Force and Stress

Force: action between matters.

There are 4 actions in nature.

Gravity

Electromagnetic

Strong action

Weak action

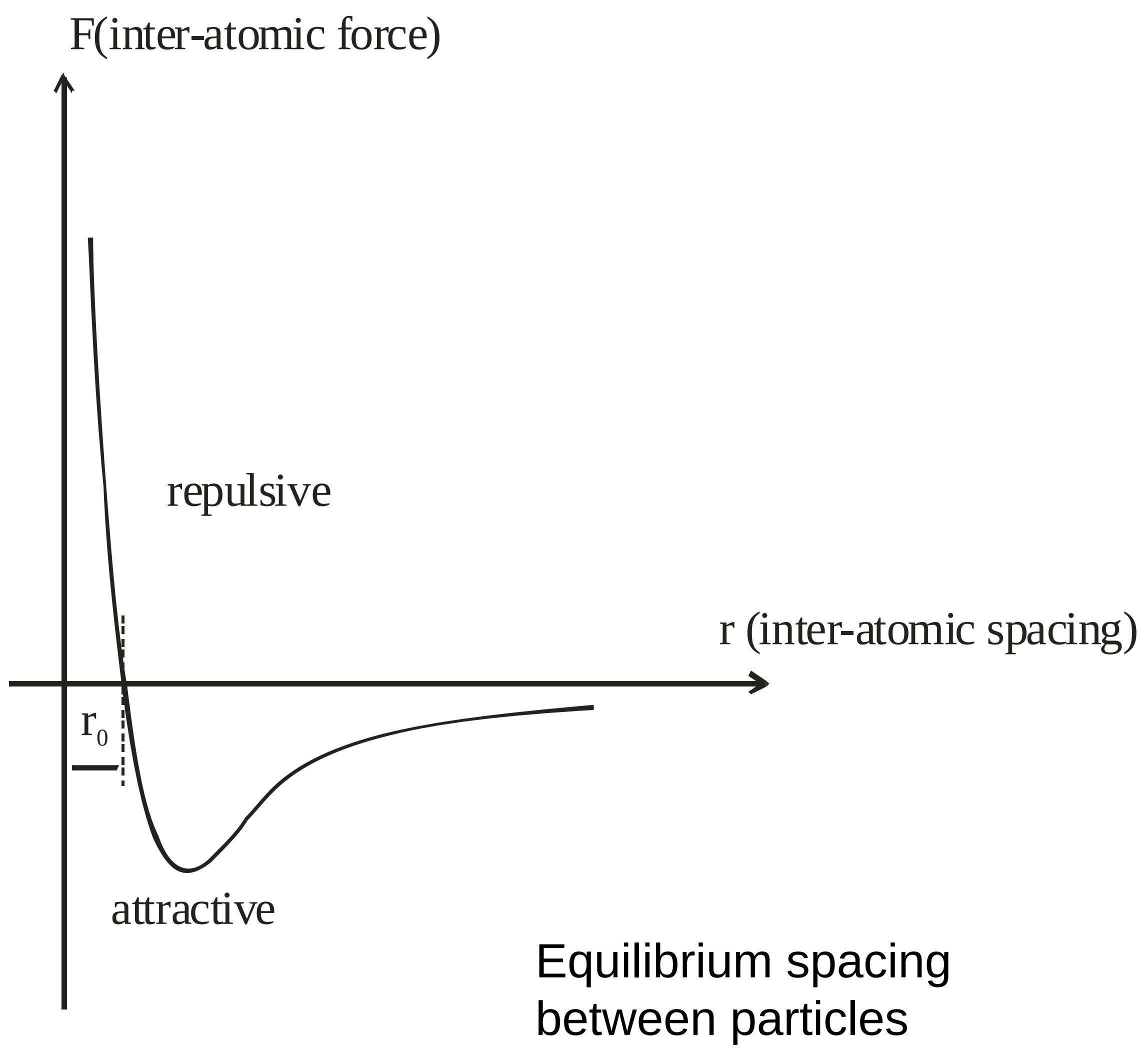
3 laws of Newtonian mechanics

1st: If an object is left alone, it will remain at rest or move at a constant speed along a straight line.

2nd: When not left alone, its velocity (speed and direction of motion) will change. That is it will accelerate.

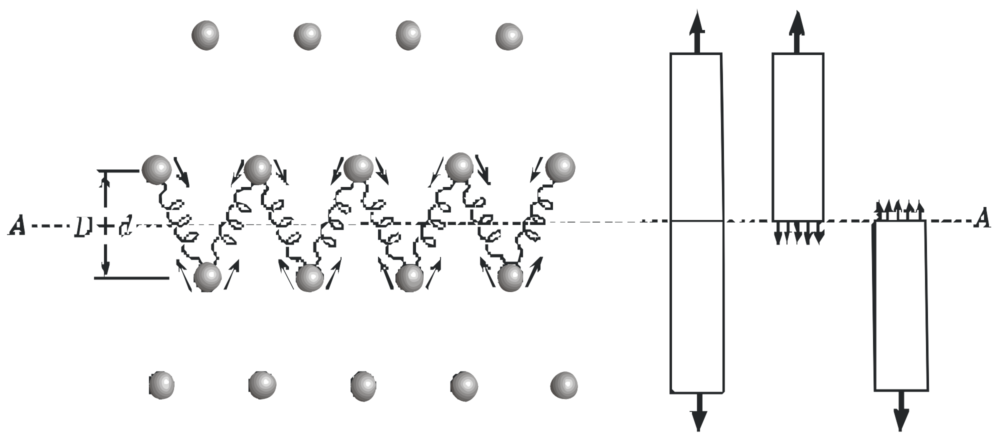
3rd: For every action, there is a reaction, equal in magnitude, opposite in direction, and acting on different objects.

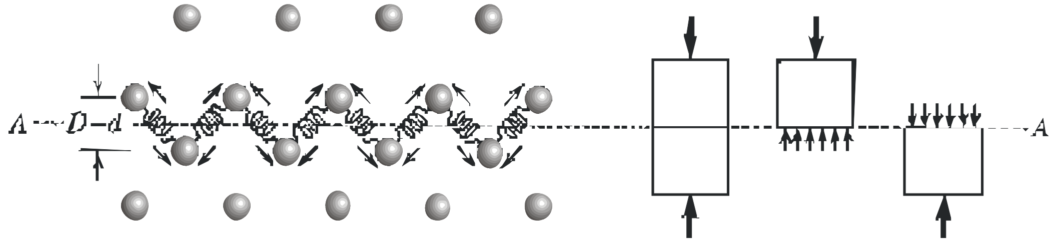
Inter-atomic (molecular) force

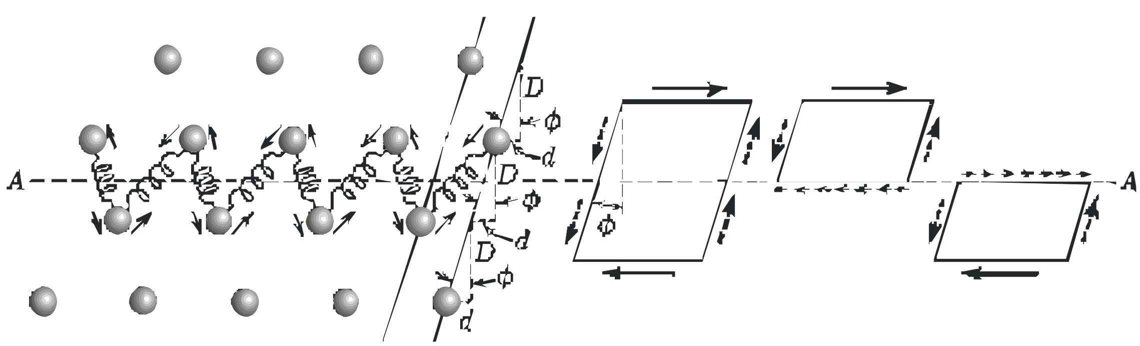


Schematic diagram showing the inter-atomic force as a function of inter-atomic spacing.

If FORCED…







Origin of Stress

- If the particles are FORCED to get closer together or farther apart, the interatomic force will increase that acts to bring the spacing between particles back to the equilibrium spacing.

This *<u>additional action</u>* is the cause of stress!

Surface force and Body force

On a macroscopic scale, force can be acted on an object either as:

body force: acting on every part of the body through a field, the strength determined by the mass or electric charge

Or

surface force: acting through the contact between objects

Uniaxial stress: vertical stress on a horizontal surface of rocks due to burial

The volume of the column down to the depth z is:

$$v = \delta A \, z$$

The overburden weight of the column down to a depth of z is:

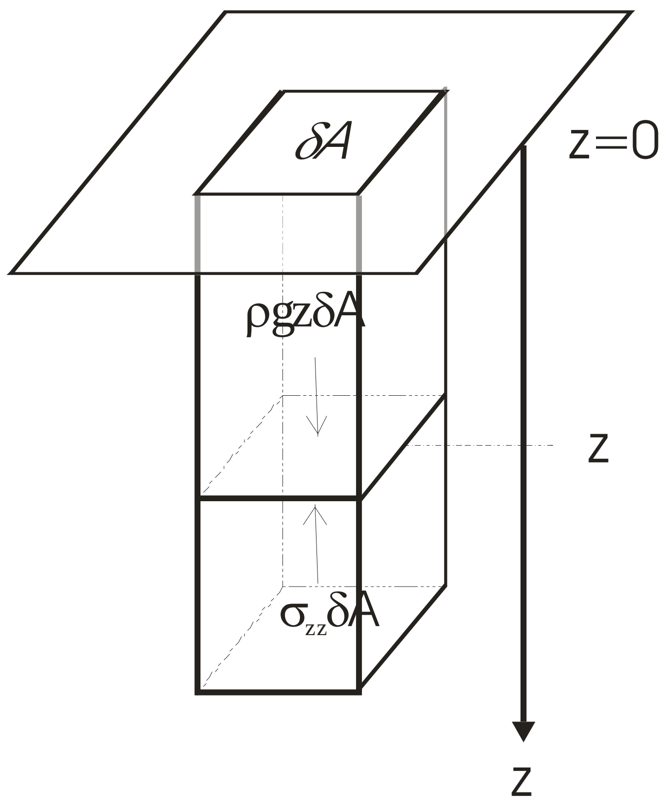
$$w = v \, \rho = \delta A \, z \, \rho$$

where ρ is the density of the overburden rock.

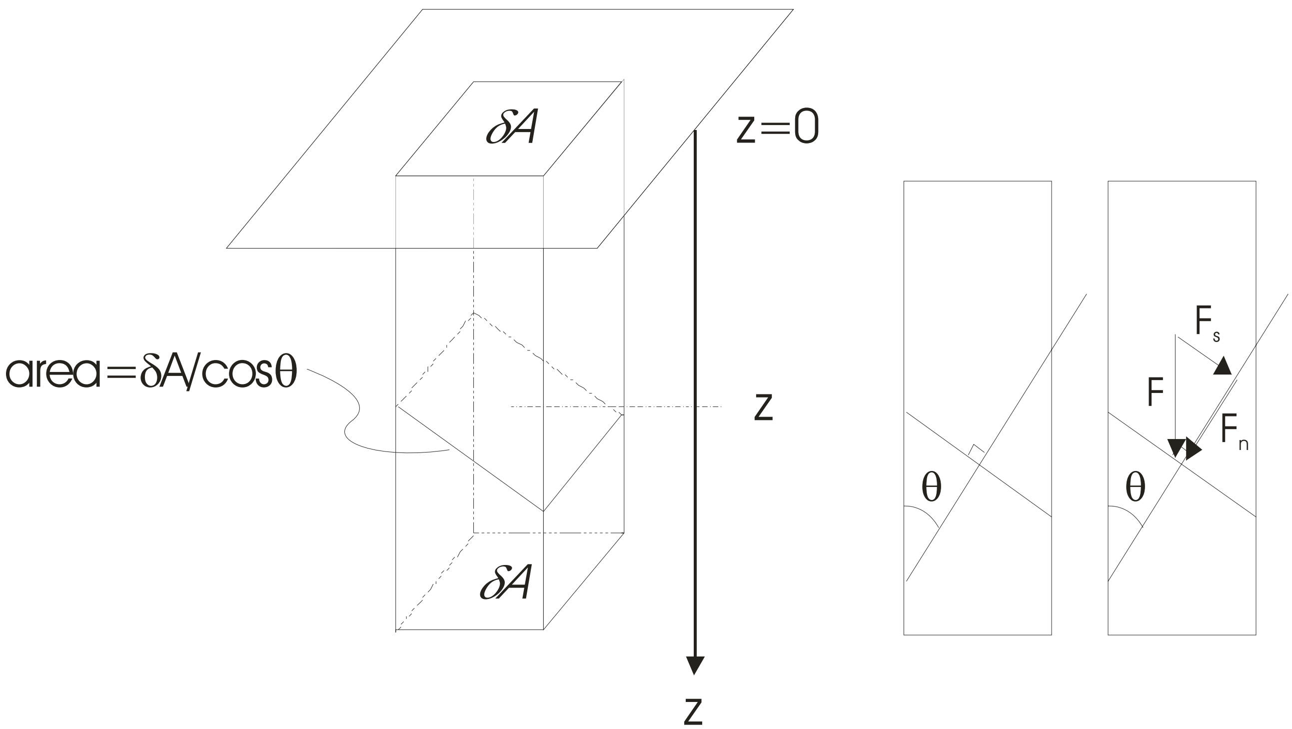
The stress acting on the horizontal plane at depth z is thus:

$$\sigma_{zz} = \delta A \, z \, \rho \, g \, / \, \delta A = \rho \, g \, z$$

Note that the stress is <u>compressive</u> and acting exactly perpendicular to the plane.
What about an inclined surface?



Shear and Normal Stress



The total force is still the same, but now the area of action is increased. In addition, the stress has both a component of pure compression and a component of shear.

$$\sigma_{\text{total}} = \frac{\delta A \cdot z \cdot \rho \cdot g}{\delta A / \cos\theta} = \rho \cdot g \cdot z \cos\theta = \sigma_1 \cos\theta$$

$$\sigma_{\text{n}} = \frac{\delta A \cdot z \cdot \rho \cdot g \cdot \cos\theta}{\delta A / \cos\theta} = \rho \cdot g \cdot z \cos^2\theta = \sigma_1 \cos^2\theta$$

$$\sigma_{\text{s}} = \frac{\delta A \cdot z \cdot \rho \cdot g \cdot \sin\theta}{\delta A / \cos\theta} = \rho \cdot g \cdot z \sin\theta \cos\theta = \sigma_1 \sin\theta \cos\theta$$

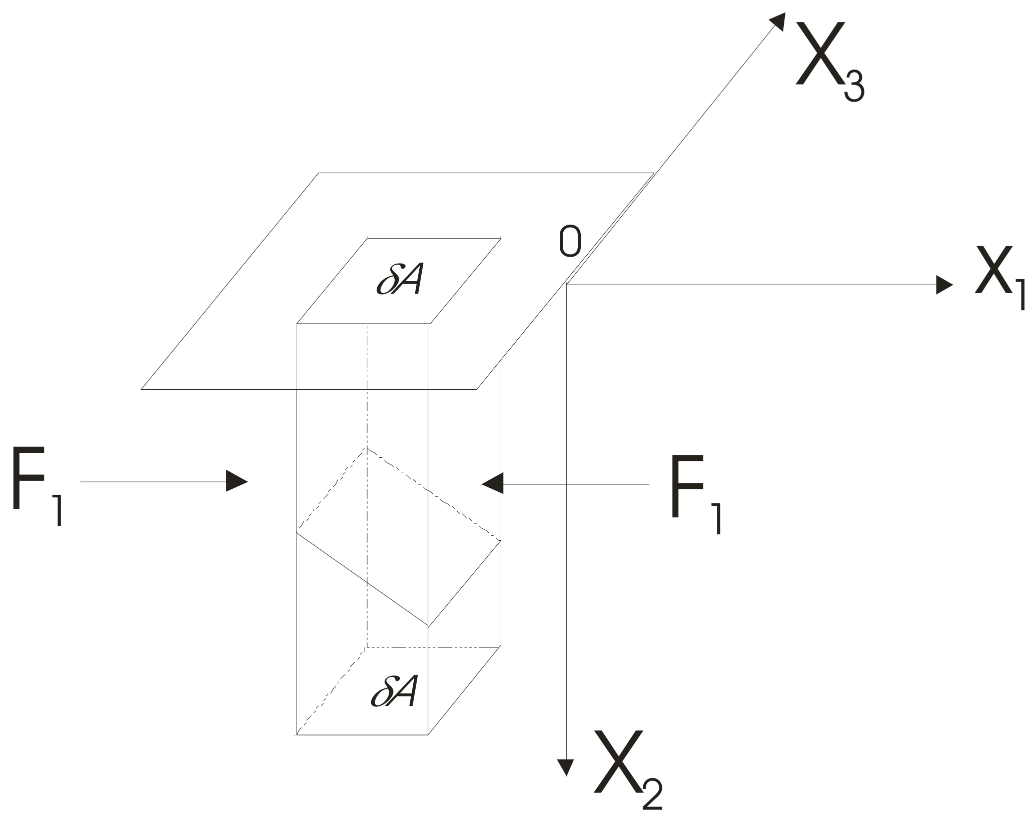
Normal stress and shear stress

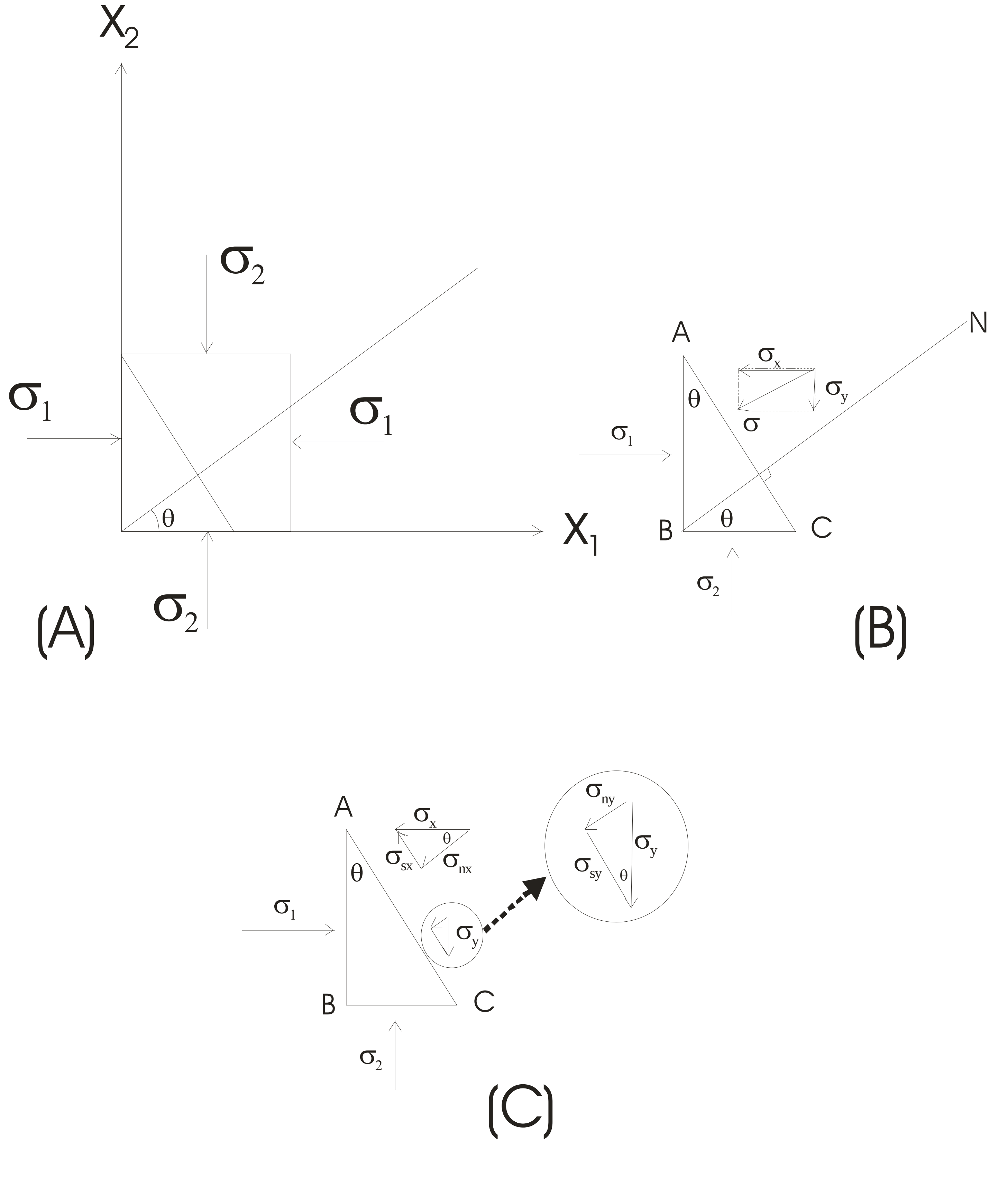
Normal stress: stress perpendicular to the plane, positive if compressive, negative if tensile.

Shear stress: stress parallel to the plane of action, positive if counterclockwise, negative if clockwise.

Scalars, Vectors, Tensors

Biaxial Stress State




X_2
σ_2
σ_1
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X_1
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(A)
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B
C
σ_2
(C)

Derivation of the normal and shear stress on a plane in biaxial stress state

$\sigma_1 \cdot AB = \sigma_x \cdot AC$ Force balance in the x-direction

$\sigma_2 \cdot BC = \sigma_y \cdot AC$ Force balance in the y-direction

$$\sigma_x = \sigma_1 \cdot \frac{AB}{AC} = \sigma_1 \cdot \cos\theta$$

$$\sigma_y = \sigma_2 \cdot \frac{BC}{AC} = \sigma_2 \cdot \sin\theta$$

$\sigma_{nx} = \sigma_x \cdot \cos\theta = \sigma_1 \cdot \cos\theta \cdot \cos\theta = \sigma_1 \cdot \cos^2\theta$ Normal stress contributed by σ_x

$\sigma_{sx} = \sigma_x \cdot \sin\theta = \sigma_1 \cdot \cos\theta \cdot \sin\theta = \frac{1}{2}\sigma_1 \cdot \sin 2\theta$ Shear stress contributed by σ_x

$\sigma_{ny} = \sigma_y \cdot \sin\theta = \sigma_2 \cdot \sin\theta \cdot \sin\theta = \sigma_2 \cdot \sin^2\theta$ Normal stress contributed by σ_y

$\sigma_{sx} = -\sigma_y \cdot \cos\theta = -\sigma_2 \cdot \sin\theta \cdot \cos\theta = -\frac{1}{2}\sigma_2 \cdot \sin 2\theta$ Shear stress contributed by σ_y

Total normal stress

$$\sigma_n = \sigma_{nx} + \sigma_{ny} = \sigma_1 \cdot \cos^2\theta + \sigma_2 \cdot \sin^2\theta = \sigma_1 \cdot \cos^2\theta + \sigma_2 \cdot \left(1 - \cos^2\theta\right) = \frac{1}{2}\left(\sigma_1 + \sigma_2\right) + \frac{1}{2}\left(\sigma_1 - \sigma_2\right) \cdot \cos 2\theta$$

Total shear stress

$$\sigma_s = \sigma_{sx} + \sigma_{sy} = \frac{1}{2}\sigma_1 \cdot \sin 2\theta - \frac{1}{2}\sigma_2 \cdot \sin 2\theta = \frac{1}{2}\left(\sigma_1 - \sigma_2\right) \cdot \sin 2\theta$$