus the integral represents the volume of that part of the rectangular solid $[0,1] \times [0,1] \times [0,4]$ which lies below the plane $z = 4 - 2y$. So

$$\iint_R (4-2y)\,dA = (1)(1)(2) + \tfrac{1}{2}(1)(1)(2) = 3$$