

Strain Ellipsoid and Deformation Path

- Strain of a line
 - Longitudinal strain
 - Elongation
 - Stretch
 - Natural strain
 - Shear strain of a line

Step 1: Identify *another* line that initially (in the undeformed configuration) at 90 degree with this line

Step 2: Find the angle between the above two lines (ϕ)

Step 3: The deviation of ϕ from 90 degree is the shear angle (ψ). Pay attention to the sign of the shear angle (+ cw, - ccw).

Step 4: The shear strain γ is $\tan \psi$.

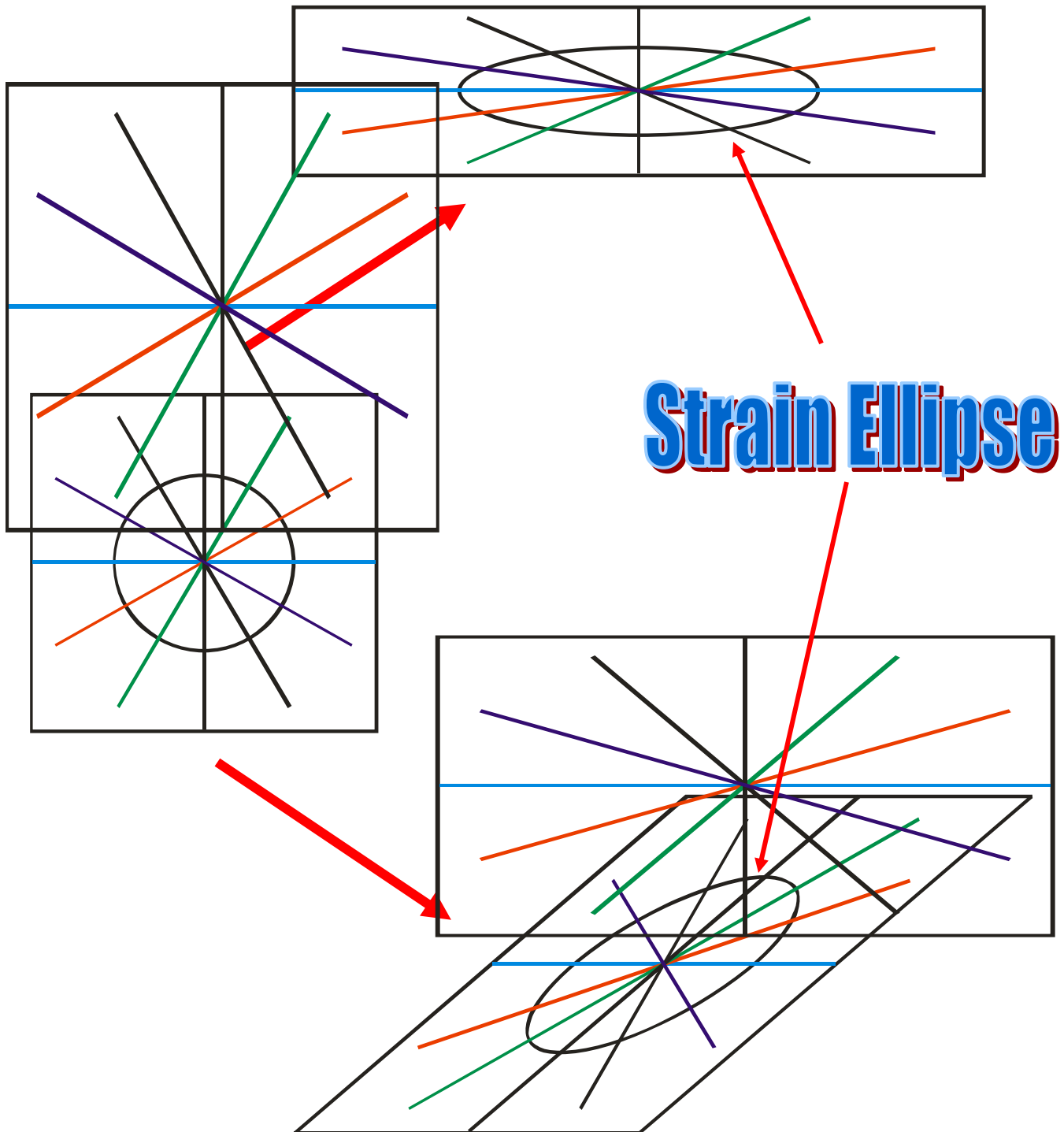
- ***How about strain of a continuous body which is 3 D?***

Volume strain

- Dilation = $(V_f - V_i) / V_i$
- Volume stretch = V_f / V_i

Natural Strain increments are additive

Strain Ellipse (Ellipsoid)



Strain Ellipse/Ellipsoid

- Lines parallel to the long axis of the ellipse have the maximum extension and stretch (e_1 , S_1)
- Lines parallel to the short axis have the least extension and stretch (e_3 , S_3)
- If the initial circle has radius of 1, the final lengths of the two semi-axes of the ellipse are respectively S_1 and S_3
- All above statements are applicable in 3D cases. In a strain ellipsoid, three semi-axes are S_1 , S_2 , S_3 .

Principal strains and Principal strain axes

- The longitudinal strains along the three axes directions ($S_1 > S_2 > S_3$) are called principal strains
- The three axes (orientation) are called principal strain axes
- **Remarkably, shear strains along principal strain axes are zero!!**

Strain measurement

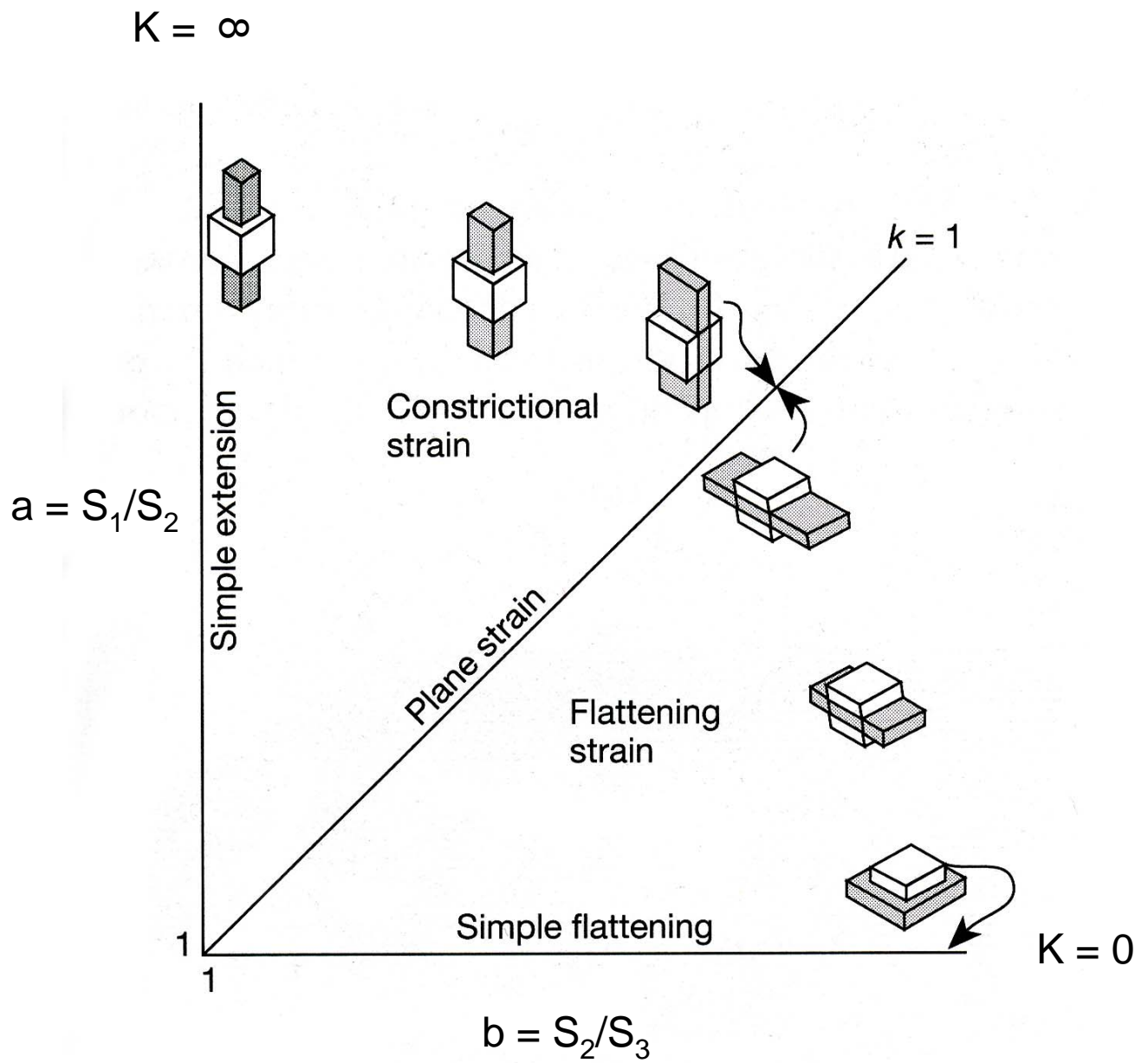
- Determination of the state of strain in the rock using various techniques including deformed fossils and other strain markers (E. Cloos 1947, deformed oolites in South Mountain, Maryland)

Deformed pebbles



Shape of Strain Ellipsoid – the Flinn Diagram

- Plane-strain: If the deformation is restricted in 2D, i.e., in the 3rd dimension, there is no strain, the deformation is said to be of plane strain.
- Uniaxial extension: extension along one direction $S_1 > S_2 = S_3$.
 - What does the strain ellipsoid look like?
- Pure flattening: $S_1 = S_2 > S_3$
 - What does the strain ellipsoid look like?



$$K = \frac{a - 1}{b - 1}$$