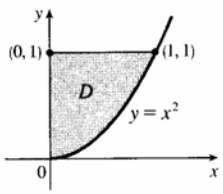
Solutions to Assignment #3

1)

**46.**




$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx = \int_0^1 \int_0^{\sqrt{y}} x^3 \sin(y^3)\, dx\, dy$$
$$= \int_0^1 \left[\frac{x^4}{4}\sin(y^3)\right]_{x=0}^{x=\sqrt{y}} dy$$
$$= \int_0^1 \tfrac{1}{4} y^2 \sin(y^3)\, dy$$
$$= -\tfrac{1}{12}\cos(y^3)\Big]_0^1 = \tfrac{1}{12}(1 - \cos 1)$$

2)

**20.** $2 = 4\sin\theta$ implies that $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so

$$A = \int_{\pi/6}^{5\pi/6} \int_2^{4\sin\theta} r\, dr\, d\theta = \int_{\pi/6}^{5\pi/6} \left[\tfrac{1}{2}r^2\right]_{r=2}^{r=4\sin\theta} d\theta = \int_{\pi/6}^{5\pi/6} (8\sin^2\theta - 2)\, d\theta$$
$$= \int_{\pi/6}^{5\pi/6} [4(1 - \cos 2\theta) - 2]\, d\theta = \left[2\theta - 2\sin 2\theta\right]_{\pi/6}^{5\pi/6} = \tfrac{4\pi}{3} + 2\sqrt{3}.$$

3)

**34.** (a) The total amount of water supplied each hour to the region within $R$ feet of the sprinkler is

$$V = \int_0^{2\pi} \int_0^R e^{-r} r\, dr\, d\theta = \int_0^{2\pi} d\theta \int_0^R r e^{-r}\, dr = \left[\theta\right]_0^{2\pi} \left[-re^{-r} - e^{-r}\right]_0^R$$
$$= 2\pi[-Re^{-R} - e^{-R} + 0 + 1] = 2\pi(1 - Re^{-R} - e^{-R}) \text{ ft}^3$$

(b) The average amount of water per hour per square foot supplied to the region within $R$ feet of the sprinkler is $\dfrac{V}{\text{area of region}} = \dfrac{V}{\pi R^2} = \dfrac{2(1 - Re^{-R} - e^{-R})}{R^2}$ ft$^3$ (per hour per square foot). See the definition of the average value of a function on page 1022 [ET 986].