Electrical Circuits II (ECE233b)

Variable-Frequency Network Performance (Part 3)

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Scaling

Often the values of circuit parameters vary by orders of magnitudes

For example:	Resistors	→	units	MΩ	→ 10^6
	Capacitors	→	units	pF	→ 10^{-12}
	Inductors	→	units	nH	→ 10^{-9}
	Time	→	units	ns	→ 10^{-9}
	Frequency	→	units	GHz	→ 10^9

Since computers are finite precision machines, scaling circuit parameters can result in numerically more accurate results.

In addition, scaling can sometimes make the results more presentable.

There are two ways to scale circuit parameters

Can scale: Magnitude (or Impedance) scaling
Frequency scaling

Magnitude Scaling

Magnitude Scaling →

$$R' \rightarrow K_M R$$

$$L' \rightarrow K_M L$$

$$C' \rightarrow \frac{C}{K_M}$$

Note magnitude scaling does not affect the frequency response

Lets verify for RLC series circuit

$$\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_M L\left(\frac{C}{K_M}\right)}} = \omega_0$$

$$Q' = \frac{\omega_0' L'}{R'} = \frac{\omega_0 K_M L}{K_M R} = Q$$

Frequency Scaling

Note resistors are frequency independent and are unaffected by this scaling

The new inductor L' and capacitor C' values must have the same impedance at the scale frequency ω_0' as the original circuit and must satisfy:

$$j\omega L = j\omega' L' \qquad \frac{1}{j\omega C} = \frac{1}{j\omega' C'}$$

where $\omega' = K_F \omega$ and K_F is the frequency scaling factor

Frequency Scaling

$$R' \rightarrow R$$

$$L' \rightarrow \frac{L}{K_F}$$

$$C' \rightarrow \frac{C}{K_F}$$

Frequency independent

Frequency Scaling

Note that frequency scaling affects the resonant frequency and bandwidth but not the Q.

Lets verify for RLC series circuit

$$\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\left(\frac{L}{K_F}\right)\left(\frac{C}{K_F}\right)}} = K_F\omega_0$$

$$Q' = \frac{\omega_0' L'}{R'} = \frac{K_M \omega_0 \left(L/K_M\right)}{R} = Q$$

$$BW' = \frac{\omega_0'}{Q'} = K_F(BW)$$

Example 14

An RLC network has the following parameters values: R = 10Ω, L=1 H and C=2F. Determine the values of the circuit elements if the circuit is magnitude scaled by a factor of 100 and frequency scaled by factor of 10 000.

Filter Networks

Two types: PASSIVE and ACTIVE circuits

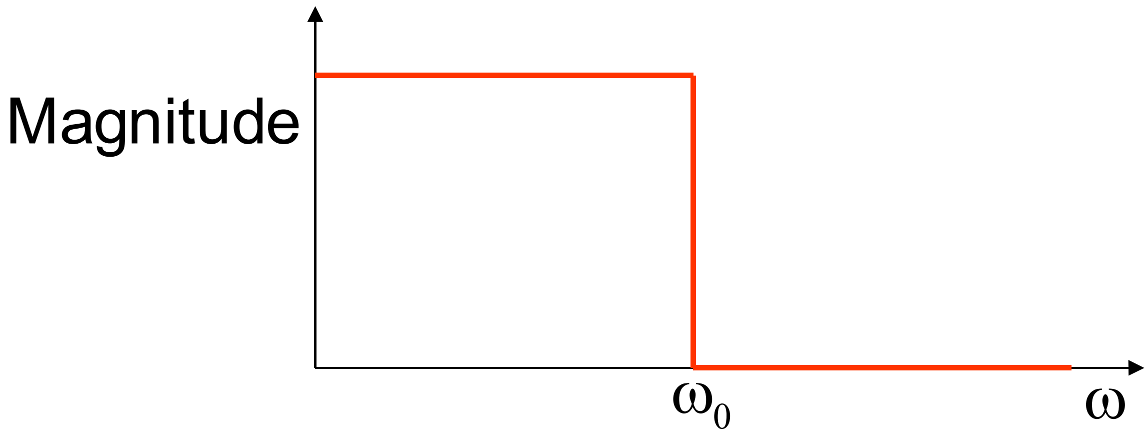
Passive Filters – circuits composed of passive RLC elements

Types of Filters

1. Low pass filters: allows low frequencies to pass and rejects high frequencies
2. High pass filters: allows high frequencies to pass and rejects low frequencies
3. Band pass filters: allows some particular band of frequencies to pass and rejects all frequencies outside the range
4. Band reject filters: rejects some particular band of frequencies and allows all other frequencies to pass

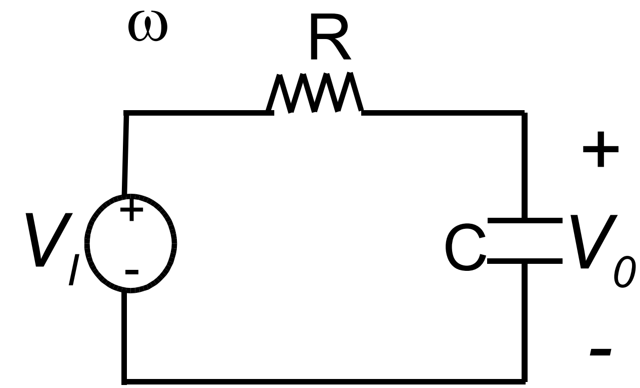
Low Pass Filters

Ideal Characteristic of Low Pass Filter

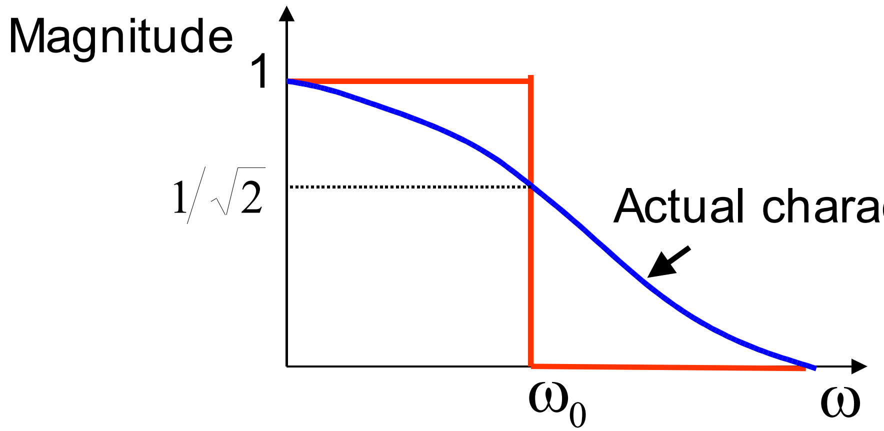


Simple Low Pass Filter

$$G_V(s) = \frac{V_0}{V_I} = \frac{1/(sC)}{R + 1/(sC)} = \frac{1}{1 + j\omega RC}$$



$$\omega_o = \frac{1}{RC}$$

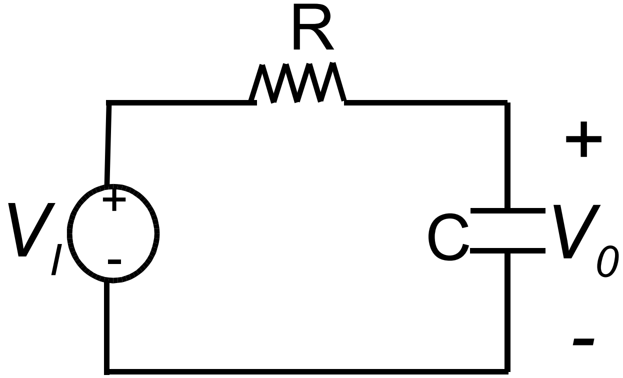


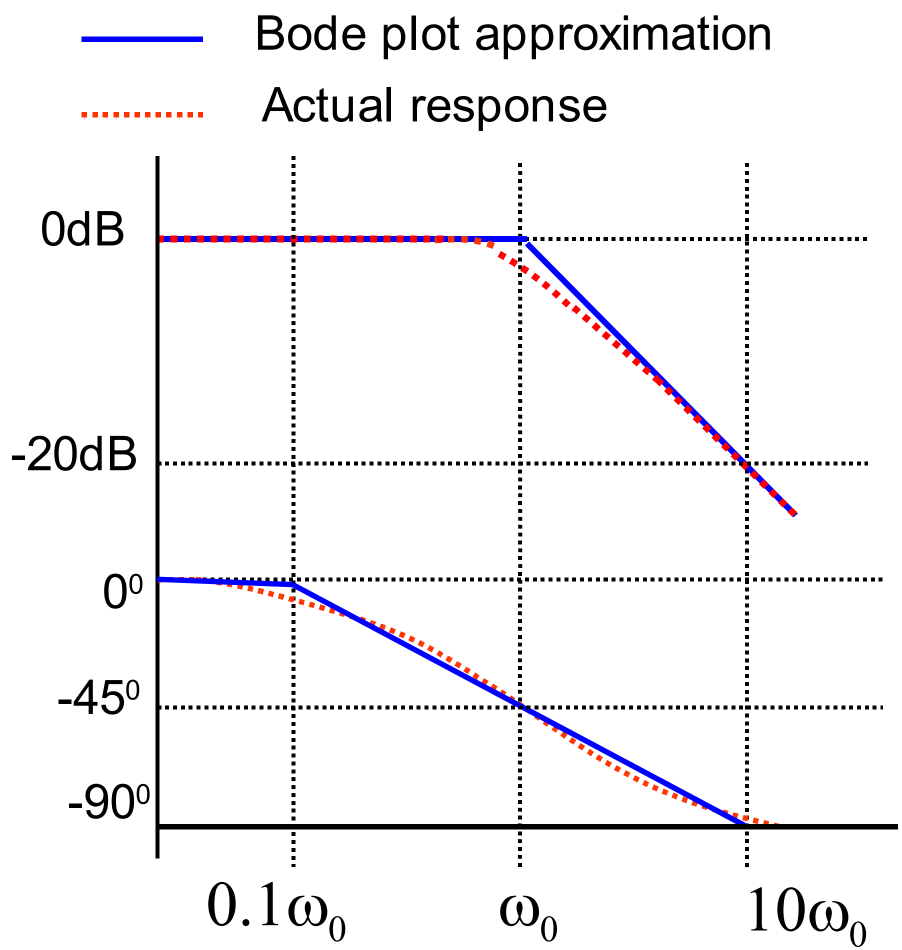
ω_o → resonant frequency
break frequency
half power frequency

Low Pass Filters

$$G_V(s) = \frac{V_0}{V_I} = \frac{1}{1 + \frac{j\omega}{\omega_0}}$$

$$\omega_o = \frac{1}{RC}$$



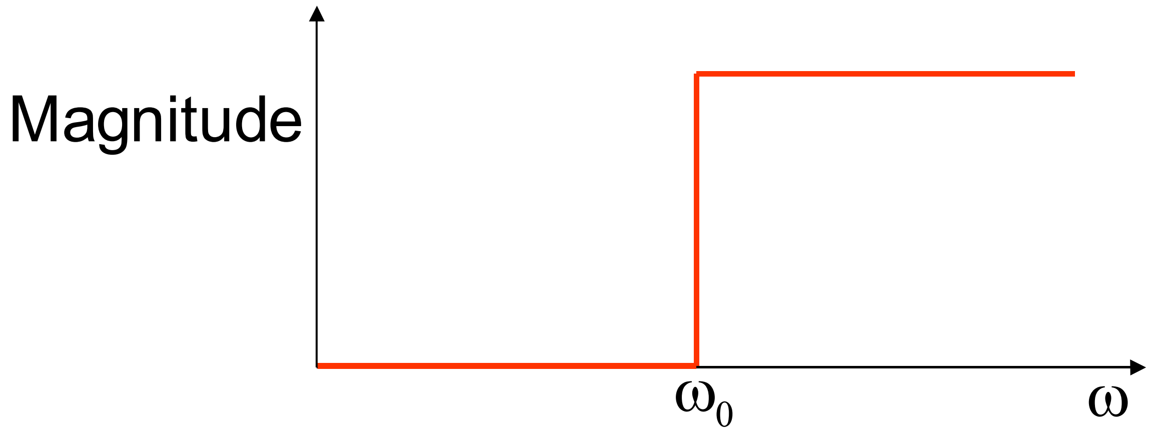


$$M(\omega) = \frac{1}{\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]^{1/2}}$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

High Pass Filter

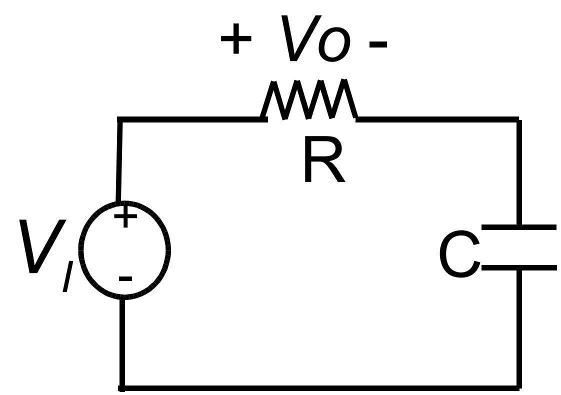
Ideal Characteristic of High Pass Filter

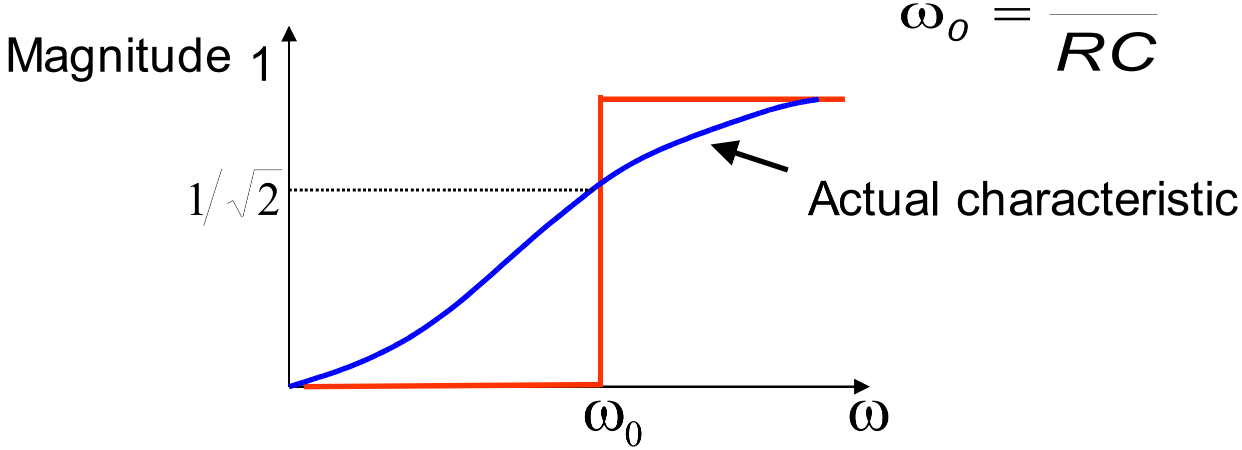


Simple High Pass Filter

$$G_V(s) = \frac{V_0}{V_I} = \frac{R}{R + 1/(sC)} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\omega_o = \frac{1}{RC}$$

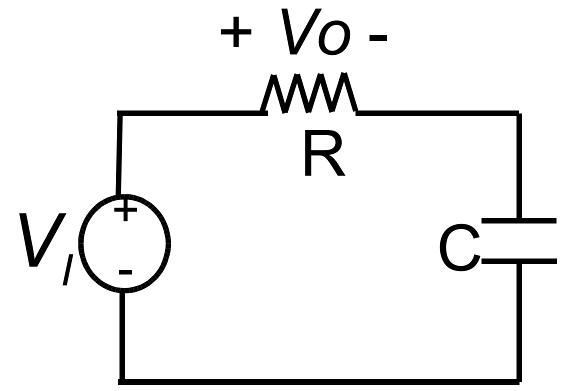


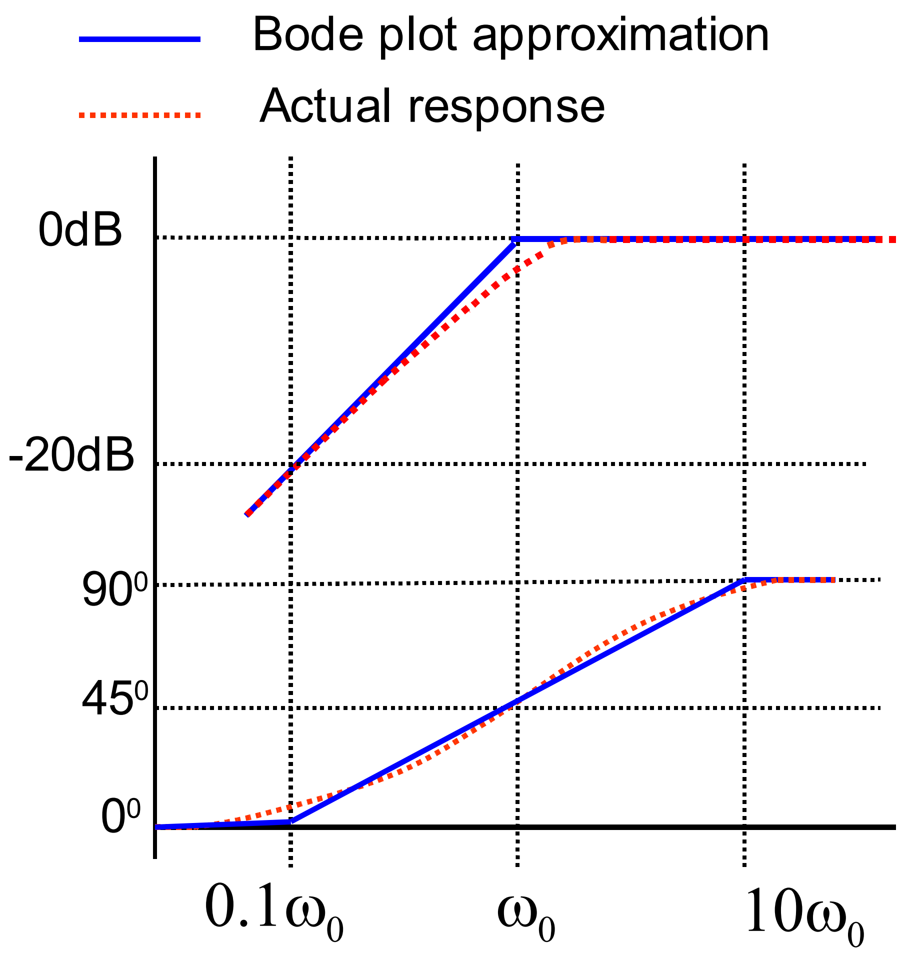


High Pass Filters

$$G_V(s) = \frac{V_0}{V_I} = \frac{\frac{j\omega}{\omega_0}}{1 + \frac{j\omega}{\omega_0}}$$

$$\omega_o = \frac{1}{RC}$$



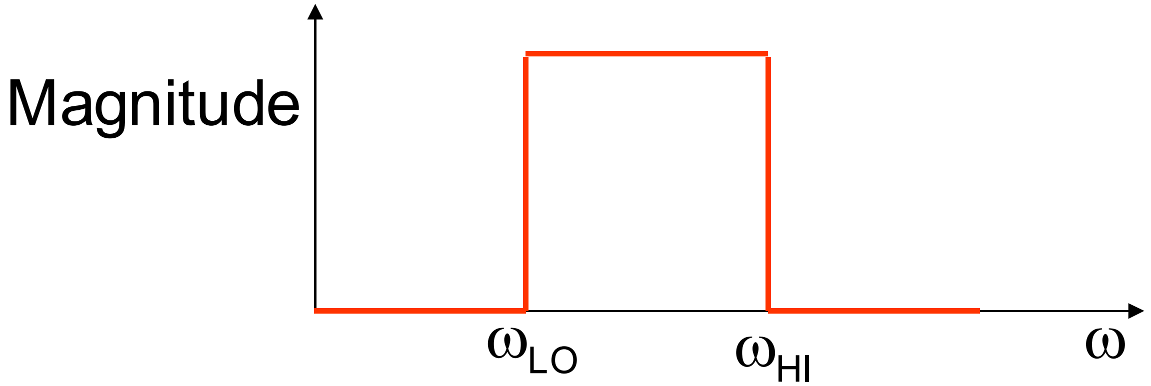


$$M(\omega) = \frac{\frac{\omega}{\omega_0}}{\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]^{1/2}}$$

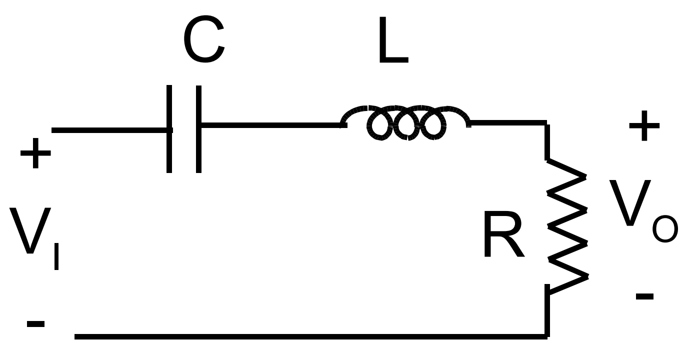
$$\phi(\omega) = 90^0 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Band Pass Filter

Ideal Characteristic of Band Pass Filter



Simple Band Pass Filter

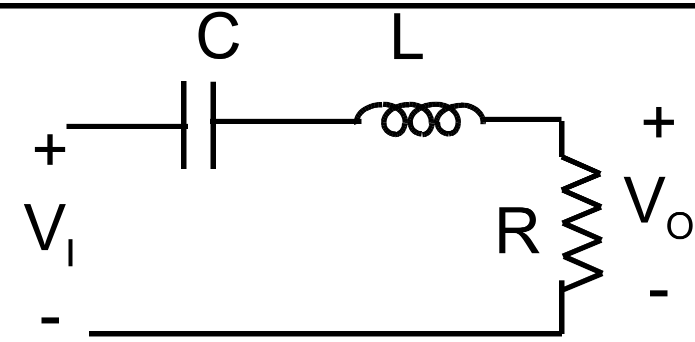


$$G_V(s) = \frac{V_0}{V_I} = \frac{R}{R + sL + 1/(sC)}$$

$$= \frac{R}{R + j\left(\omega L - 1/(\omega C)\right)}$$

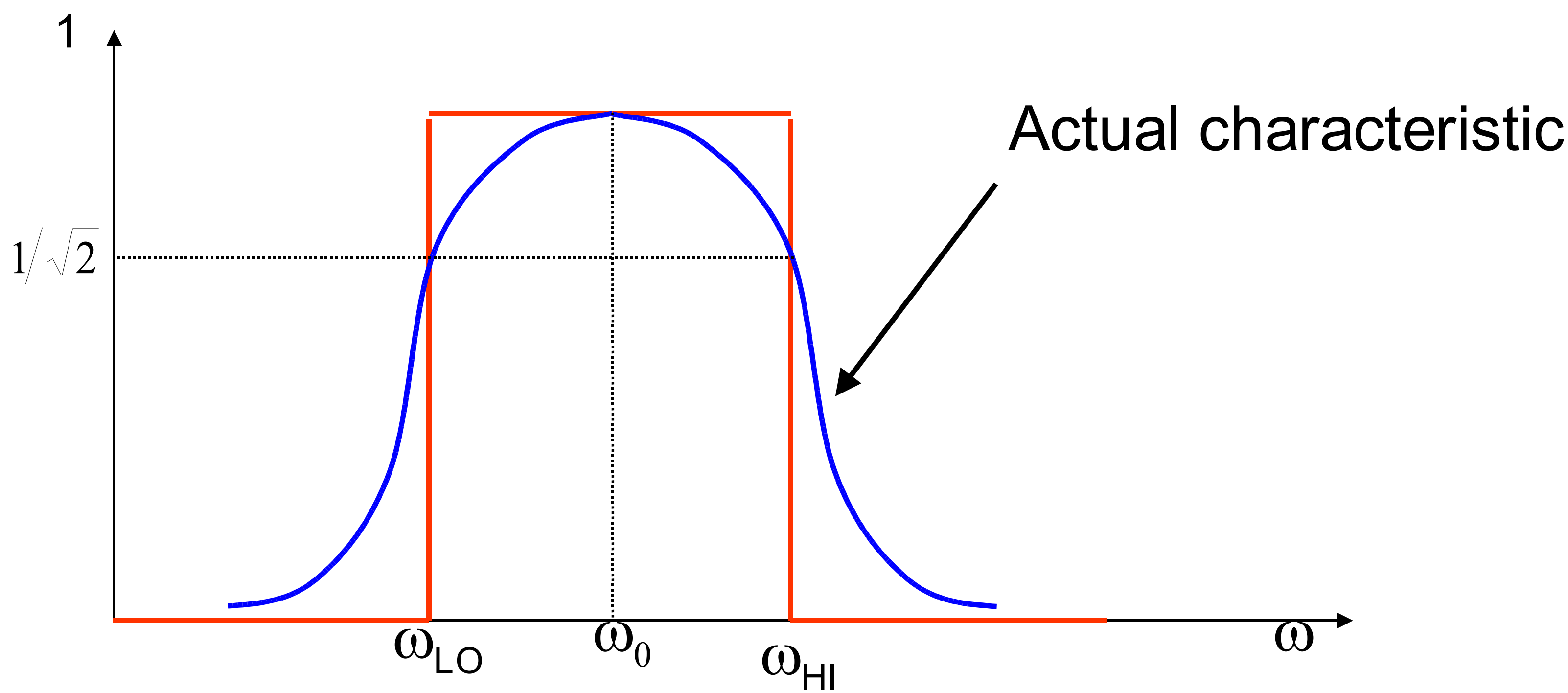
Band Pass Filter

$$G_V(j\omega) = \frac{V_0}{V_I} = \frac{R}{R + j\left(\omega L - 1/(\omega C)\right)}$$

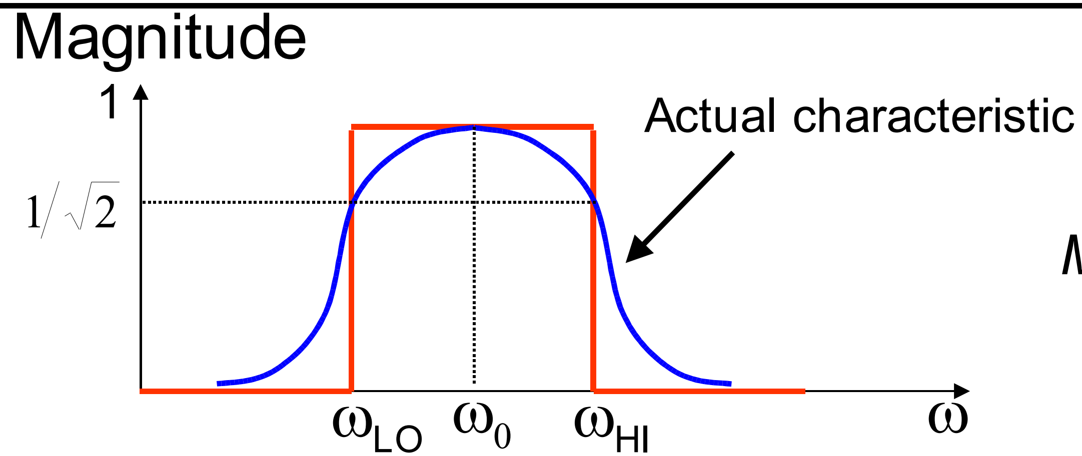


Magnitude

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$



Band Pass Filter



$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

The center frequency ω_0

$$\omega^2 LC - 1 = 0 \quad \rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Lower cut off frequency

$$\omega RC = -\left(\omega^2 LC - 1\right)$$

$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L) + 4\omega_0^2}}{2}$$

Upper cut off frequency

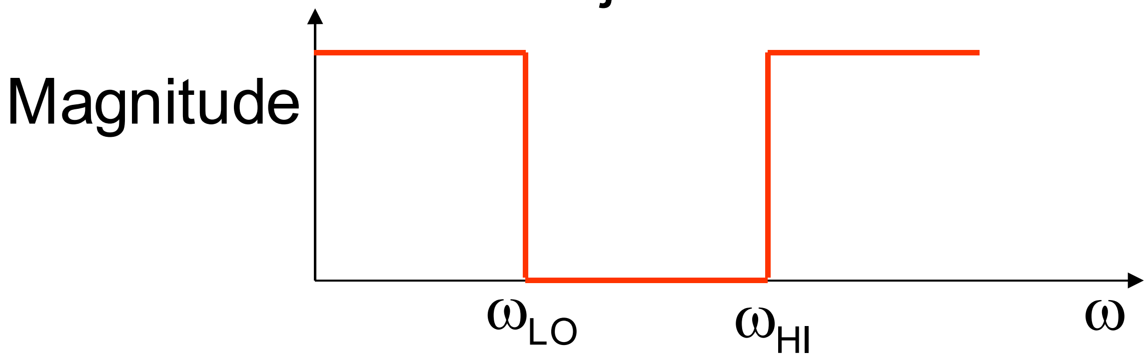
$$\omega RC = \left(\omega^2 LC - 1\right)$$

$$\omega_{LO} = \frac{(R/L) + \sqrt{(R/L) + 4\omega_0^2}}{2}$$

Bandwidth → $BW = \omega_{HI} - \omega_{LO} = \dfrac{\omega_0}{Q} = \dfrac{R}{L}$

Band Reject Filter (Notch Filter)

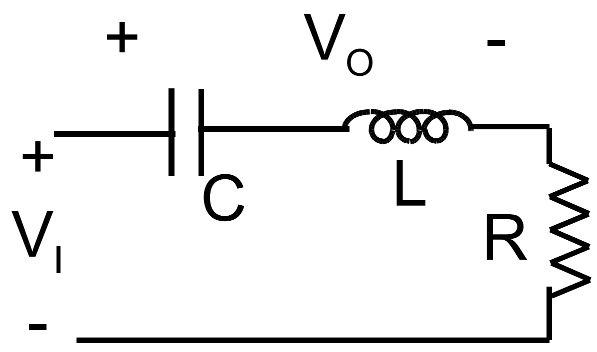
Ideal Characteristic of Band Reject Filter

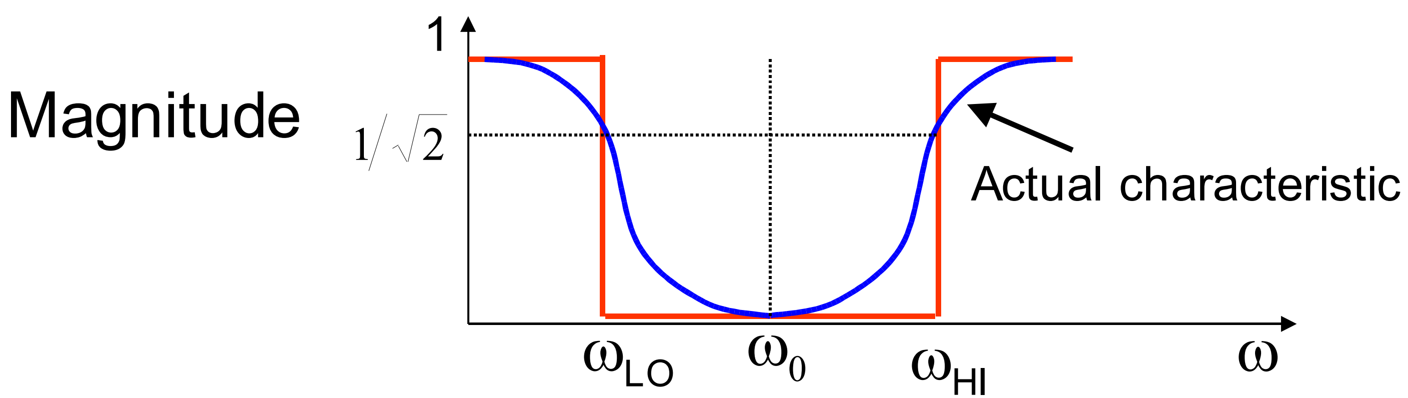


Simple Reject Pass Filter

$$G_V(s) = \frac{V_0}{V_I} = \frac{sL + 1/(sC)}{R + sL + 1/(sC)}$$

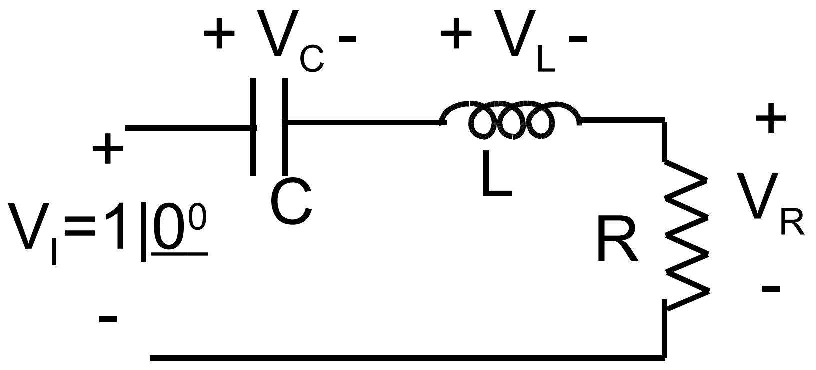
$$= \frac{j(\omega L - 1/(\omega C))}{R + j(\omega L - 1/(\omega C))}$$





Example 15

Given the following circuit parameter values: L=159mH, C=159mF and R=10W. Demonstrate that this circuit can be used to produce a low-pass, high-pass, or band-pass filter.



Active Filters

Drawbacks of Passive Filters

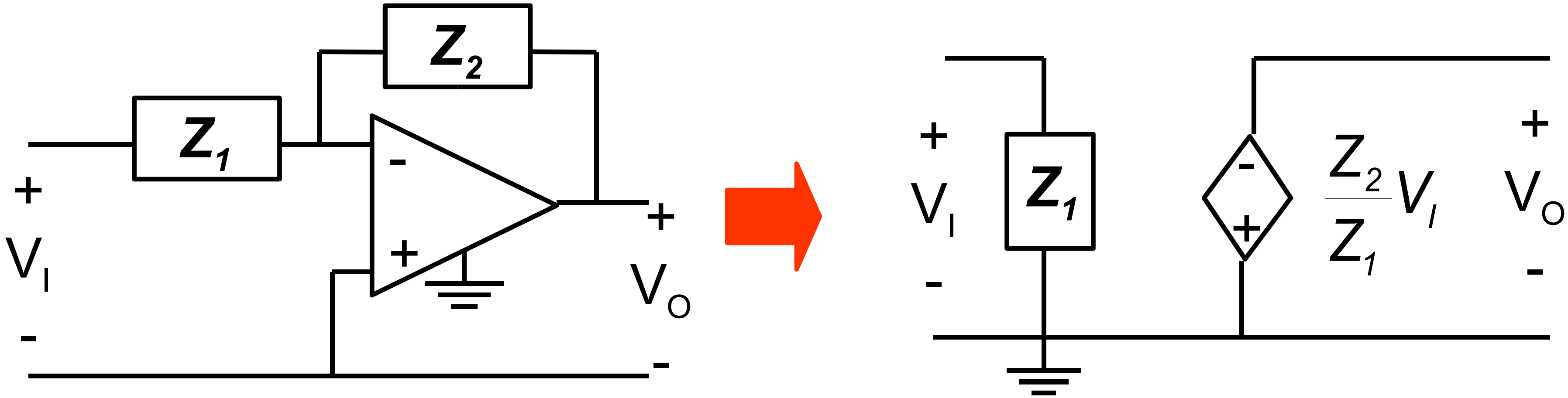
1. Inability to generate a network with a gain greater than one since passive elements cannot add energy to signals
2. Inductors are generally expensive and occupy to much space

Advantages of Active Filters

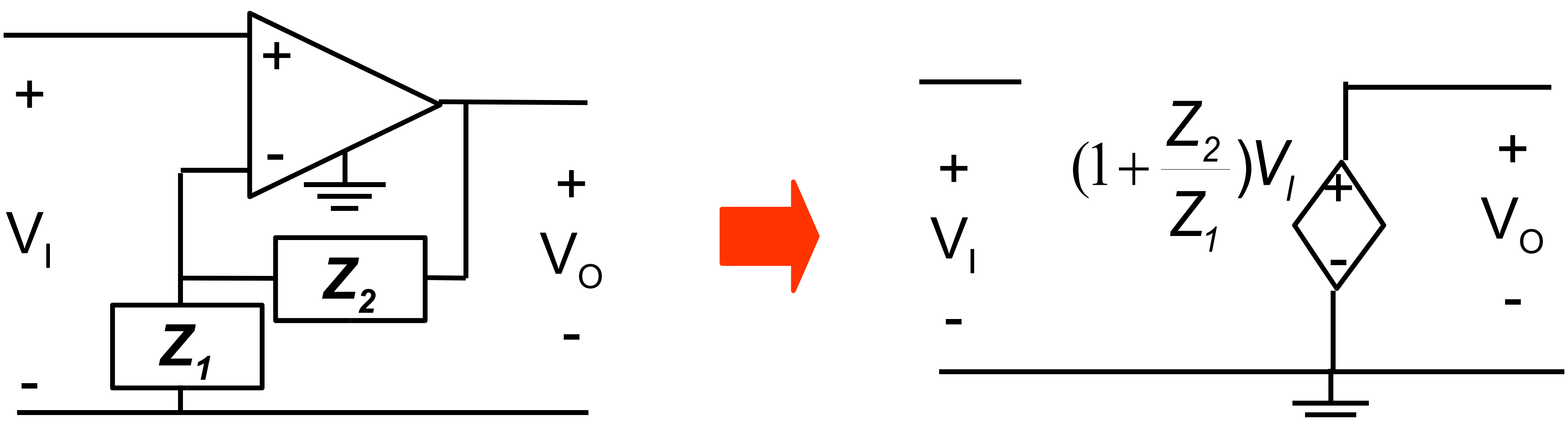
1. Active Filters are able to add energy to signals
2. Can construct inductors using resistors, capacitors and operational amplifiers (Op-amps).

Active filters

Inverting operational amplifier



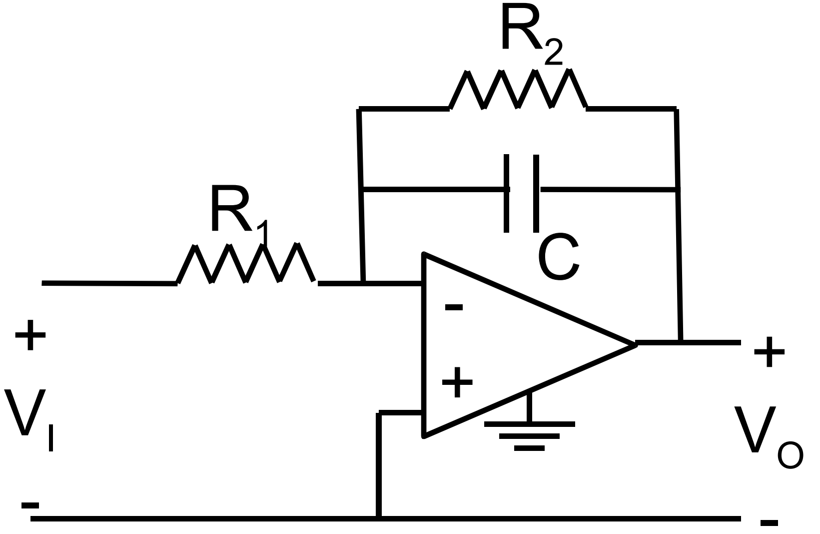
Noninverting operational amplifier



Filter characteristics are determined by the choice of Z_1 and Z_2.

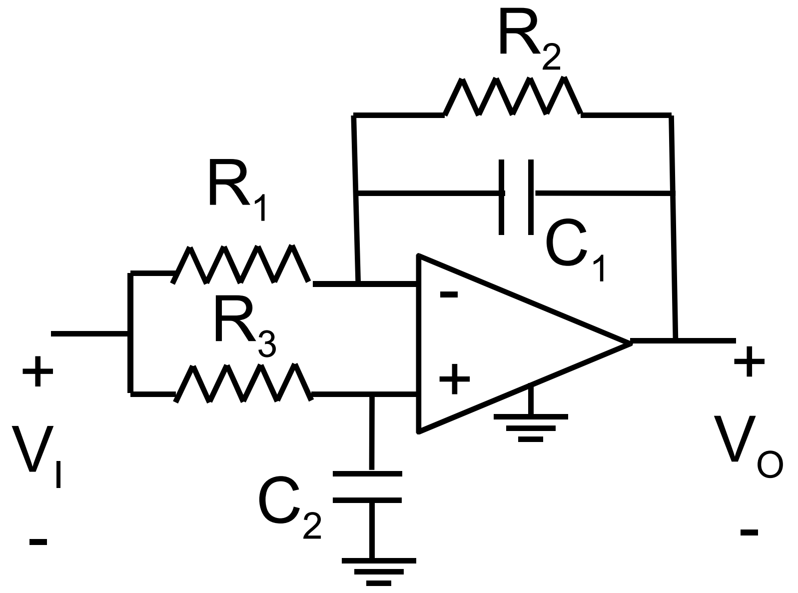
Example 16

Find the voltage gain V_o/V_I for the following circuit



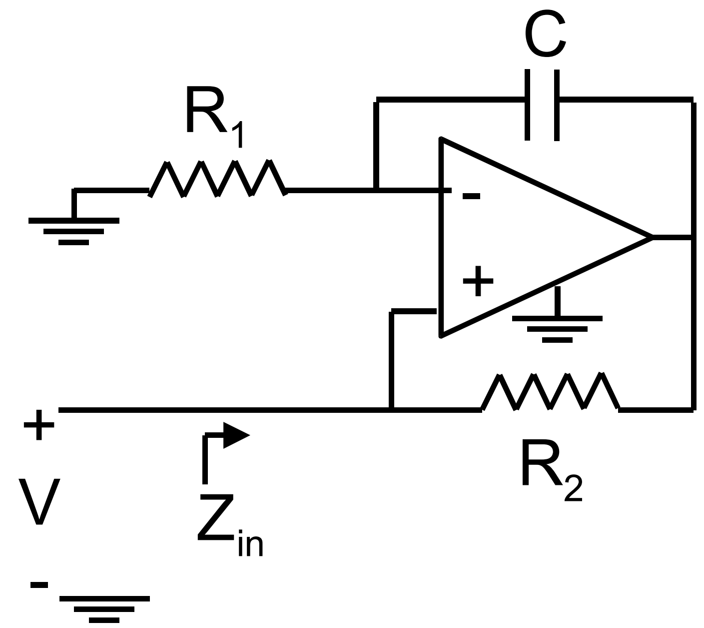
Example 17 (Difference Amplifier)

Find the voltage gain V_o/V_I for the following circuit



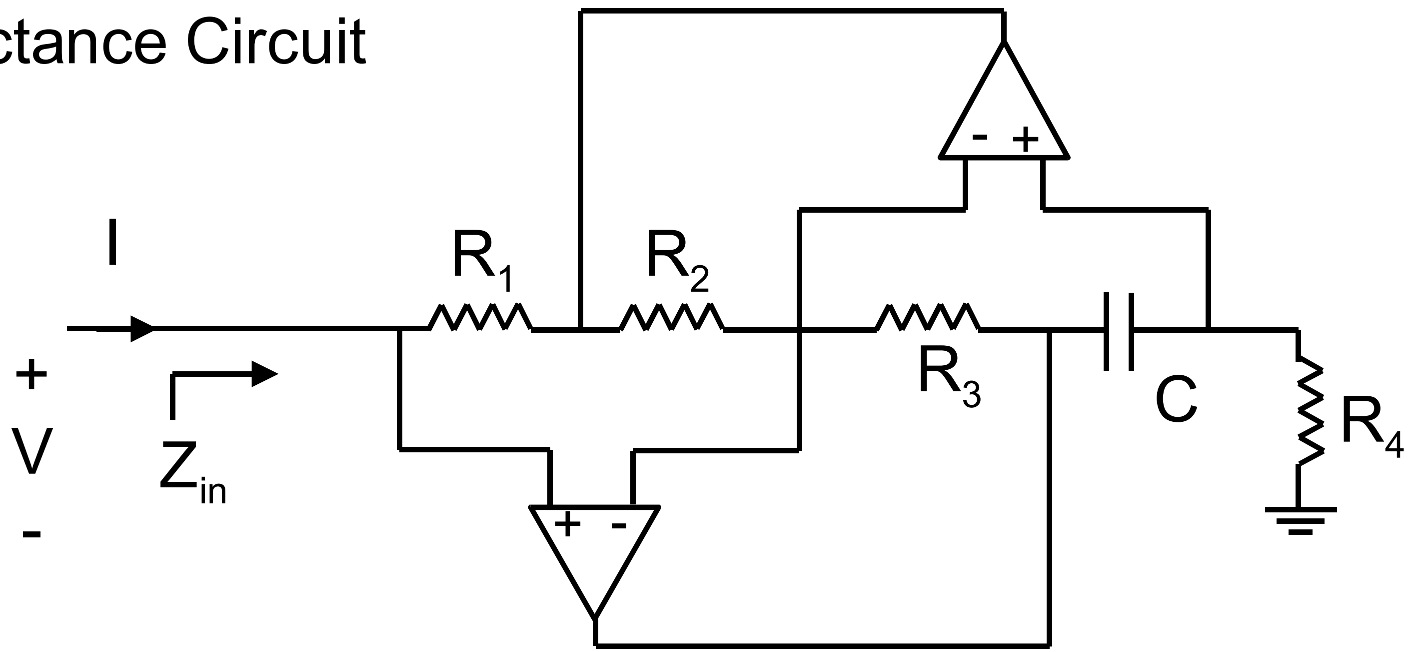
Example 18

Find the input impedance for the following circuit



Inductor Replacement

Antoniou Inductance Circuit

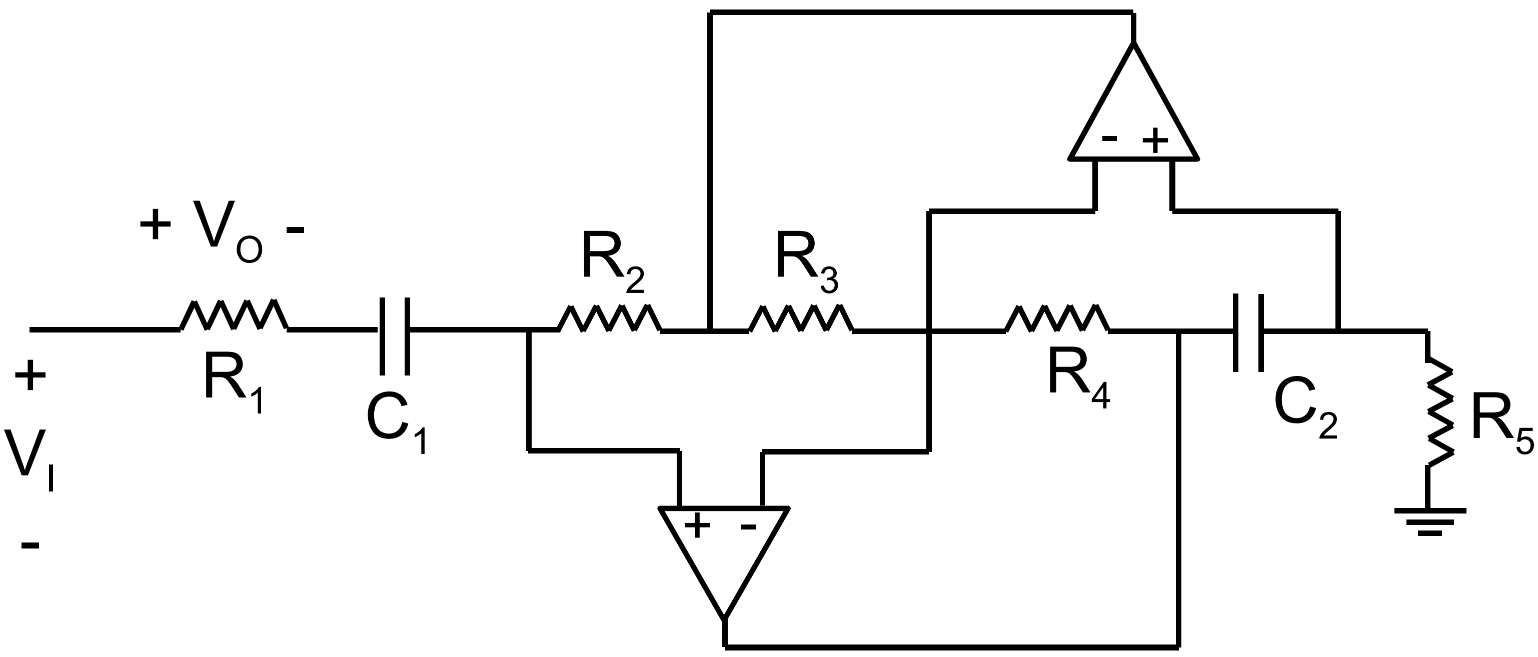


$$Z_{in} = \frac{V}{I} = sCR_1R_3R_4/R_2 = sL$$

where $L = CR_1R_3R_4/R_2$

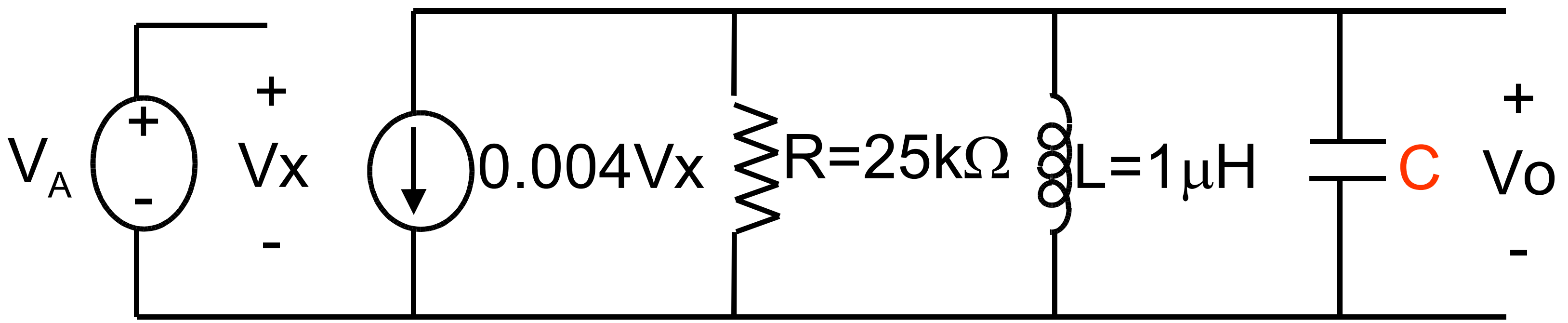
Example 19

Find the transfer function *V_o/V_I* for the circuit shown below and state what type of filter this transfer function represents



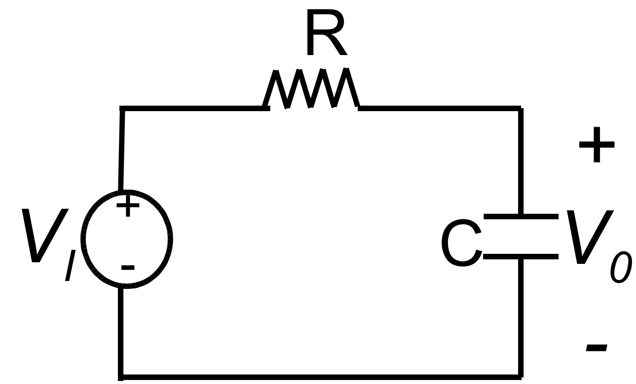
Example 20

The network shown is a circuit model for a single stage tuned transistor amplifier. Find the transfer function Vo/V_A, and the value of C so that the center frequency is 91.1 MHz.

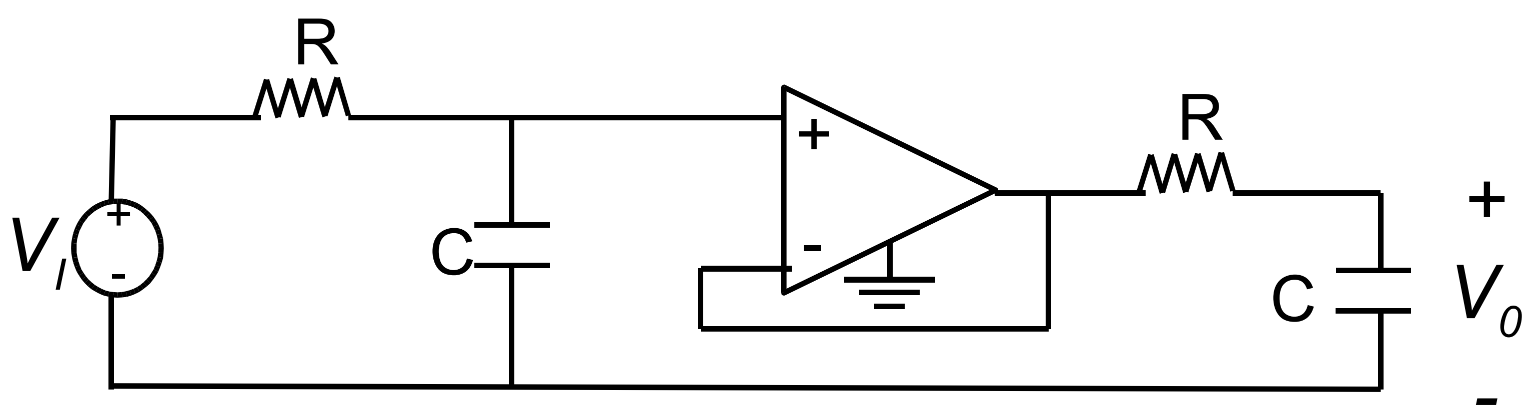


Example 21

Design 20db attenuation at 22.05 KHz for the following two circuits



a) Single pole low pass filter



b) Two-stage buffered low pass filter