Rheology -- mechanical properties of materials

What is Rheology?

- The relationship between stress and kinematic variables (strain, rate of strain etc.)

Rheological equations (or Constitutive equations)

- Mathematical statement of the relationship

History of Rheology

The term was CREATED by Bingham in 1928 at Easton PA.

Markus Reiner recalled his conversation with Bingham: “When I arrived [in 1928 at Easton, PA, the birthplace of rheology], Bingham said to me “Here you, a civil engineer, and I a chemist, are working together at joint problems. With the development of colloid chemistry, such a situation will be more and more common. We therefore must establish a branch of physics where such problems will be dealt with.”

Reiner said: “This branch of physics already exists; it is called mechanics of continuous media, or mechanics of continua.”

“No, this will not do” Bingham replied. “Such a designation will frighten away chemists.”

$\pi\alpha\upsilon\tau\alpha\rho\epsilon\iota$ = Everything flows

Some Rheological Equations

Elasticity: Stress and strain are uniquely related. Knowing the stress, you know the strain and vice versa. This is the famous Hooke's law:

$$\sigma_n = Ee_l$$

where E is called Young's modulus of the material.

Poisson's ratio (ν): the ratio of radial contraction (expansion) strain to longitudinal extension (shortening) strain.

$$\nu = -\frac{e_{radial}}{e_{longitudinal}}$$

Therefore, in a uniaxial extension or compression experiment, there will be lateral (radial) stress:

$$\sigma_r = Ee_r = -\nu Ee_l$$

In an isotropic material, the elastic behavior is completely described if the <u>Young's modulus</u> and <u>Poisson's ratio</u> of the material are known by experiment.

One may use another set of two parameters: <u>shear modulus</u> G and <u>bulk modulus K</u>:

$$\sigma_s = G\gamma$$

$$K = P / e_v$$

$$G = \frac{E}{2(1+\nu)}, \quad K = \frac{E}{3(1-2\nu)}$$

Elastic Limit and Plastic Solid Behaviors

Elastic Limit: As the stress is increased, a limit is reached beyond which the material ceases to behave elastically. The threshold stress is called the elastic limit.

Beyond the elastic limit, the material may maintain continuity (plastic deformation) or it fractures.

A **solid** can support principal stress difference as long as it is below the elastic limit.

Viscous Fluid Behavior

Viscosity: In a fluid, stress is proportional to the **rate** of strain, not strain itself!

$$\sigma_n = \eta \cdot \frac{d\varepsilon}{dt} = \eta \cdot \dot{\varepsilon}$$

$$\sigma_s = \eta \cdot \frac{d\gamma}{dt} = \eta \cdot \dot{\gamma}$$

Viscoelastic Behavior

On a short time scale, the behavior is predominantly elastic; on a long time scale it is fluid-like.

Rheological models:

Elastic spring,

Plastic (frictional sliding)

Elastio-plastic

Viscous : dashpot

Viscoelastic: dashpot in serial with a spring

Rheology of the Earth's Mantle

Elastic? (1922 AAPG in New York)

$$V_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}}$$

$$V_s = \sqrt{\frac{G}{\rho}}$$

YES!

Viscous?

YES as well

Time-effect: on a short time scale, it is mainly elastic, on a long time scale it is mainly viscous!

Rheological "Models"

- The elastic behaviour is similar to a <u>spring</u> – the applied stress is proportional to the resultant strain. Once the stress is released, the strain is recovered.
- The viscous behaviour is similar to a <u>dashpot</u> – the stress is proportional to the RATE (how fast or slow) of strain. The higher the rate the higher the stress.
- The plastic behaviour (a solid beyond elastic limit) is like a <u>solid block sitting on a horizontal surface</u> – before the applied stress reach a critical value the solid block is deformed elastically. After the critical value, the block slides on the surface. Here the critical value of stress is like the elastic limit whereas the sliding is like a plastic yielding.

What is the best analogue for earth's plate?

It seems that the mechanical behavior of lithospheric plate is analogous to an elastic spring connected in series with a dashpot. The behavior of this device is 'viscoelastic'. Applying a high stress to it or deforming it at a fast rate, it behaves essentially elastically. Applying a low stress to it or deforming it slowly, it essentially behaves viscously. There is mixed behavior depending on the time scale.