Electrical Circuits II (ECE233b)

Variable-Frequency Network Performance (Part 3)

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Scaling

Often the values of circuit parameters vary by orders of magnitudes. For example:

- Resistors:  $\text{units } \Omega \rightarrow 10^6$
- Capacitors:  $\text{units } \text{pF} \rightarrow 10^{-12}$
- Inductors:  $\text{units } \text{nH} \rightarrow 10^{-9}$
- Time:  $\text{units } \text{ns} \rightarrow 10^{-9}$
- Frequency:  $\text{units } \text{GHz} \rightarrow 10^9$

Since computers are finite precision machines, scaling circuit parameters can result in numerically more accurate results.

In addition, scaling can sometimes make the results more presentable.

There are two ways to scale circuit parameters:

- Magnitude (or Impedance) scaling
- Frequency scaling
Magnitude Scaling

Note magnitude scaling does not affect the frequency response

Lets verify for RLC series circuit

$$\omega'_0 = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_ML\left(\frac{C}{K_M}\right)}} = \omega_0$$

$$Q' = \frac{\omega'_0L'}{R'} = \frac{\omega_0K_ML}{K_MR} = Q$$
Frequency Scaling

Note resistors are frequency independent and are unaffected by this scaling.

The new inductor $L'$ and capacitor $C'$ values must have the same impedance at the scale frequency $\omega'$ as the original circuit and must satisfy:

$$j\omega L = j\omega' L'$$
$$\frac{1}{j\omega C} = \frac{1}{j\omega' C'}$$

where $\omega' = K_F \omega$ and $K_F$ is the frequency scaling factor.

Frequency Scaling

$$R' \rightarrow R$$
$$L' \rightarrow \frac{L}{K_F}$$
$$C' \rightarrow \frac{C}{K_F}$$

Frequency independent
Frequency Scaling

Note that frequency scaling affects the resonant frequency and bandwidth but not the $Q$.

Let's verify for RLC series circuit

$$\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\left(\frac{L}{K_F}\right)\left(\frac{C}{K_F}\right)}} = K_F \omega_0$$

$$Q' = \frac{\omega_0'L'}{R'} = K_M \omega_0 \left(\frac{L}{K_M}\right) = Q$$

$$BW' = \frac{\omega_0'}{Q'} = K_F (BW)$$
Example 14

An RLC network has the following parameters values: $R = 10\, \Omega$, $L=1\, \text{H}$ and $C=2\, \text{F}$. Determine the values of the circuit elements if the circuit is magnitude scaled by a factor of 100 and frequency scaled by factor of 10 000.
Filter Networks

Two types: PASSIVE and ACTIVE circuits

Passive Filters – circuits composed of passive RLC elements

Types of Filters

1. Low pass filters: allows low frequencies to pass and rejects high frequencies
2. High pass filters: allows high frequencies to pass and rejects low frequencies
3. Band pass filters: allows some particular band of frequencies to pass and rejects all frequencies outside the range
4. Band reject filters: rejects some particular band of frequencies and allows all other frequencies to pass
Low Pass Filters

Ideal Characteristic of Low Pass Filter

\[ |G(j\omega)| = 1 \]

Simple Low Pass Filter

\[ G_v(s) = \frac{V_0}{V_i} = \frac{1/(sC)}{R + 1/(sC)} = \frac{1}{1 + j\omega RC} \]

\[ \omega_0 = \frac{1}{RC} \]

magnitude vs frequency diagram

Actual characteristic

- Resonant frequency
- Break frequency
- Half power frequency
Low Pass Filters

\[ G_v(s) = \frac{V_0}{V_i} = \frac{1}{1 + \frac{j\omega}{\omega_0}} \]

\[ \omega_0 = \frac{1}{RC} \]

Bode plot approximation

Actual response

\[ M(\omega) = \frac{1}{\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]^{1/2}} \]

\[ \phi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right) \]
High Pass Filter

Ideal Characteristic of High Pass Filter

Simple High Pass Filter

\[ G_v(s) = \frac{V_0}{V_i} = \frac{R}{R + 1/(sC)} = \frac{j\omega RC}{1 + j\omega RC} \]

\[ \omega_0 = \frac{1}{RC} \]
High Pass Filters

\[ G_v(s) = \frac{V_0}{V_0} = \frac{j\omega}{\omega_0} \frac{1}{1 + \frac{j\omega}{\omega_0}} \]

\[ \omega_0 = \frac{1}{RC} \]

Bode plot approximation

Actual response

\[ M(\omega) = \frac{\omega}{\omega_0} \left[ 1 + \left( \frac{\omega}{\omega_0} \right)^2 \right]^{1/2} \]

\[ \phi(\omega) = 90^\circ - \tan^{-1} \left( \frac{\omega}{\omega_0} \right) \]
Band Pass Filter

Ideal Characteristic of Band Pass Filter

Simple Band Pass Filter

\[ G_v(s) = \frac{V_o}{V_i} = \frac{R}{R + sL + 1/(sC)} \]

\[ = \frac{R}{R + j(\omega L - 1/(\omega C))} \]
Band Pass Filter

\[ G_v(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j(\omega L - 1/(\omega C))} \]

**Magnitude**

\[ M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}} \]

Actual characteristic
Band Pass Filter

**Magnitude**

The center frequency $\omega_0$

$$\omega^2 LC - 1 = 0 \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Lower cut off frequency

$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L) + 4\omega_0^2}}{2}$$

Upper cut off frequency

$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L) + 4\omega_0^2}}{2}$$

Bandwidth

$$BW = \omega_{HI} - \omega_{LO} = \frac{\omega_0}{Q} = \frac{R}{L}$$

$$M(\omega) = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$
Band Reject Filter (Notch Filter)

Ideal Characteristic of Band Reject Filter

Simple Reject Pass Filter

\[ G_v(s) = \frac{V_o}{V_i} = \frac{sL + 1/(sC)}{R + sL + 1/(sC)} = \frac{j(\omega L - 1/(\omega C))}{R + j(\omega L - 1/(\omega C))} \]

Magnitude

Actual characteristic
Example 15

Given the following circuit parameter values: L=159mH, C=159mF and R=10W. Demonstrate that this circuit can be used to produce a low-pass, high-pass, or band-pass filter.
Active Filters

Drawbacks of Passive Filters

1. Inability to generate a network with a gain greater than one since passive elements cannot add energy to signals
2. Inductors are generally expensive and occupy too much space

Advantages of Active Filters

1. Active Filters are able to add energy to signals
2. Can construct inductors using resistors, capacitors, and operational amplifiers (Op-amps).
Active filters

Inverting operational amplifier

Noninverting operational amplifier

Filter characteristics are determined by the choice of $Z_1$ and $Z_2$. 
Example 16

Find the voltage gain $V_o/V_i$ for the following circuit

![Circuit Diagram]

Example 17 (Difference Amplifier)

Find the voltage gain $V_o/V_i$ for the following circuit

![Circuit Diagram]
Example 18

Find the input impedance for the following circuit

![Circuit Diagram]
Inductor Replacement

Antoniou Inductance Circuit

\[ Z_{in} = \frac{V}{I} = sCR_1R_3R_4/R_2 = sL \]

where \( L = CR_1R_3R_4/R_2 \)
Example 19

Find the transfer function $V_o/V_i$ for the circuit shown below and state what type of filter this transfer function represents.
Example 20

The network shown is a circuit model for a single stage tuned transistor amplifier. Find the transfer function $V_o/V_A$, and the value of $C$ so that the center frequency is 91.1 MHz.
Example 21

Design 20db attenuation at 22.05 KHz for the following two circuits

a) Single pole low pass filter

b) Two-stage buffered low pass filter