Page 1: **Euler made a grave mathematical mistake**
(posted 16 March 2011)

Euler postulated the stress tensor in 1776. Why?

He was probably the first to seek a solution to spatiality, and found it necessary to describe the properties of a vector in space as the function of another vector in space. So he chose a point of interest Q, let planes pass through it in all directions, and let force vectors change direction as a function of the orientation of the plane. The concept looks so simple that one wonders why it should be wrong. But it is. Let's look at scalars first.

That 1 + 1 = 2 is known for as long as people counted objects. The recognition that 1 – 1 = 0 is much younger. Originally it was thought that 1 – 1 = nothing. To understand that "nothing" is a number, (1) one needs to separate objects from numerals, (2) one must understand that 1 and –1 are two different numbers, (3) one must accept that zero is a number too. This was done in India only 1500 years ago, when zero was *defined* as the sum 1 + -1 = 0. Zero is the only number without a sign.

In the 17th century CE the necessity arose to describe objects which had more than one property – one relating to arithmetic, the other relating to geometry, that is: magnitude and direction, the vector. For us it is worth noting that these vectors were all *free vectors*, or discrete vectors. A discrete vector can be given like [sin pl cos pi], and *observed* at a point P; a *vector field* requires a generating function which *assigns* a vector to every point in space. They cannot be transformed into one another, mathematicians use separate notations for them. Whether a vector quantity is a field vector or a discrete vector is decided by the physical problem. Newton's mechanics involves free vectors, and this was the only concept Euler knew. He died in 1783. Vector fields were invented by Lagrange in 1784.

Vectors describe directions in space, but not space itself, this is done through coordinates. Lack of precise rules made people handle them intuitively, and with great liberty. Cauchy's writings show a conceptual innocence that was paradisiacal. None of this would be permitted today. This stopped when Hermann Grassmann discovered linear algebra and the rules for vector spaces, matrices and tensors in 1860. Cauchy died in 1857.

The Euclidean space is the space in which every point can be given by its coordinates. Grassmann realized that we need a unique correlation of notation and object such that

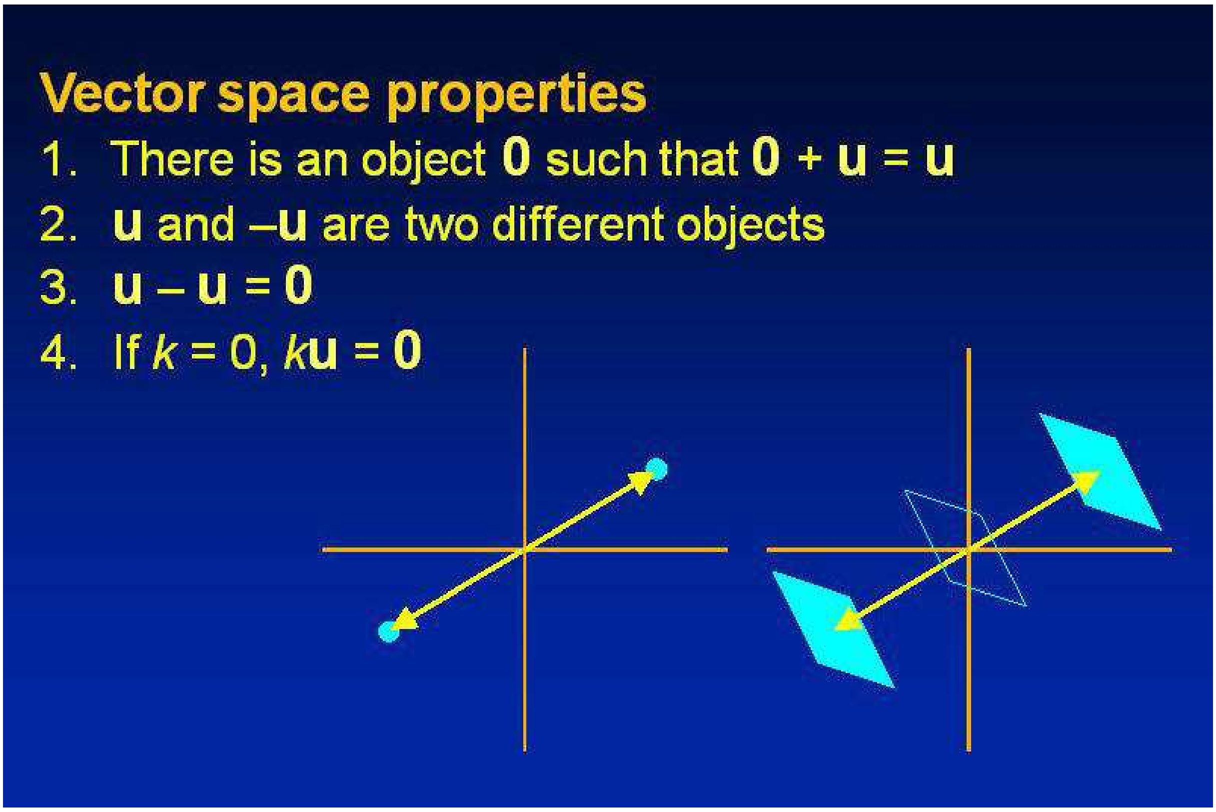
- no notation can describe more than one object,
- no object can be described by more than one notation, and
- that the zero object exists.

"Objects" may be points, vectors or planes if the latter are given by the normal vector emanating from the coordinate origin. In the most common notation used today (found by Otto Hesse ca.1845) the vectors [2 3] and [-2 -3] are two different vectors, and their sum is the zero vector [0 0]. If they indicate planes, they are at opposite sides of the origin Q, and parallel to one another.

Every coordinate system has a singularity at its origin: an object with the notation [0 0] has no properties. The zero vector is by definition a vector without magnitude and direction or sign – the equivalent to the scalar number zero. It follows that any plane that passes through the origin Q has the zero vector as notation; its direction cannot be defined. *We cannot calculate with it.*

The rules of vector spaces, known since 1860, collide head-on with Euler's stress concept from 1776. Euler innocently used the convention that a plane passing through the point of interest Q and perpendicular to the x-axis has the notation v = [x 0 0] in 3D. But so does the vector –v = [-x 0 0], the notation is non-unique; and the zero object does not exist in his convention, the operation v – v is meaningless. Moreover, planes at points other than Q cannot be described, we can do this only in the Hesse notation. But we cannot use two mutually exclusive conventions simultaneously. Euler's convention for planes in space violates the properties of Euclidean space. His notation of planes in space, and thus his concept of the stress tensor, is invalid.

"Well, but … why has this never been said before?" – – "Who are you to question Euler?"
It is high time to do it. Science is not and should not be a church.



The vectors **v** and –**v** indicate two different objects, left: points, right: planes. The plane parallel to these planes passing through the origin cannot be indicated. In Euler's convention both vectors could indicate the plane through the origin, but the planes not passing through the origin cannot be indicated. Euler's convention for planes violates four rules for vector spaces.

Page 2: **Euler made a grave physical mistake**
(posted 18 March 2011)

Why did Euler define stress as a form of pressure? Why did he contradict Newton?

Newton's mechanics is the mechanics of discrete bodies in freespace which interact by collision. The forces involved are readily and correctly described by free vectors which act upon a point.

This differed strongly from the situation within distributed matter where discreteness does not exist. The only example of a force distribution known then in the 18^{th} C was that of pressure, force per area. The ratio f/A is known to be scale-independent. Thus Euler began to ponder force distributions on planes, and planes in space; the result was the stress tensor. At this point it was just a postulate.

There is much to say about distributions, but that must wait. Here I discuss the Euler-Newton contrast.

The force **f** acting on a plane can be decomposed into the plane-normal component and the plane-parallel component using the plane-normal vector. The magnitude of the latter is unconstrained, it was taken to be an unit vector **n**. Here Euler contradicted Newton. Let's compare:

Newton considered discrete bodies of given size and shape, and he realized that the center of mass Q is an unique point about which a freely spinning body rotates. Any mechanical force acting upon the body must have its point of action P on the surface, hence there is a radius **r** = QP with which the force **f** interacts. The force may be decomposed relative to **r**, and the body does not experience an angular acceleration if the sum of all torques **r**x**f** = 0. **r** is a function of the shape, and **r** is a *lever*, that is: **r** is mechanically significant. Note that neither the surface nor its orientation is important in Newton's mechanics, only the points P of which it is made, and their spatial relation to Q.

A lever is a distance in space *within a solidly bonded body*. The absolute requirement for the lever to exist is that there is continuity of bonds – not mass continuity, bond continuity matters. But the earliest paper known to me that mentions bonds is from ca.1850, neither Euler nor Cauchy knew about them.

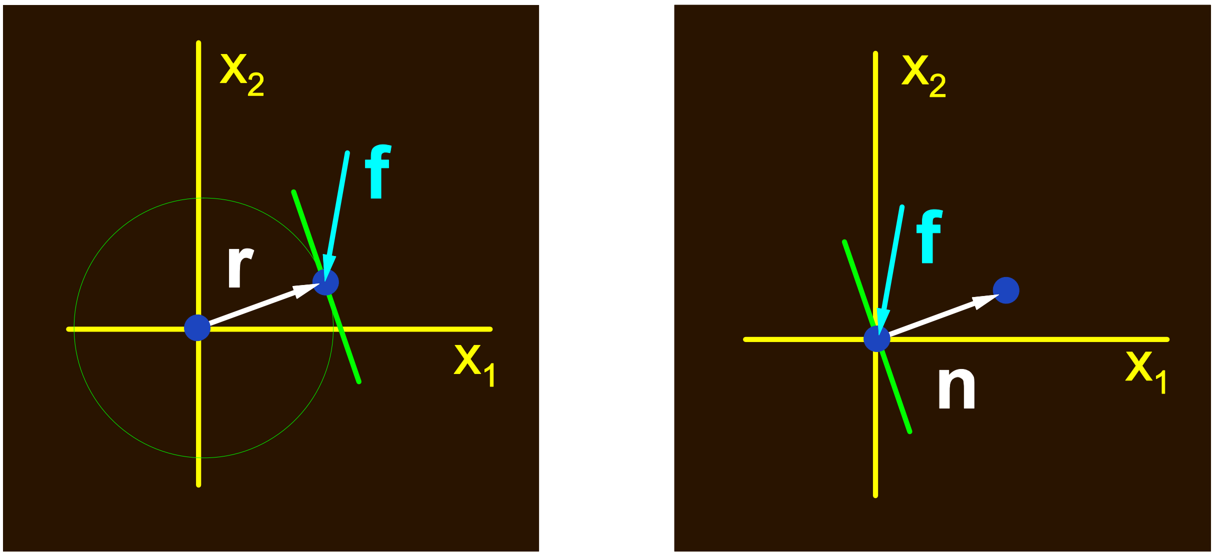
Euler's concept could be physically valid if it were possible to transform Newton's **r** and Euler's **n** into one another. This is not possible. Nobody has ever shown that **n** is a physically meaningful term, i.e. a lever. It follows that shear force and normal force, or shear stress and normal stress, following Euler, are physically irrelevant terms. This should not be surprising, because the product **|f||r|** is a [Joule] term which relates to the work done; whereas **|f||n|** is just **|f|**.

There are various ways in the literature how Newton's torque **f**×**r** is to be handled; none of them was convincing, especially not in Truesdell's works, because the distance term involved is not explicitly a lever, i.e. a distance within a solid. These distance terms may be distances in freespace since bonds are never mentioned anywhere in the textbooks – literally, I have searched dozens of textbooks, and have not found mention of bonds. *Thus, continuum mechanics has forgotten to define what a solid actually is.* This is neither continuum nor mechanics, it is bogus.

Why did this concept survive for so long? Because authority can mislead. There are two giants of science involved here who both erred – Leonhard Euler who has some 900 papers to his name, plus his most prominent victim Augustin Cauchy who wrote only 800 papers. Cauchy mentioned the torque **|f**×**r|** in his paper on stress theory, but perfunctorily set it to zero, no reason given. He completely ignored the fact that **r** is shape-dependent. No freshman would get away with this today. Then he turned to the relation of **f** to **n**, thereby assigning a physical importance to it which it does not have. Since **n** assumed the place of **r**, the radius of the volume element was assigned unit magnitude. This has the effect of an unrecognized boundary condition – which generally does not hold.

It is time to state it openly and publicly: Newton is right, and Euler was wrong. **f**×**r** is sound physics, **f**×**n** is not. Believing Euler is an exercise in self-deception.

"But why are we taught all this?" – – "Because. Do you want to pass or fail?"



Left: Newton's systematics. A force **f** (blue) acts upon a body (green circle) at the point whose position vector is **r** (white). The radius is a mechanical lever. The rotational equilibrium condition is sensitive to the shape of the body. The plane perpendicular to **r** is not of interest in Newton's mechanics. Right: Euler's systematics. A plane passing through the origin has an orientation indicated by the vector **n**. The magnitude of **n** is said to be unity, but it could be any length; in reality **n** is a ray and not a vector. **n** is not a physical lever. The cross product **f**×**r** is physically meaningful, whereas the cross product **f**×**n** is not. The systematics of Newton and Euler cannot be transformed into one another.

Page 3: **Continuum mechanics is a perpetuum mobile theory (1)**
(posted 21 March 2011)

In order to assess Cauchy's theory properly it helps to realize that in all the 16 papers by Cauchy which I have studied I have not found one single mention of *physical work*. That's too bad; he would have found out all by himself that he wrote a perpetuum mobile theory.

I have published three demonstrations to show that current texbook theory always leads to the conclusion that the work done in a volume-neutral deformation is zero <www.elastic-plastic.de/koenemann2008-2.pdf>. (Today I could quote a fourth source.) Why is this so?

All of classical physics (this side of Einstein & Planck) can be grouped into two very fundamental categories. Every system – a volume in space containing mass, e.g. a kinetic system with n bodies – contains a certain energy U. A process that does not change U is called *conservative* since U is "conserved", i.e. invariant. Commonly the system is isolated, which means that there is no exchange of mass or energy between system and surrounding across the system boundary.

The energy conservation law $E_{kin} + E_{pot} = U = const$ defines a conservative process. Any process that observes this law will turn E_{kin} into E_{pot} and vice versa; the change is the work w. Hence all the work is done *within the system*. There is no exchange with a surrounding. Classical examples of a conservative process are the revolution of planets about the sun, or diffusion in water at rest at constant T.

If a process changes the system energy U, it is called non-conservative. It requires that energy fluxes take place between the system and its surrounding across the system boundary. Thus we need a new energy conservation law that takes account of the fluxes; this is the First Law of thermodynamics, $dU = dw + dq$. Non-conservative processes may be reversible or irreversible. In this case the work is done *upon the system*. It is commonly known as PdV-work.

The difference between a conservative and non-conservative process can be given by a simple mathematical condition.

- If there are no fluxes **f**, the divergence div **f** = 0. This is called the Laplace condition.
- If there are net fluxes **f**, the divergence div **f** = $\varphi \neq 0$. This is called the Poisson condition.

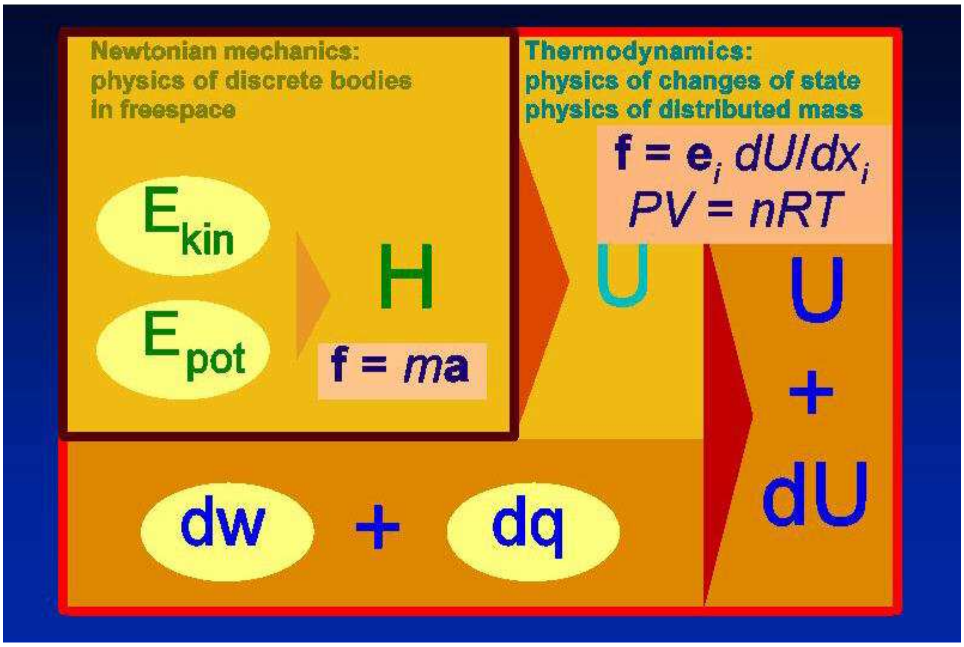
φ is the charge, which is a measure of the work done upon the system. Now, is elastic deformation a conservative or a non-conservative process? Clearly work is done upon the system such that it deforms, and energy is stored in the system, the elastic potential.

But it is well known, taught in every intro class, and found in every textbook, that the trace of the stress tensor $\text{tr } s = s_{11} + s_{22} + s_{33} = 0$ for a volume-constant deformation. This is the Laplace condition. That is, *the no-work condition is solidly built into Cauchy's stress theory.*

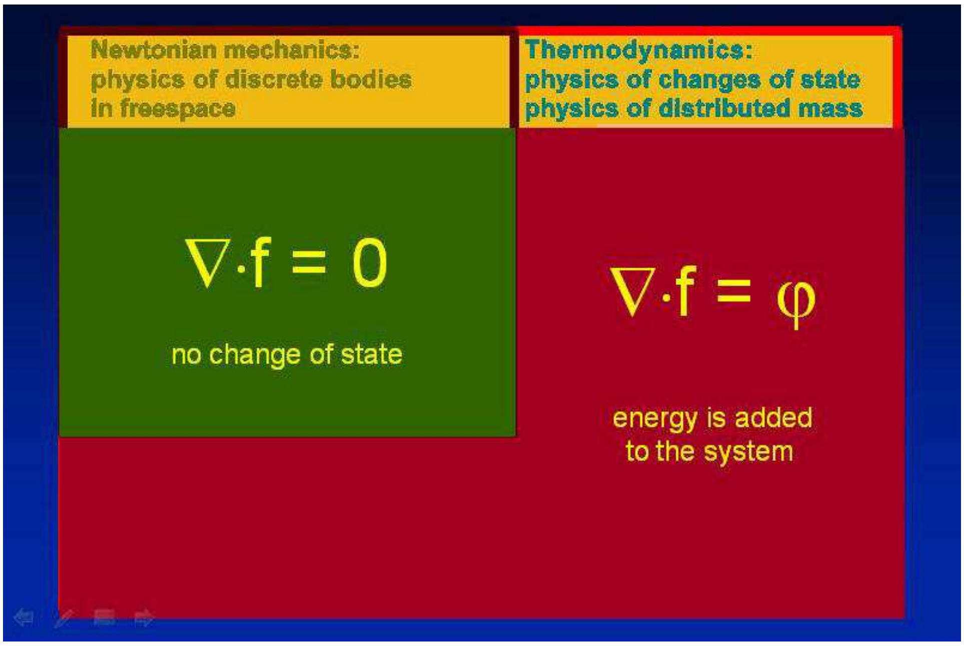
How could this happen? Very simply: in the mid-18th C when Euler thought about all this, Poisson's condition and the First Law of thermodynamics were still many decades in the future. How could he know that the energy of a system can be a variable? That was understood only after 1845. The only theoretical template Euler knew was Newton's mechanics *which is rightfully conservative*, but unsuited for any understanding of elasticity.

Today we would start by asking: is elasticity conservative or non-conservative? Of course the latter. Thus we would turn to thermodynamics, assume a system with a given amount of mass, e.g. one mol, and then study the energetic fluxes between system and surrounding. And we would use div **f** = $\varphi \neq 0$ as a test to see whether we made a mistake – because it must be non-zero.

If the Gauss divergence theorem is the entrance gate into potential theory, the Laplace and Poisson conditions are its door wings. Potential theory is a wonderful and incredibly powerful theory, the backbone of all of classical physics, and the core of a myriad of methods in applied mathematics. Continuum mechanics has managed to ignore it completely and entirely.



Systematics of classical physics. Black box: Newtonian mechanics, the *entire mechanical energy* of the system is H. The equation of motion **f** = ma is the correct force definition. H is called the *internal energy* U in thermodynamics. If fluxes are added from outside – dw or dq – the state of the system changes from U_0 to U_1. We need an equation of state because the energetic state of the system is a variable, and the correct force type is a field force which is derived from a potential, here the internal energy (correctly from ΔU).



Newtonian mechanics is conservative because there are no fluxes from outside. Thermodynamics is non-conservative because there are fluxes from outside.

Page 4: **Continuum mechanics is a perpetuum mobile theory (2)**
(posted 23 March 2011)

For reasons beyond my knowledge, consideration of physical work has never found much attention in continuum mechanics. But the condition that a volume-constant deformation – according to the current theory – must be zero, is so obvious that I noticed it right in my own intro class. (Of course, my question went unanswered.)

Consider a volume element of spherical shape. This is also a thermodynamic system. By convention work done upon the system is negative, hence compression does negative work, and stretch does positive work. If the system is shortened along x_3 and stretched along x_1 such that the volume remains constant, and the dimensions in x_2 are constant, necessarily the work must cancel.

This applies to several prominent theoretical outlines in the literature, for example Landau & Lifshitz. This is also the error in Euler's continuity equation $d\rho/dt + \rho dv_i/dx_i = 0$. Here ρ is the density of the inertial mass, v is the velocity, and x_i are the coordinates. If ρ is constant with time, the first term LHS = 0; the rest has the form of the Laplace condition as explained on Page 3.

In fact, there is even a natural situation where this logic is perfectly correct: in case of a gas – because a gas is incoherent, a volume-constant process does not cost any work. But a gas cannot develop an elastic potential in such a case, it does not reconstitute if it is let go. For a solid the zero work result is nonsense – but how is a gas and a solid distinguished in a theory of elasticity and stress that never mentions bonds? The problem in Euler's continuity equation is the continuity of mass distribution, not the continuity of bonds – which is the defining property of a solid.

So is it clear now just how hopelessly obsolete the Euler-Cauchy theory really is?

But I am not done yet. Shortening and stretching must be done by normal forces. Where is the contribution of the shear forces? Do they or don't they contribute to the work done in a deformation?

It turns out that the Euler-Cauchy theory is highly equivocal about this point, but in the end no work is done. The question cannot really be asked because the Euler-Cauchy theory never explains where the forces come from, and how they are spatially arranged. (A modern vector field theory is much more specific here.) It is just *assumed* that the sum of the shear forces is zero, because it must be zero in equilibrium. Thus it seems that the shear work cancels too.

If the stress tensor is written out in a full 3×3 matrix, the diagonal terms are normal components, and the off-diagonal components are then said to be the shear components. BUT a symmetric tensor can always be oriented such that the off-diagonal terms are zero. Hence there are no shear components in this orientation – or did I misunderstand something? And if so, why doesn't anybody dare address these questions?

Let's summarize the last two pages: the Euler-Cauchy theory of stress and deformation always produces a zero result for the work done in a volume-constant elastic deformation; it is unspecific about if and how shear forces contribute to the work done, but probably they don't; it does not make a difference between a gas and a solid because it does not consider the existence of bonds – which complies with the properties of a gas only, but not with those of a solid.

It requires the blind faith of some obscure religious sect to be oblivious to these giant gaps in the Euler-Cauchy theory. Moreover, I tend to believe that the very many colleagues who refuse to get into a theoretical discussion do so – not because they believe the Euler-Cauchy theory, but precisely for the opposite reason: they don't admit it to themselves, but they harbor a distrust against the theory which is so all-pervasive and deep that the Mariana trench looks like a pothole in comparison.

Folks, wake up, grow up, stand up, ask questions. *Blind faith is self-inflicted damage.* Whether my theory is right is yet to be seen, but one thing is plainly obvious: there must be a better way than the current continuum mechanics theory. The flat earth theory at least starts with the assumption that the Earth exists. The first assumption in mechanics of solids is that the solid does not exist. What is your definition of "ridiculous"?

Equilibrium condition $$\frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

Work of internal tensions per unit volume $$\delta R = -\sigma_{ii}\,\delta u_{ii}$$

such that $$\int dR\, dV = \int \frac{\partial \sigma_{ij}}{\partial x_j} du_i\, dV$$

Landau & Lifshitz

Hence $\int dR\, dV = \int 0\, du_i\, dV = 0$ No work is done

This argument is from Landau & Lifshitz. Note that the integrand in the equation in line 3 is zero if the equilibrium condition is to hold.

Thermodynamics

$$dU = - PdV + TdS$$
$$dU = - \sigma_{ij} d\varepsilon_{ij} + TdS$$
$$dU = - PdV - \sigma_{ij} d\varepsilon_{ij} + TdS$$

reversibility condition: $TdS = 0$
volume-neutral deformation: $PdV = 0$

volume-neutral condition: $\sigma_{ii} = 0,\ d\varepsilon_{ii} = 0$

Thus $dU = 0$ No work is done

These are the terms that are commonly used in material science. Textbooks always insist that σ_{ii} and $\varepsilon_{ii} = 0$ for a volume-neutral deformation, but they are clearly not aware of the implications in the context of potential theory: this is the mathematical definition that no work is done.

Page 5: **Conceptual contrasts between Newtonian mechanics and thermodynamics**
(posted 25 March 2011)

When I was in my first graduate year, I had thermodynamics in the morning and stress & deformation in the afternoon. The two just would not match at all, when clearly they should match. Here is the reason: to this day, the theory of elasticity has a conservative structure. **NM** stands for Newtonian mechanics, **TD** for thermodynamic concepts. Irreversible processes are not considered here.

Nature of mass:
NM: the mass is the inertial mass, measured in [kg].
TD: the mass is the thermodynamic mass which is counted in atoms, and dimensionless. (The mol is a number, 6×10^{23}.)

Nature of work:
NM: work is done against inertia. It is acceleration work, the change of E_kin and E_pot the sum of which, the total energy of the system U, is always constant. It is work done **within a system**.
TD: work is done by the surrounding **upon a system** and vice versa such that its energy U_0 changes to U_1. It is commonly known as PdV-work. (This is for isotropic boundary conditions.)
It follows that Newtonian work and thermodynamic work cannot be summed.

Nature of force:
NM: the definition of a force is the equation of motion $\mathbf{f} = m\mathbf{a}$ where m is the inertial mass. It is not possible to define force and mass separately. **f** is a discrete vector, it cannot be derived.
TD: Forces are field forces $\mathbf{f} = dU/dx$, and derived from the electromagnetic potentials U of the atoms in the system such that the magnitude of the potential is proportional to mass (in atoms, not in kg).

Time as a parameter:
NM: Time is an indispensable parameter because work is acceleration work.
TD: Time is not a parameter in thermodynamics, there is no time term in PV = nRT; a reversible thermodynamic process (e.g. elastically stretching a spring) is time-independent.

Path-independence:
NM: work depends only on the starting point and the end point of a path. Newtonian work is path-independent in Euclidean space, or the common geographic 3D-space.
TD: work is path-independent in energy space, or PV-space, of which the common 3D-space is only a subspace. A term whose magnitude only depends on the starting point and the end point of a path, is called a state function.

Purpose of theory:
NM: the intent is to understand the physics of discrete bodies in freespace. A discrete body is a body of mass which can be enveloped by a closed surface S such that A nowhere runs through mass.
TD: designed to understand the physics of mass distributions which may or may not be solidly bonded with their surrounding. Continuity of mass distribution is a precondition, so S must run through mass.

Governing equation:
NM: the equation of motion $\mathbf{f} = m\mathbf{a}$; the total energy of the system is invariant.
TD: the First Law dU = dw + dq and the equation of state PV = nRT.

Equilibrium condition:
NM: two forces balance if they are equal in magnitude and opposite in direction. Disequilibrium reaction: acceleration of the body in some particular direction.
TD: the forces exerted by the system at the surrounding and those exerted by the surrounding at the system balance. Disequilibrium reaction: reconstitution into the lowest energy state.

Energy conservation law:
NM: E_kin + E_pot = U = const.
TD: dU = dw + dq.

That is, NM and TD represent mutually exclusive realms in physics. Mixing them – which is routinely done in material science – produces a hilarious mix of apples and oranges.

Page 6: **Proper relation of Newtonian mechanics to thermodynamics**
(posted 30 March 2011)

Keeping apples and oranges apart sounds trivial, except if people are unaware of the difference. This sounds sassy, but it isn't funny at all. In <www.elastic-plastic.de/koenemann2008-2.pdf> I have published three textbook examples where the First Law of thermodynamics is turned upside-down. Here I give a fourth one: Holzapfel, Nonlinear solid mechanics, Wiley, 2000. (Like all other books known to me, it never mentions bonds.)

On p.164, eqn.4.122 the First Law dU = dw + dq, the *energy conservation law for non-conservative processes*, is implied in integrated form, U = w + q, and then given as time derivative dU/dt = dw/dt + dq/dt. This is not wrong, any thermodynamic process is a historical process, so the time derivative is legitimate. But why, if reversible thermodynamic processes are time-independent, including elasticity?

On p.153, eqn.4.85 the rate of external work is given as the sum of the rate of kinetic energy plus rate of internal work, $dE_{kin}/dt + d(\sigma ii\ \varepsilon_{ii})/dt = dw/dt$. All terms in this equation are derived from conservative, Newtonian concepts, i.e. *the conservative energy conservation law* $E_{kin} + E_{pot}$ = const is implied. (Sorry for the obtuse terminology, these are the terms we must use.)

Then dw/dt of eqn.4.122, a thermodynamic term, is plugged into the conservative eqn.4.85. Literally, the First Law of thermodynamics is thus interpreted as subordinate to the conservative energy conservation law, in effect making the First Law conservative. What's left is an empty name, but the substance, the very nature of the First Law is gone.

Again, note that both σ_{ii} and ε_{ii} (sum over i) are zero for a volume-neutral deformation, thus the work is zero. – Again, why is the First Law above given as a time-derivative? It is an attempt to make the First Law nominally compatible with Newton's equation of motion **f** = m**a** which cannot exist without time dependence, but it is the only force definition known in continuum mechanics. It is an unmistakable tracer to the conservative roots of continuum mechanics, and a reference to E_{kin}. It is characteristic for continuum mechanics that the obvious alternative, f = dU/dx, has never been used (with one single exception known to me, by Helmholtz 1902, but this book is forgotten.)

When I searched for a better relation between $E_{kin} + E_{pot}$ = const and dU = dw + dq, I thought of atoms jetting around in a thermodynamic system, solid or gaseous, observing $E_{kin} + E_{pot}$ = const and Newton's mechanics; their kinetic energy is mv^2, where the velocity v is an average term. I interpreted E_{pot} as an expression that stands for bonds in solids, so E_{pot} became fr where f is a force, and r is a bond length. Their sum is then the total energy U of the system in the unloaded state, say U = PV, such that the conservative energy conservation law turns into

$$mv^2 + fr = PV$$

where the first term LHS is the heat (atomic motions only), and the second term LHS is the bonds. But this law is in effect an equation of state, and accessible to the First Law.

That was in 1986. Nobody liked it. In 2006 I found my law in a standard thermodynamics textbook – it is known as the *virial law* of Rudolf Clausius (<www.elastic-plastic.de/clausius1870.pdf> for Clausius' original paper in English, or look up the law in Wikipedia). Gloria Victoria!

A couple things now fall into place. Whatever work is done by the atoms upon one another, bumping around and following Newton's laws, is all *within the system* and interesting only for the nature of heat. BUT the equation of motion **f** = m**a** and the entire rest of the conservative Newtonian toolbox are now used up. Whenever work is done *upon the system* to cause a change of state, such as an elastic deformation, the cause of the deformation is outside the system, and we must use the First Law.

An one-line intro to my deformation theory is now possible: for practical purposes we can ignore mv^2, we are left with fr = PV or $f_{int}\ r = f_{ext}\ r$ (which is the thermodynamic equilibrium condition in vector form) where f_{int} is a force exerted by the system at the surrounding, f_{ext} is the opposite, and r is a typical distance, be it a bond length or the radius of the thermodynamic system. Furthermore, $f_{int}r = E_{pot}$, hence f can be understood as f = dU/dx, *a force field derived from a potential U* – which can be done only from a potential energy term, not from kinetic energy. Now everything is wide open. We have a system of finite size, which has a radius of unit length in the unloaded state when the forces are zero. If the external forces are non-zero, they do work on the radius by changing its length – an one-dimensional equivalent to PdV-work. It all pans out.

Page 7: **Classification of physical processes**
(posted 2 April 2011)

I received some private emails showing that there are questions on the terminology. In particular, the terms "reversible" and "conservative" are taken as largely identical. This confusion is at the core of all the problems in continuum mechanics, there is now a long tradition to make this mistake in good faith. That good faith needs to be broken. Some Wikipedia wisdom is worked in below.

A conservative system is an *isolated system* which does not exchange mass or energy with a surrounding. Any processes that can be studied under this condition must necessarily take place *within the system*. It is therefore a characteristic property of a conservative system that its mass and energy are proportional. *A conservative process does not change the energy of a system.*

A conservation law states that a particular measurable property of an isolated physical system does not change as the system evolves. These are in particular (for the context here): (1) Conservation of mass-energy, (2) Conservation of linear momentum, (3) Conservation of angular momentum, (4) Conservation of probability density.

Items 1-3 are applicable to a kinetic system of n discrete bodies, e.g. a natural gas, or Newton's mechanics of discrete bodies in freespace. Classical examples of a conservative process are the revolution of planets about the sun, or the motion of molecules in a drop of water at rest. The conservative energy conservation law is E_kin + E_pot = const.

Item 4 implies that the system is in its highest entropy state, i.e. it is in equilibrium with itself. This item leads into statistical mechanics, which is interesting for diffusion and dissipation.

All of this is good to know, but irrelevant for elastic deformation. The latter implies interaction of system and surrounding, hand and coffee cup, such that *work is done upon a system*, and the energy of the system changes. The simplest elastic deformation is therefore the volume change of an ideal gas. Hence we are in the field of thermodynamics. Its subject is not the interior of the system, but the interaction of system and surrounding. The law that takes account of these fluxes is the First Law of thermodynamics, dU = dw + dq. Such a process is therefore *non-conservative*.

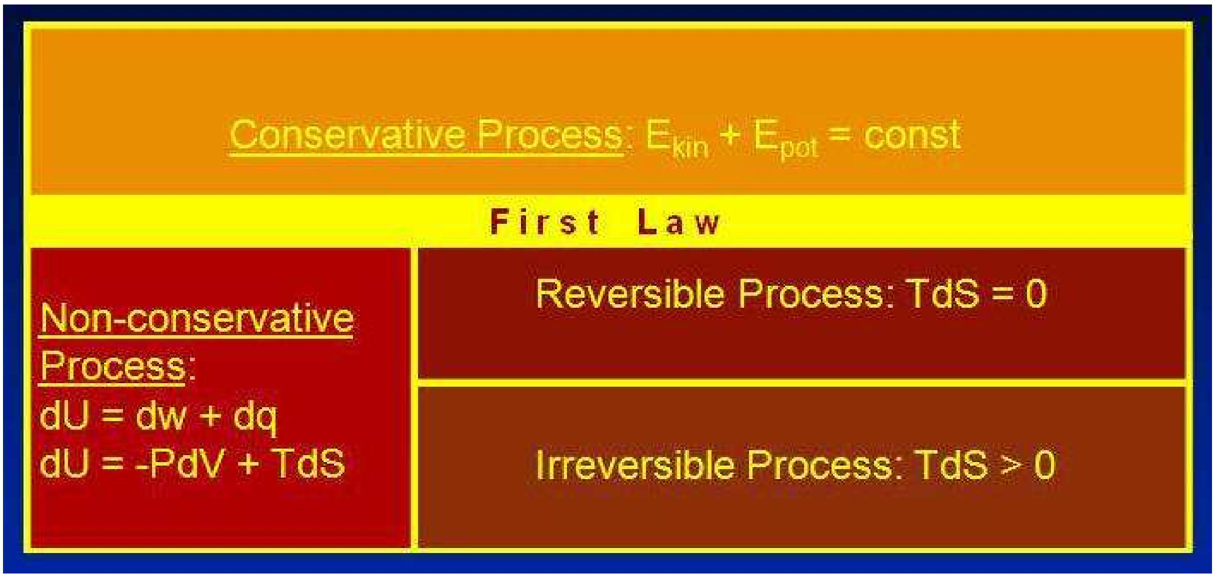
Thermodynamics cannot say anything about processes in the system's interior, it only considers fluxes across the system-surrounding interface. Thermodynamics does not even know atoms, its properties are average properties. The continuity assumption in thermodynamics is that *the mass distribution is twice differentiable*. This condition must break down at the atomic scale.

The conservative energy conservation law E_kin + E_pot = const and the non-conservative energy conservation law, or energy flux account law dU = dw + dq cover different subjects, and they have nothing in common. I have shown yesterday how they are properly related – through Clausius' virial law, mv^2 + rf = PV. *The attempt to plug one into the other is arguably the silliest mistake one can make in classical physics.* It is routinely done in continuum mechanics, hence it is wrong.

There is confusion in the literature regarding circular paths. A process is said to be conservative if a body can describe a circular path in space without gaining or loosing energy. This path is meant to take place in *freespace*! An elastic loading and unloading is not a circular path in this sense because it takes place in *PV-space* – in the isotropic case (volume change only; for anisotropic loading there has not been a theory so far, this is mine, to be explained soon; but now you can see where all this is going to). If a system can be loaded and unloaded such that no energy has been dissipated, the process is said to be *reversible*. A dissipative process is *irreversible*, it produces entropy.

Summary:
If a process is *conservative*, its energy conservation law is E_kin + E_pot = const.
If a process is *non-conservative*, its energy flux conservation law is dU = dw + dq.
If a non-conservative process takes place without entropy production it is *reversible*.
If a non-conservative process produces entropy, it is *irreversible*.

The conservative-non-conservative contrast is the most fundamental categorical difference in classical physics.



Classification of physical processes. The distinction of conservative and nonconservative processes is the sharpest categorical difference in classical physics.

Page 8: **Why the stress theory is wrong, and why this matters so much**
(posted 5 April 2011)

"There are few parts of mechanics in which theory has differed more from experiment than in the theory of elasticity." This is the opening sentence of Maxwell 1850. The reason, from today's point of view: elasticity is not a part of mechanics, try thermodynamics.

This page has three purposes: (1) it shows why the Cauchy theory is wrong; (2) it shows that P = f/A and P = U/V are not identical terms; (3) it offers a great avenue to new solutions. *The chapter "Refutation of Cauchy stress" from <www.elastic-plastic.de/koenemann2008-2.pdf> is shown below because the full mathematical argument matters.* It is simple enough to be understood by everybody. The Gauss divergence theorem is applied to a volume element within a solid. Poisson 1813 used this very same argument to show that the gravitational force at the center of the Earth is zero.

Why does this argument prove Cauchy wrong? He assumed that the ratio f/A is scale-independent. Therefore he assumed that f/A reaches a finite value as A vanishes. However, this is true only on *free planes* which divide the universe into left & right. The divergence theorem instead considers a *closed surface* which distinguishes inside and outside. Thus f/A must reach infinity as the volume vanishes.

Three cases are possible: (1) A is a free plane. Then f/A is scale-independent. (2) A part ΔA of a closed surface A → 0. f/A is scale-independent because V is not itself subject to the limit operation. (3) A is the surface of a volume V. In that case V → 0, thus f/A → infinity. This proof is *necessary and sufficient*. Beyond that, if V is the volume of a thermodynamic system, and U is its internal energy, then U and V are extensive parameters. P = U/V is intensive, and scale-independent. Therefore P = f/A and P = U/V differ in their mathematical properties. We can use U/V, but not f/A.

The argument shows that within masses, the force exerted by the mass and its radius are proportional,

f/r = const.

If r vanishes, so does f; if r = 0, f = 0 (cf. Poisson's result above.) This relation within masses is the flip side of the relation $f = 1/r^2$ in freespace, both are derived the same way. Both are among the most fundamental relations in physics, each in its own realm. Since f/r is scale-independent, we can build a new theory on this; f/r can be taken geometrically as the one-dimensional equivalent of the spatial U/V.

The distance term r has tremendous significance in potential theory. It is the *zero potential distance*, required to define work. It can be infinite or finite, but it cannot be zero: the distance from infinity to the sun, or from a Lagrange point to either Earth or Sun, the half-distance between two capacitors, the length of a spring, – hasn't anybody noticed that Cauchy let Hooke's spring vanish? – or, in thermodynamics, the radius of the thermodynamic system. It has unit magnitude in the unloaded state, and changing it at constant mass costs work. V_0 in scalar thermodynamics serves the same purpose.

The fact that Cauchy's V must reach zero for the stress tensor to exist, makes it incompatible with *Euclidean vector space rules, thermodynamics, and potential theory.* The thermodynamic system is finite and cannot vanish. Thermodynamics distinguishes inside and outside, the stress theory does not and cannot. The EOS for a gas is PV = nRT; if V → 0, n → 0 such that V/n = const, and in the end 0 = 0. This jives with the properties of f/r = const, but not with Cauchy's argument. The missing zero potential distance is so badly missing that it had to be reinvented, and fudged back into the theory. That's the purpose of FEM; without Cauchy's error we could use Fourier series methods instead.

In his derivation of the stress tensor, Cauchy 1827 assumed a volume element V upon which forces act from outside. *He never mentions forces acting from inside.* The condition that forces act on both sides of a plane is fulfilled only in the moment when upper and lower plane merge, when V vanishes identically while its surfaces are still there – like the cat that vanished, excepting its grin. The shape of V must change during the continuity approach, disregarding equilibrium conditions. He took the lateral faces as "terms of higher order", and ignored them. No undergraduate would get away with this today.

"Proof of existence" in the mathematical language is nothing metaphysical, but simply the proof that a postulate or theorem is in accord with common logic. If a volume element exerting forces is let vanish identically (= reach zero) and forces are still left, that's a contradiction with reality, and this is exactly the core of Cauchy's stress theory. It is therefore correct to say that Cauchy's tensor does not exist. This has provoked unpleasant reactions, but *no one ever claimed that it is wrong or otherwise wanting*, and no one will. Therefore: it is simply professional, and intellectually honest to accept proof of an error if it is presented. Anything else is self-deception, denial or worse.

From: Falk H. Koenemann, On the systematics of energetic terms in continuum mechanics, and a note on Gibbs (1877). Int. J. modern Physics B, **22**, 4863-4876 (2008)

Refutation of Cauchy stress

Above (eqn.09-11) it is shown that Landau's tensor is insufficient. The refutation of Cauchy's tensor is repeated here from Koenemann (2001a). Consider a system of distributed mass within a larger volume of distributed mass. For simplicity, the system is assumed to be sperical, and the distributed mass may be bonded as in a solid, or unbonded as in a gas. Its pressure is given by $P = \Delta U/\Delta V$ which is an explicit statement of the proportionality of mass and energy in some given state. Both the externally applied forces and the forces exerted by the system upon the surrounding form radial force fields, one directed inward, one outward, such that at every point on the system surface A the equilibrium condition is $\mathbf{f}_{syst} + \mathbf{f}_{surr} = 0$, which translates into the thermodynamic equilibrium $P_{syst} + P_{surr} = 0$. Since the system contains mass, and since it interacts with the surrounding through exchange of work, it acts as a source of forces. An existence theorem in potential theory requires that if there is a function f of a point Q such that

$$\int f(Q)dV = \kappa, \qquad \text{(eqn.12)}$$

both sides must vanish simultaneously with the maxium chord of V if $V \to 0$ (Kellogg 1929:45). The relation can thus be represented by the Gauss divergence theorem,

$$\int \mathbf{f} \cdot \mathbf{n}\, dA = \int \nabla \cdot \mathbf{f}\, dV = \kappa \qquad \text{(eqn.13)}$$

where $\mathbf{f}$ is either one of the forces mentioned above, $\nabla\cdot\mathbf{f} = \varphi$ is the source density or charge density which is a constant that characterizes the state in which the system is, and $\kappa = \varphi V$ is the charge which is known to be proportional to mass in a given state. Thus in eqn.13, LHS $\propto \kappa \propto V$. Since $V \propto r^3$ where $r = |\mathbf{r}|$ is the radius of the system, but $A \propto r^2$, for LHS $\propto V$ to hold it follows that

$$\frac{|\mathbf{f}|}{|\mathbf{r}|} = const \qquad \text{(eqn.14)}$$

This result is known since Poisson derived it in 1813 (Kellogg 1929:156). Thus if $V \to 0$, both f and r vanish such that eqn.12 is observed. However, as $V \to 0$, $\Delta U/\Delta V \to const$, but

$$|\mathbf{f}|/A \to \infty. \qquad \text{(eqn.15)}$$

The limit does not exist. The argument is necessary and sufficient proof that the Cauchy stress tensor does not exist.

Thus, the two definitions of pressure known to us, Newton's $P = |\mathbf{f}|/A$ and the thermodynamic $P = \Delta U/\Delta V$, are not equivalent, and only the latter can be used here. The continuity approach in Cauchy's theory of stress is based on the assumption that $|\mathbf{f}|/A \to const$ as $V \to 0$. This is not the case. Newton's definition only applies to free surfaces A, or to sections of closed surfaces at constant V, but not if A is closed and V is a variable. When Cauchy (1821) worked out his theory, potential theory was still in its infancy, and the importance to distinguish system and surrounding was not understood yet (it is in fact missing entirely from continuum mechanics literature up to this day, cf. Koenemann 2001b). Thus he used Newton's third law as equilibrium condition – as did Euler – whereas the correct equilibrium condition is that of thermodynamics, of system vs. surrounding. Cauchy believed that in his continuity approach, P is independent of $|\mathbf{r}|$. He did not consider that $|\mathbf{r}|$ is a measure of the scale of the system, and mathematically related to mass, which is a variable in his limit operation. Today $\mathbf{r}$ must be equated with the zero potential distance [Kellogg 1929:63] which may be infinite or finite, but it cannot be zero. In thermodynamics, $\mathbf{r}$ is the radius of the thermodynamic system, and thus finite, like n and V in $PV = nRT$. Cauchy's continuity approach is understood to be the proof of existence of the stress tensor. His reasoning violates the condition in eqn.12. (Batchelor [1967:9f] noted the difference in behavior of volume terms and surface terms as $V \to 0$, but took the scale-independence of $|\mathbf{f}|/A$ for granted.)

Page 9: **Strain is not a physically meaningful term**
(posted 7 April 2011)

The all-important question in deformation studies – up to now – is the relation of stress to strain. Enough has been said about stress; it is energetically empty, and it violates the rules for vector spaces – planes passing through the origin of a coordinate system cannot be assigned a notation in Euclidean vector space. Since Cauchy concentrated on these, his stress theory can only consider the chosen point Q of interest, but not any other point in space.

Promptly Cauchy produced contradictions when he had to consider two points in space, say, the two end points of Hooke's spring, to consider the effect of stress. This distance in real space made him develop thoughts that are indeed compatible with vector space rules (which – not to forget – became known in systematic form only 30 years later, Cauchy's views were still intuitive). Thus, from today's point of view, Cauchy's theory of stress and his theory of strain are incompatible with one another. Strain is indeed geometrically well-defined. The question is rather: is strain physically relevant? This may sound absurd to some, but it has never been asked. *How do we know that strain is meaningful?*

Cauchy did not understand the significance of physical boundary conditions, he never considered them. His ideas are incompatible with the conditions of simple shear. (Promptly, it is simple shear for which current continuum mechanics fails systematically.) Today we distinguish strain and displacement, but these terms are much younger. Cauchy used 'condensation & dilatation' which he meant to be a displacement, then ended up as strain, for two reasons: he was seduced by the rectangular relation of plane and plane orientation vector to consider only non-diagnostic situations with orthogonal eigendirections, not lesser symmetry states – plus, he knew beforehand where he wanted to end up: in ellipsoids. Cauchy developed the concept of principal axes. This was a first, and insufficient, attempt to come to grips with a mathematical phenomenon which only Grassmann 1860 would fully explore: eigendirections. But these may or may not be mutually orthogonal (they can even be parallel). Cauchy's mathematical work in this matter is obsolete since we have linear algebra.

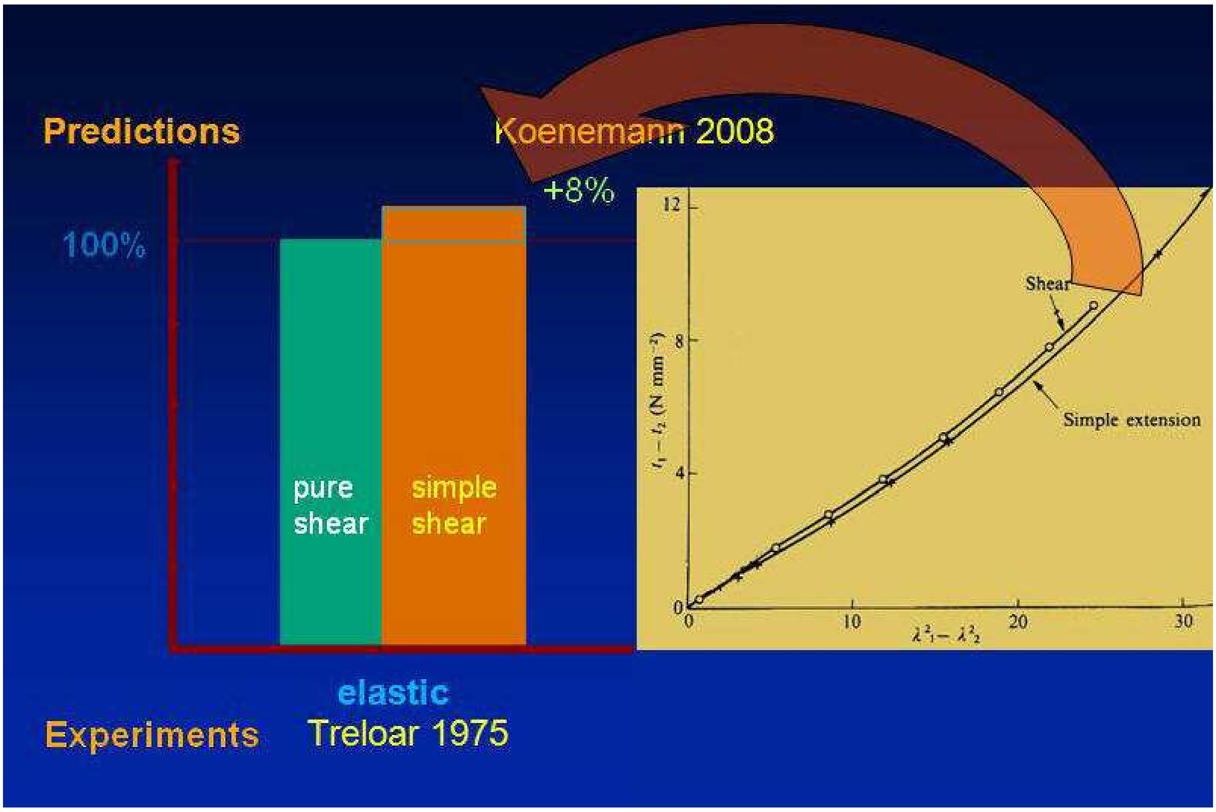
In reading Cauchy's original papers <www.elastic-plastic.de/Cauchy_draft_100301.pdf> I discovered what really drove him. *It was not physics.* He was fascinated by elliptical-hyperbolic surfaces, and "knew" without a trace of doubt before he started that these are the key to understanding deformation. For his mathematical mind they were the juiciest bait, he mentioned them already in a short note in 1821, long before his ca.20 papers on deformation 1827-1829. By handling ellipsoids he deeply impressed his contemporaries, and led generations down the garden path.

Is strain really a physically relevant term? Or is it displacement? Since elastic deformation is a reversible, i.e. a thermodynamic process, the better choice of words is: *Is strain a state function?*

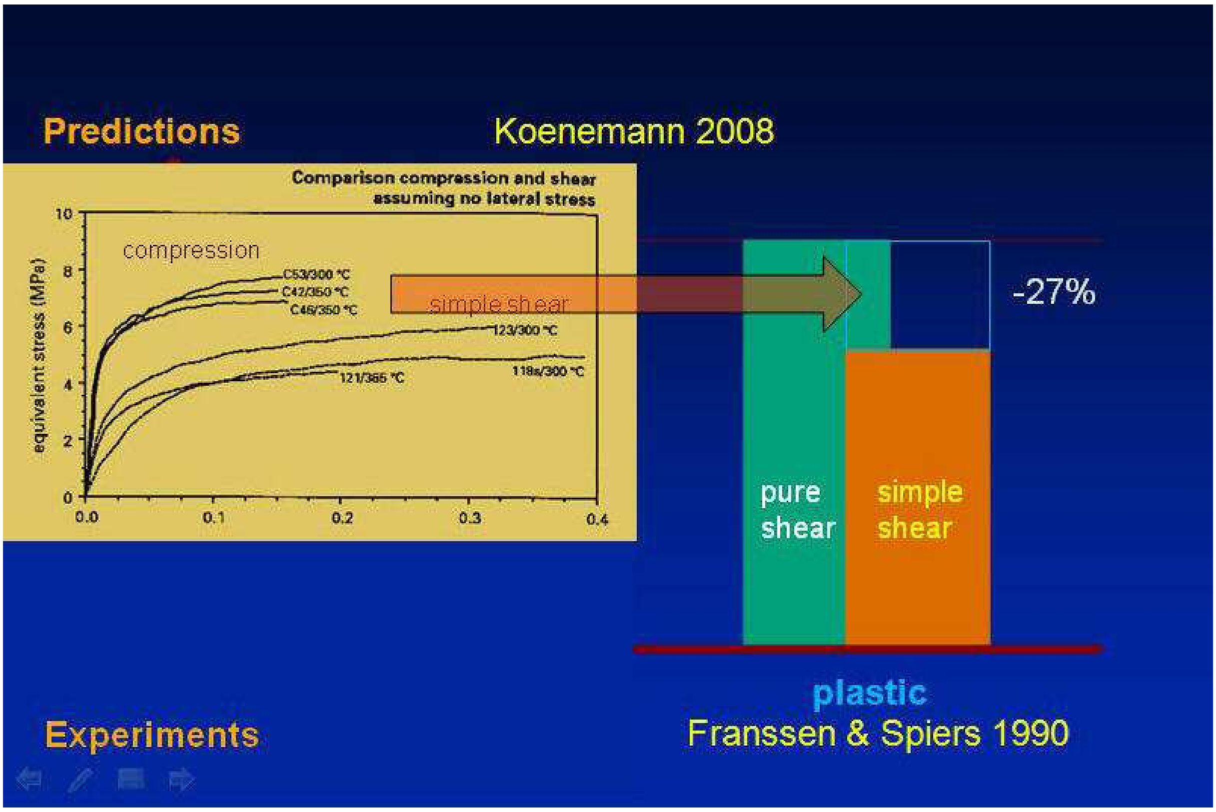
State functions were recognized as a fundamental term only in 1850-1870 (Clausius, Gibbs). They are path-independent. Process variables, such as work or heat, are path-dependent, unless one of them is zero. Excluding heat as a variable ($dq = 0$) means that the process in question requires maximum work; this is the case for a reversible process, and $dw = dU$. If strain were a state function, all identical states of strain should cost the same amount of work. However, the experiments show clearly that a given strain achieved through a plane pure shear and a plane simple shear cost different amounts of work – elastic simple shear always costs ca.10% more than pure shear. Therefore *strain cannot be a state function, whereas displacement is one* <www.elastic-plastic.de/energetics_draft_110102.pdf, and the last figure in <www.elastic-plastic.de/experimentaldata.pdf>.

This experimental fact alone will put the physical understanding of deformation on entirely new feet: we have been barking up the wrong tree all the time! But the question can still be sharpened: do we need a strain theory? What we want is a proper cause-effect relation. Thermodynamics provides one: the EOS gives the material properties, and the pressure difference ΔP is the cause. The work equation PdV delivers the result, ΔV, such that – addressing a reader's question to an earlier page – *the sign of the effect is determined by that of the cause*; but we do not need an extra theory. From the paragraph above we only know that ΔV must be the displacement and not the strain. Elasticity should work in the same way. How this is done, that is: how the scalar theory of thermodynamics (P, V, T) is transformed into a vector field theory (f, r, T), will be shown next time.

Strain is easy to measure, but it contains insufficient information. Didn't we know this all along?



Prediction (left) and experimental observation (right) for the difference in the work done in elastic pure and simple shear deformation. Simple shear is predicted and observed to require ca. 8-10% more work than pure shear. This is experimental proof that strain e cannot be a state function.



Prediction (right) and experimental observation (left) for the difference in the work done in plastic pure and simple shear deformation. Simple shear is predicted to require ca. 30% less work than pure shear. Observed are -18% in cold copper (not shown) and –40% in hot salt (Franssen & Spiers).

Page 10: **Outline for a new understanding**
(posted 8 April 2011)

"If you make only one step out of the familiar coordinate system the world looks entirely different, and often quite frightening" (Douglas Adams, A Hitchhiker's Guide to the Galaxy). But then – it just takes some getting-used-to to feel better again. So let's do that.

1. *We need a geometrical concept that can handle force distributions.* Linear algebra does that. The fundamental equation **Ax** = **b** assigns a vector **b** to any point P whose position vector relative to a point of interest Q is **x**. **A** is a matrix, the field property tensor. For **x** = **0**, **b** = **0**. **x** and –**x** are different vectors, and **x** – **x** = **0** is a meaningful operation, hence we are in Euclidean space. The equation **Ax** = **b** is the mathematical definition of a vector field.
2. *Newton's mechanical principles are observed,* above all the lever is resurrected. Newton let forces act on the surface of a discrete body of solid, its shape determines its macroscopic mechanical properties. The concept is adopted in the sense that a thermodynamic system within an infinitely extended solid is assumed; its shape represents the mechanical properties of the material. A spherical system represents an isotropic material.
3. *Euler's planes are abandoned.* Newton's definition of normal force and shear force (rotating force) relative to the radius vector **r** = **x** is adopted.
4. *Elasticity is by nature a part of thermodynamics.* The task is to transform the common theory of thermodynamics which is written in scalars (P, V, T) into a vector field theory while leaving its basic properties and its structure untouched. The thermodynamic equilibrium condition is in scalar form $P_{syst} + P_{surr} = 0$. In vector form it is $\mathbf{f}_{syst} + \mathbf{f}_{surr} = 0$.
5. *Two coordinate sets are required.* The set X_i defines the geographic space in which the point of interest Q is located. The position of the thermodynamic system in space is determined by its center of mass at Q. Q is then taken as the origin of a local coordinate set x_i, and the surface points P of the system are then given by **x** relative to Q.
6. *Two force vector fields are defined.* The internal potential must be demonstrated to exist; it is equated with the internal energy or the Helmholtz free energy U, hence $\mathbf{f}_{syst} = \mathbf{e}_i\, dU/dx_i$. This is the physical definition of a vector field. The external potential can be postulated, hence **f**_surr = $\mathbf{e}_i\, dU_{surr}/dx_i$. **A** translates into $d^2U/dx_i\, dx_j$, to be integrated over the length of **x** = QP, which yields **b** = **f**. **A** for **f**_syst defines the material properties. **A** for **f**_surr can be freely defined.
7. *Bonds are expressed as internal pressure.* The internal pressure $(dU/dV)_T$ is defined as the pressure that would be observed in a mol of gas if it is compressed to the molar volume of the solid. This pressure is internally balanced by the bonds; the solid is therefore in equilibrium with itself in a vacuum. In solids this pressure is in the order of a few kbar.
8. *Bonds ascertain equilibrium.* The two fields $\mathbf{f}_{syst}$ and $\mathbf{f}_{surr}$ may be incompatible with one another. Also, tensional forces and shear forces can do work upon the system only if bonds exist. It is assumed that equilibrium is always ascertained by constraint forces to the effect that system and surrounding are solidly bonded to one another. Disequilibrium is therefore impossible. (Constraint forces are forces which make a certain situation possible, but they do not do work. Example: if a sphere of mass m is accelerated to roll horizontally on a table, the table exerts a constraint force upon the sphere to balance the earth acceleration downwards.)
9. *An equation of state for solids is needed.* PV = nRT is valid for a gas. $P^kV = z$ is an universal EOS for a solid, where k = $(\ln V_{solid})/(\ln V_{gas})$. k is the Grüneisen exponent and relates the internal pressure (#7) to the atmospheric pressure outside. (The universality of this EOS is demonstrated for the alkali elements, using experimental data by Bridgman; Approach, Fig.1.) For modelling purposes k is set to be unity, and PV = const applies.
10. *We need a work equation in vector form.* The thermodynamic work equation is PdV. In vector form it is interpreted as fdr, such that each external force $\mathbf{f}_{surr}$ interacts with the position vector **r** of its point of action P by shortening or stretching it. The work done upon the volume is then found by integrating fdr over all directions in space around Q.
11. *Shear forces and normal forces both do work.* It is postulated that Joule terms are isotropic, such that work cannot have anisotropic properties, because work is always done upon a volume. The condition PV = const is thus interpreted as $|\mathbf{f}||\mathbf{r}|$ = fr = const which must be invariant in all directions. This condition is needed to find a constraint for the relative spatial magnitudes; there cannot be a direction in which no work is done.
12. *Scale independence.* U/V is scale-independent, so is f/r. The model can be normalized either to V = 1 or to r = 1, for the purposes here the latter option is preferred.
13. *The effective force field:* $\mathbf{f}_{syst}$ and $\mathbf{f}_{surr}$ interact with one another. Using the constraint condition (#8) they merge to form a third force vector field $\mathbf{f}_{eff}$ which is the "cause". Applying the work

equation fdr to $\mathbf{f}_{eff}$ yields the "effect", the displacement field (also a vector field), they have identical eigendirections.

14. *Elastic deformation is a Poisson problem.* That is, there are energetic net fluxes into the system such that an elastic potential dU builds up. This is done by partitioning $\mathbf{f}_{eff}$ in two: (a) an inward-directed average operative field $\mathbf{f}_{op}$ which is isotropic, and which causes a volume contraction; and the deviatoric field $\mathbf{f}_{dev}$ the average of which is zero. The deviatoric field is then partitioned again into a normal force component field $\mathbf{f}_{n}$, and a shear force component field $\mathbf{f}_{s}$. The energetic and volumetric net effect of the normal field is zero. Because of $\mathbf{f}_{op}$ the Poisson condition div $\mathbf{f} \neq 0$ is observed. (This does not include the work done by shear forces because the divergence theorem considers only normal forces.)
15. *The work done by shear forces has only one sign.* It follows from the geometric relations between $\mathbf{f}_{s}$ and $\mathbf{r}$ that a shear force always has a dilating effect.

Thus, all work done in an elastic deformation has a volume-changing effect. The total work done on the volume is then found by integrating the fdr-work over all directions. It turns out that for an isotropic material subjected to pure shear conditions the volume changes by the various components sum to zero, i.e. the deformation is indeed volume-neutral. *But in contrast to the Euler-Cauchy theory, this is now a prediction and not a precondition.* With work it is different: it does not sum up to zero (as in the Euler-Cauchy theory), net work is done in an elastic deformation.

This is the general outline. The rest I must leave to the serious reader. Predictions come next time.

For isotropic boundary conditions this vector field theory (in f, r, T) is identical to thermodynamics in the common scalar form (in P, V, T). This theory provides a regular differential approach (which the Euler-Cauchy approach does not), just as in thermodynamics: once the boundary conditions are defined, the force field and finite displacement field can be calculated by integration. The strain can then be readily calculated if desired. No more fuddling with infinitesimal strain and finite strain!

The theory has been meticulously examined twice without my knowledge, and approved. A gas dynamics professor from the Academy of Sciences at Gdansk was asked in 1995 by a colleague to develop an opinion. He invited me in 1997, and returned my scripts saying, "I have read everything, and checked and accepted every conceptual thought. I have not checked the correctness of the numerical calculations, this is your job." – In 2003 a student at the University of Tehran discovered my home page. Another student was then assigned the task to translate my scripts into Farsi, and to check everything mathematically and physically for her bachelor thesis. She must have worked on it for a few months, and found one sign error in the text, in the calculations it was correct.

The links to the offprint pdf's are:

- Critique of the Euler-Cauchy theory: <www.elastic-plastic.de/koenemann2008-2.pdf>
- The new approach: <www.elastic-plastic.de/koenemann2008-1.pdf>

Page 11: **Predictions**
(posted 9 April 2011)

The new theory offers a fair number of **predictions** which can be compared with **observations**.

1. In continuum mechanics volume constancy is a precondition ("incompressible flow" in fluid dynamics). In this theory, no assumptions are made, but volume constancy is a prediction for an isotropic material subjected to pure shear.
2. For elastic simple shear a volume expansion is predicted. It is observed, and known since the experiments of Poynting 1909.
3. Elastic simple shear is predicted to require ca.10% more work per chosen strain than pure shear. This is also observed (experiments with rubber). Prediction and observation match perfectly.
4. Plastic simple shear is predicted to require 30% less work per chosen strain than pure shear. Experiments are between –18% and –40%. This result – both experiment and prediction – indicates that the reason for shear concentration in mylonites is the energetics. It is an error to assume that all strains cost the same amount of work (which should be the case if strain were a state function); simple shear is preferred by the law of least work in the plastic field to such a degree that other deformation types are impossible at the scale at which the deformation mechanism works. A pure shear therefore consists of two conjugate simple shear systems. This implies that the deformation must be heterogeneous by nature at the scale at which the deformation mechanism works. The assumption of a perfectly homogeneous deformation state, which is at the core of the theory of plasticity, cannot be right. It is not at all helpful to think in terms of strain.
5. Homogeneous elastic deformation requires an infinite bonded continuum, is at a maximum in the interior of the body and reaches a minimum along free faces. This effect is also observed.
6. Simple shear is predicted to have a contracting eigendirection at ca.110° to the shear direction (foliation plane in the direction of shear). This direction is identical to (a) the orientation of the measured max compressive direction along the San Andreas fault, (b) the joints in deactivated metamorphic shear zones which opened during uplift, (c) the fabric-dividing line in porphyroclast studies in mylonites (sigma-delta clasts), (d) standing waves in water subjected to simple shear.
7. Simple shear is predicted to have an extending eigendirection at ca.11° above the foliation plane. This is identical to the direction of S-planes in SC-fabric, and porphyroclasts align in this direction <www.elastic-plastic.de/sc_draft_091103.pdf>. I concede that a thorough discussion of the details of SC is badly overdue, there is much confusion in the literature. Contrary to Berthé et al 1979 I do not believe that the S-plane is parallel to the macroscopic foliation, my field experience is different. The 11° direction is also found in LPO diagrams of simple shear tectonites in minerals with only one glide plane (mica, ice).
8. Simple shear is predicted to have a max shear direction at ca.28° below the foliation plane; it should shear synthetically, but simultaneously rotate antithetically, and in addition the shear plane should stretch parallel to its own extent during flow. This is in full accord with the properties of C-planes in SC-fabric as they are known to me. (N.B. I did <u>not</u> build my expectations into my theory; this result is strictly derived.)
9. Perfectly spherical porphyroclasts (the garnets in Vermont?) may rotate during plastic simple shear, but clast rolling is expected to be impossible from an axial ratio of a/b = 1.15 on, whence they should align with the 11° direction. This could be tested experimentally.
10. The most fascinating prediction is a bifurcation at the reversible-irreversible transition which controls the orientation of irreversible structures. The geometrical properties of the bifurcation match closely with the orientation of joints in mylonites, and with the geometric properties at the initiation of turbulence in fluid flow. This instability offers itself as the cause of turbulence in fluid flow, and for the generation of sheath folds.
11. The theory provides a material-independent EOS for solids. This is verified for the alkali elements. In practice I expect the EOS to vary with bonding types, i.e. the alkalies and the silicates should differ. But here I have no data.

The most important general conclusion is: *the eigendirections of the displacement field are fabric-forming*. All the evidence points in the direction that the predicted properties of the loaded state (the eigendirections of the effective force field/displacement field) are reflected in the physical reality of the rock structure.

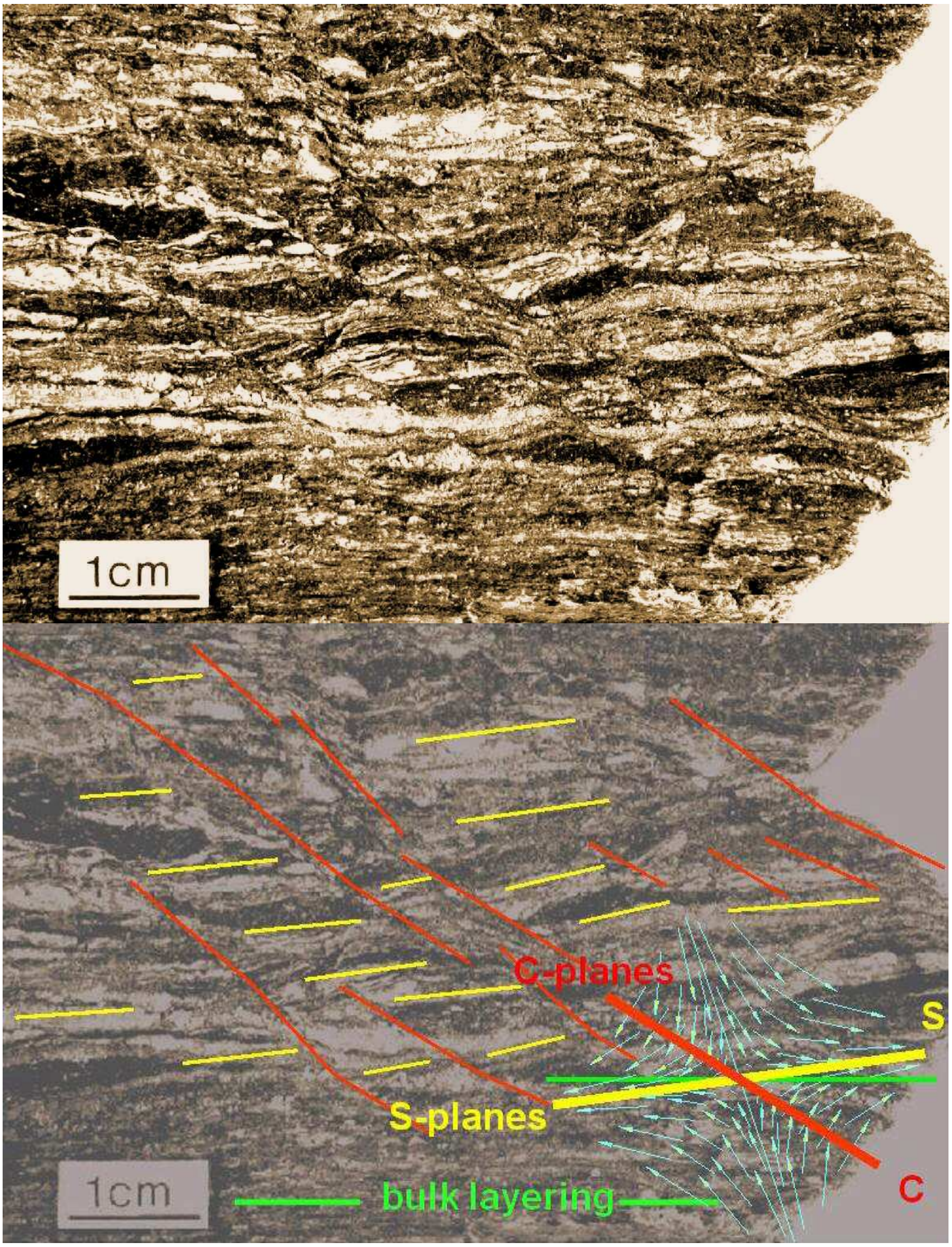
I am not aware of conflicts between theory and reality. (There is one curious paradox.)

I consider the conceptual flaws in the Euler-Cauchy theory so grave that its predictions are irrelevant, even if they look very good.

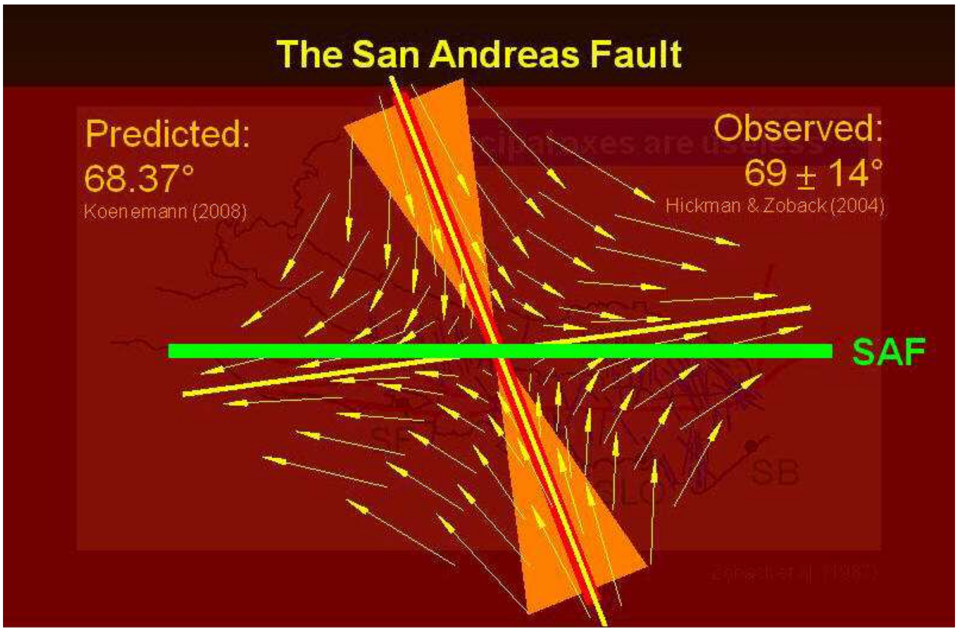
Some readers expressed their disagreement with my views in general terms early in this discussion. I would be glad if they formulate their critique in more concrete form.

Would anyone still claim that my work has nothing to do with structural geology? Or wouldn't you rather have preferred to know about this in 1993?

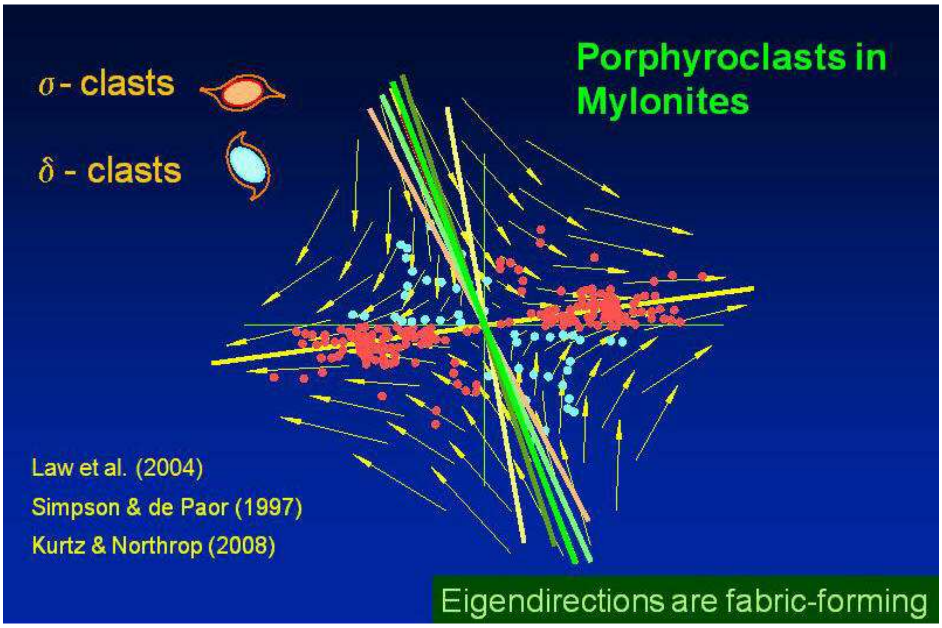
Falk H. Koenemann



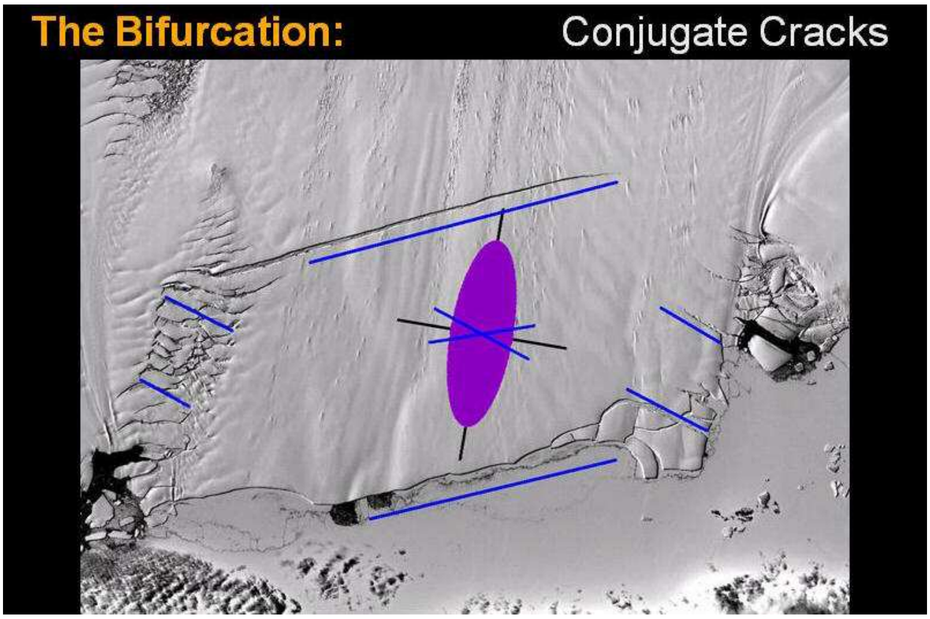
Dextral S-C fabric in meta-igneous gneiss, Insubric Line, Western Alps. Lower panel: the diagram in the lower right corner is the prediction, the yellow and red lines over the photograph accentuate the natural features.



Red and orange: observed max stress orientation along the San Andreas with error range. Yellow: predicted flow field for simple shear, contracting eigendirection coincides with observations.



Yellow: predicted flow field for simple shear, red and blue dots: porphyroclast orientations from Law et al, the σ-clasts align with the predicted extending eigendirection. Observed σ-δ dividing lines from Law et al, Simpson & de Paor and Kurtz & Northrop (various green lines) coincide with the predicted contracting eigendirection (covered by the green lines).



A glacier in the Antarctic. The blue lines are predicted orientations for joints, they are all exactly parallel to the cracks in the glacier.