

# Lecture 04

## CHAPTER 3 - CMOS MODELS

### Chapter Outline

- 3.1 MOS Structure and Operation
- 3.2 Large signal MOS models suitable for hand calculations
- 3.3 Extensions of the large signal MOS model
- 3.4 Capacitances of the MOSFET
- 3.5 Small Signal MOS models
- 3.6 Temperature and noise models for MOS transistors
- 3.7 BJT models
- 3.8 SPICE level 2 model
- 3.9 Models for simulation of MOS circuits
- 3.10 Extraction of a large signal model for hand calculations from the BSIM3 model
- 3.11 Summary

### Perspective

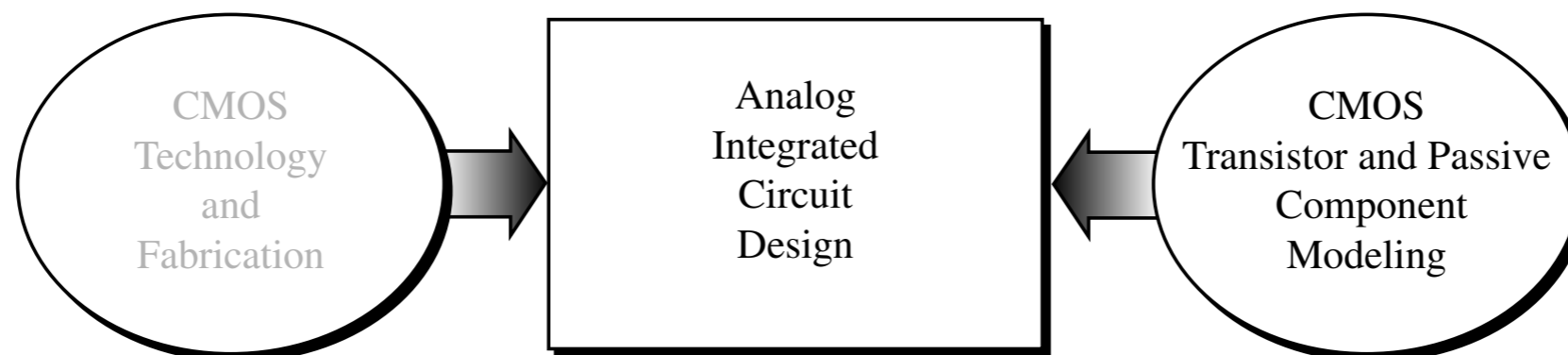


Fig.3.0-1

## Philosophy for Models Suitable for Analog Design

The model required for analog design with CMOS technology is one that leads to understanding and insight as distinguished from accuracy.

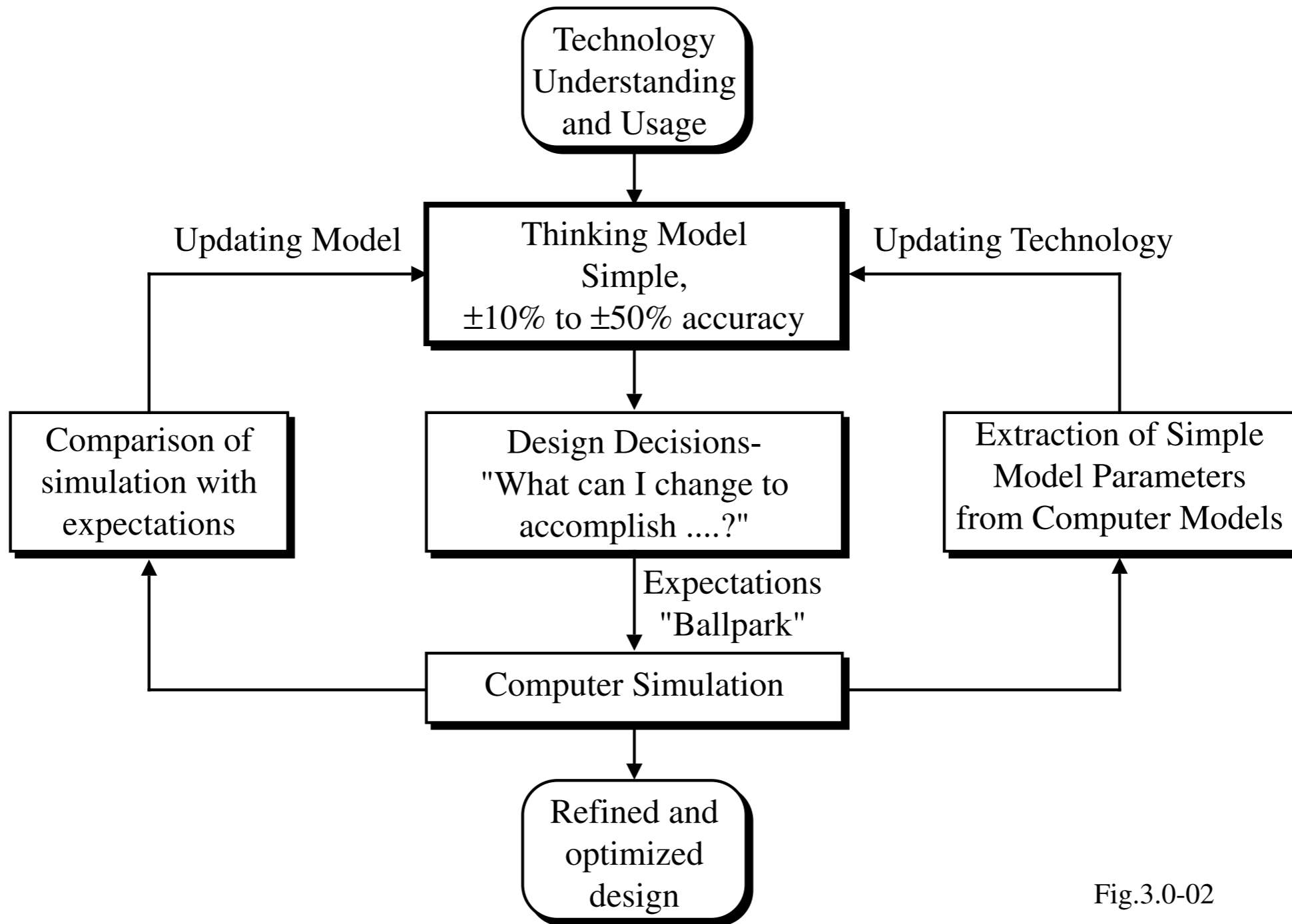


Fig.3.0-02

This chapter is devoted to the simple model suitable for design *not* using simulation.

## Categorization of Electrical Models

		Time Dependence	
		Time Independent	Time Dependent
Linearity	Linear	Small-signal, midband $R_{in}, A_v, R_{out}$ (.TF)	Small-signal frequency response-poles and zeros (.AC)
	Nonlinear	DC operating point $i_D = f(v_D, v_G, v_S, v_B)$ (.OP)	Large-signal transient response - Slew rate (.TRAN)

Based on the simulation capabilities of SPICE.



## 3.1 - MOS STRUCTURE AND OPERATION

### Metal-Oxide-Semiconductor Structure

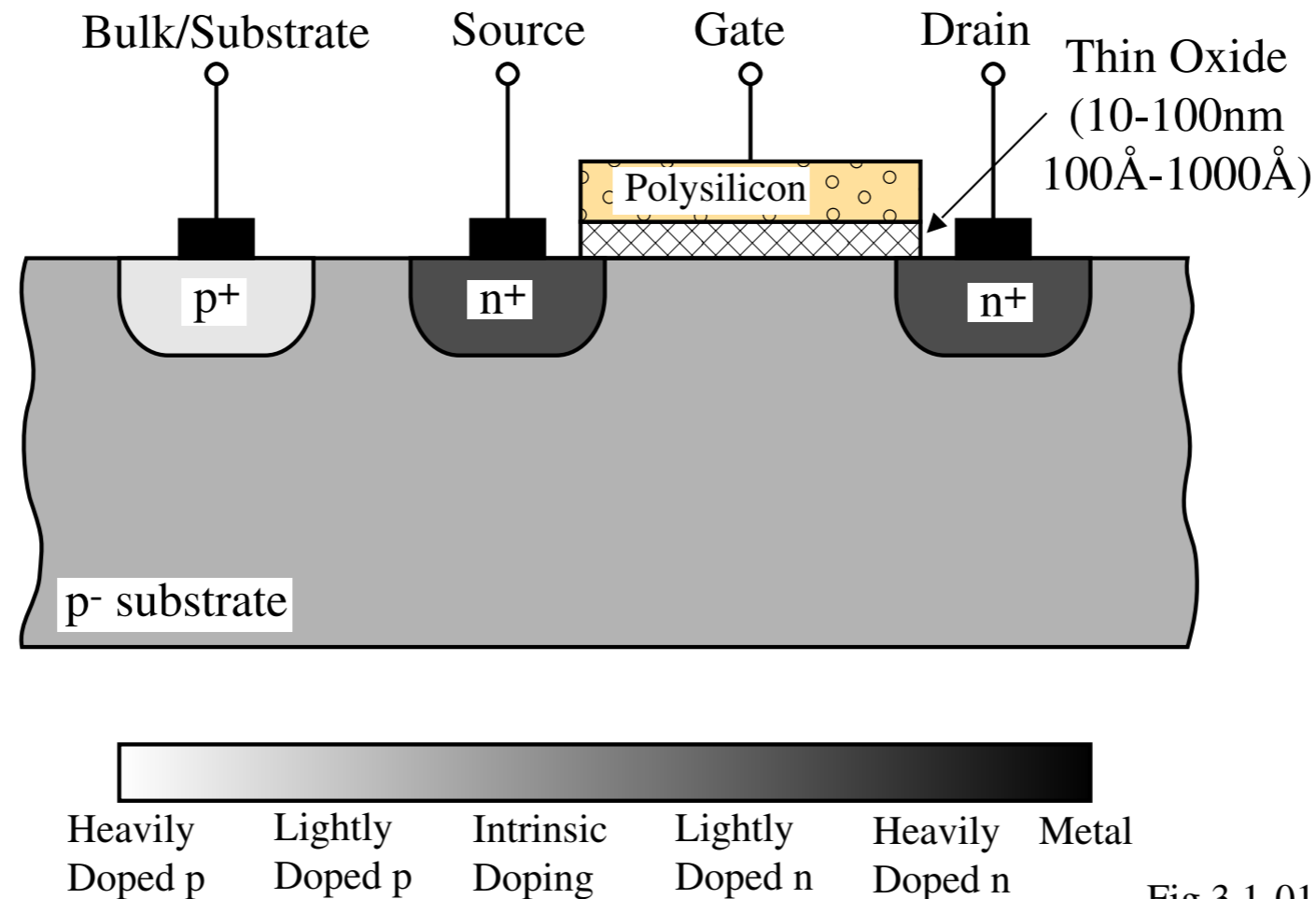


Fig.3.1-01

#### Terminals:

- Bulk - Used to make an ohmic contact to the substrate
- Gate - The gate voltage is applied in such a manner as to invert the doping of the material directly beneath the gate to form a channel between the source and drain.
- Source - Source of the carriers flowing in the channel
- Drain - Collects the carriers flowing in the channel

## Formation of the Channel for an Enhancement MOS Transistor

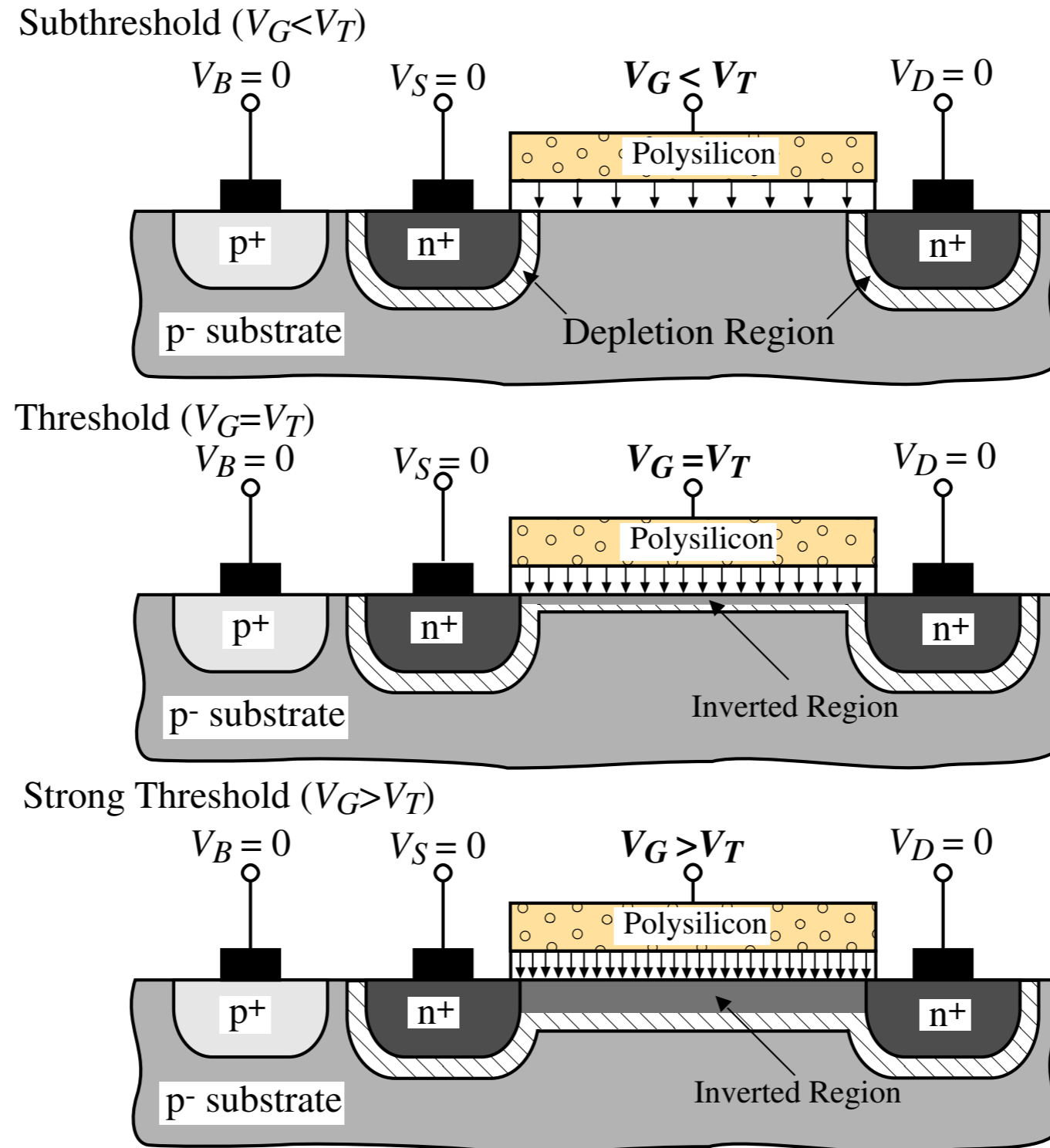
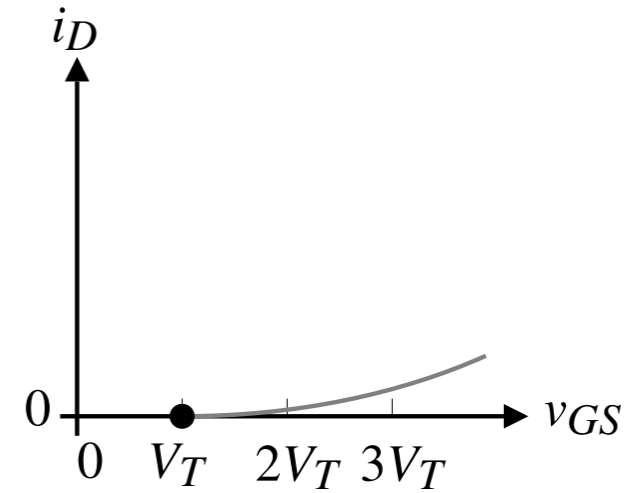
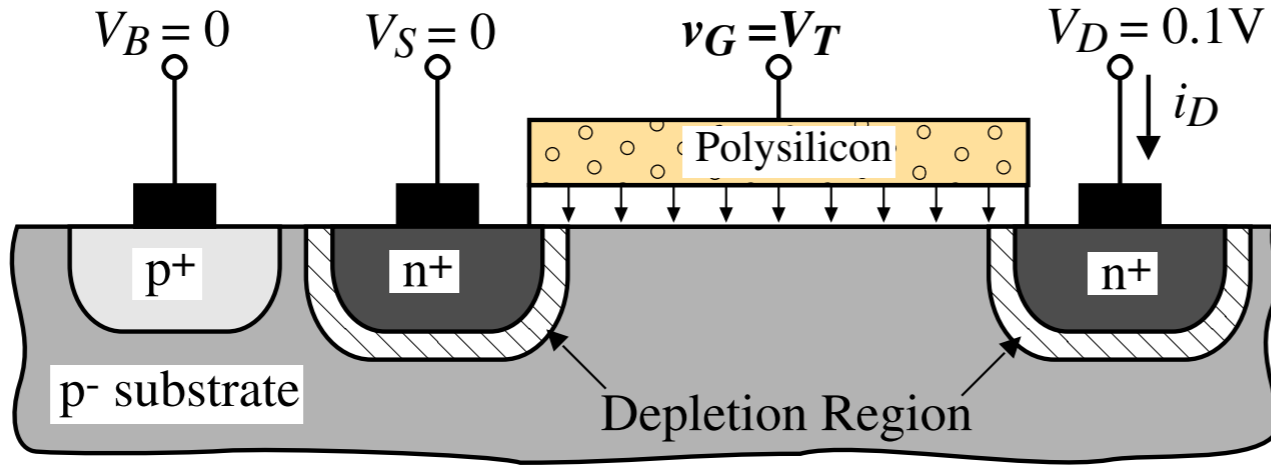


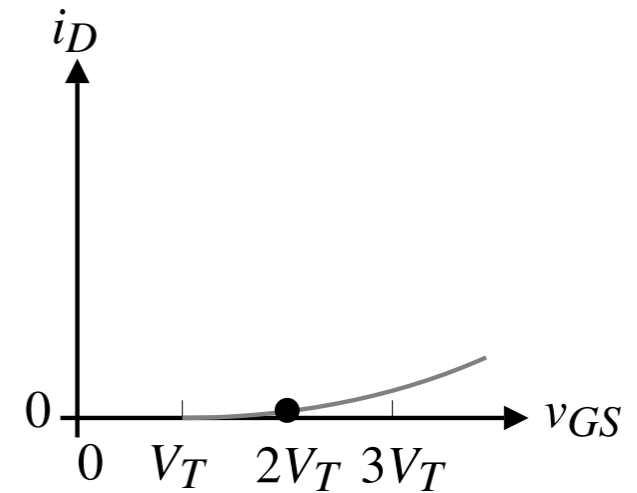
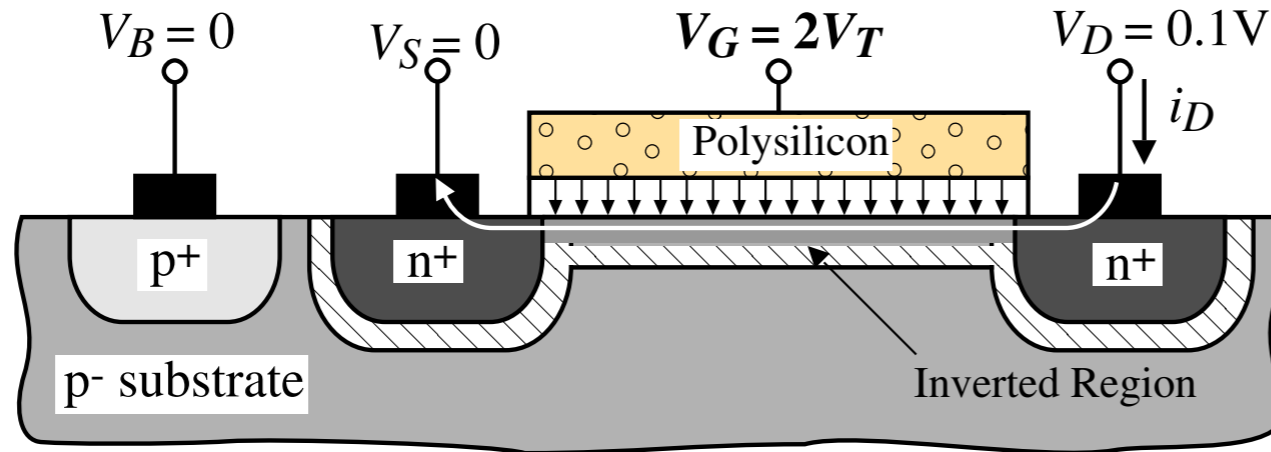
Fig.3.1-02

# Transconductance Characteristics of an Enhancement NMOS FET when $V_{DS} = 0.1V$

$V_{GS} \leq V_T$ :



$V_{GS} = 2V_T$ :



$V_{GS} = 3V_T$ :

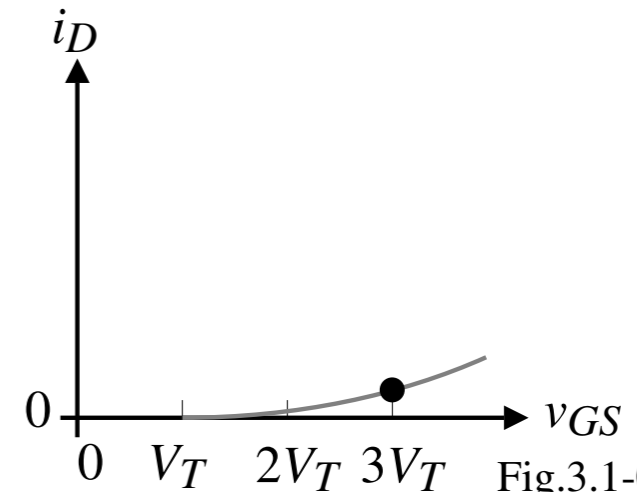
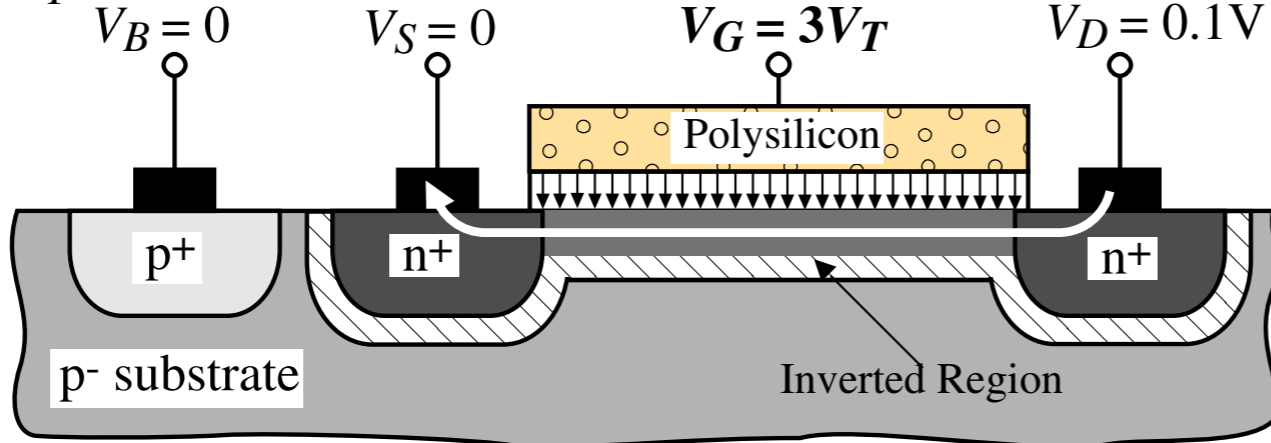
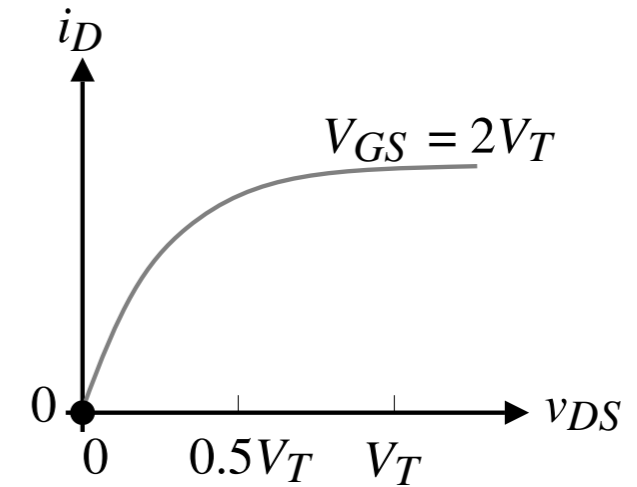
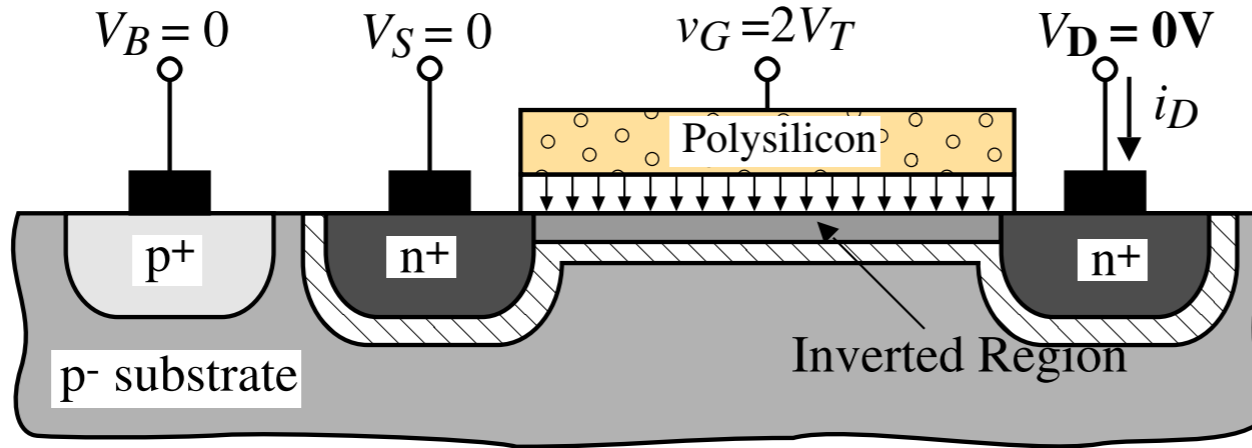


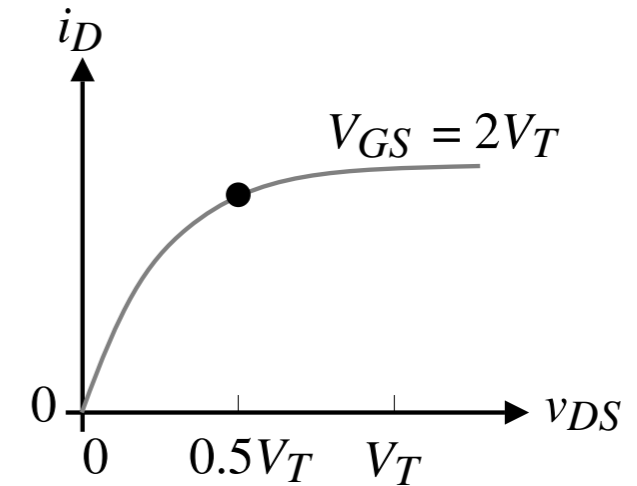
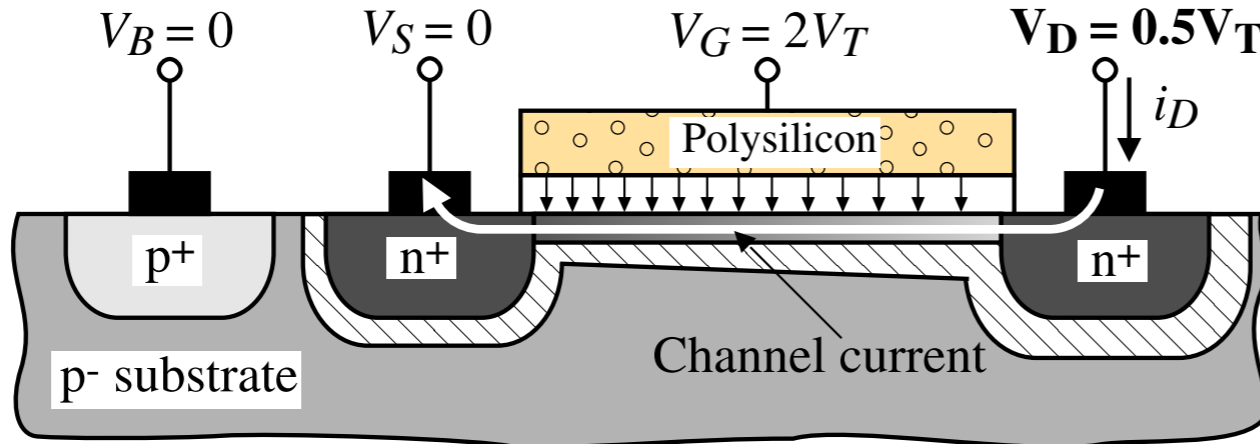
Fig.3.1-03

# Output Characteristics of the Enhancement NMOS Transistor for $V_{GS} = 2V_T$

$V_{DS}=0$ :



$V_{DS}=0.5V_T$ :



$V_{DS}=V_T$ :

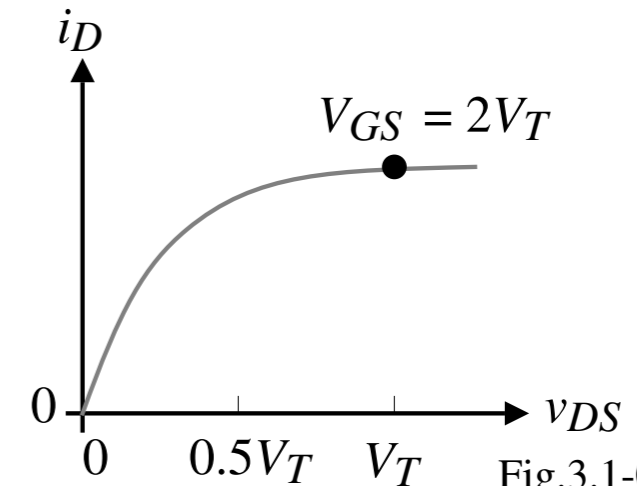
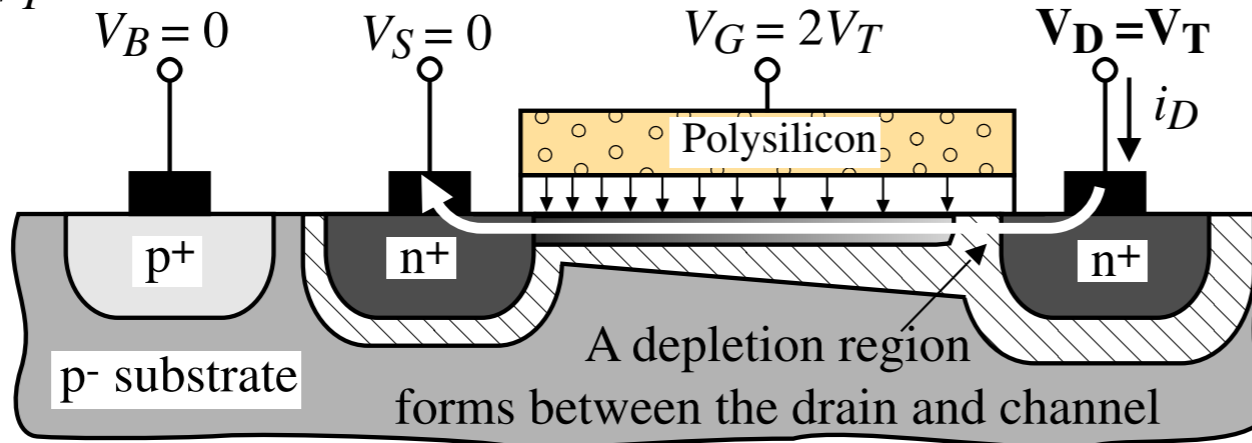
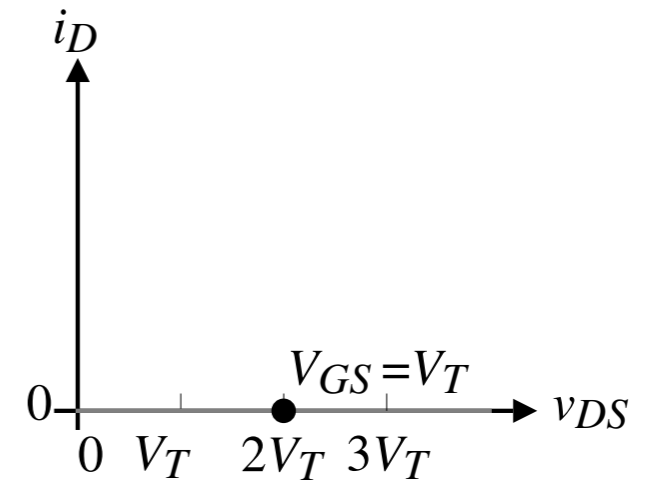
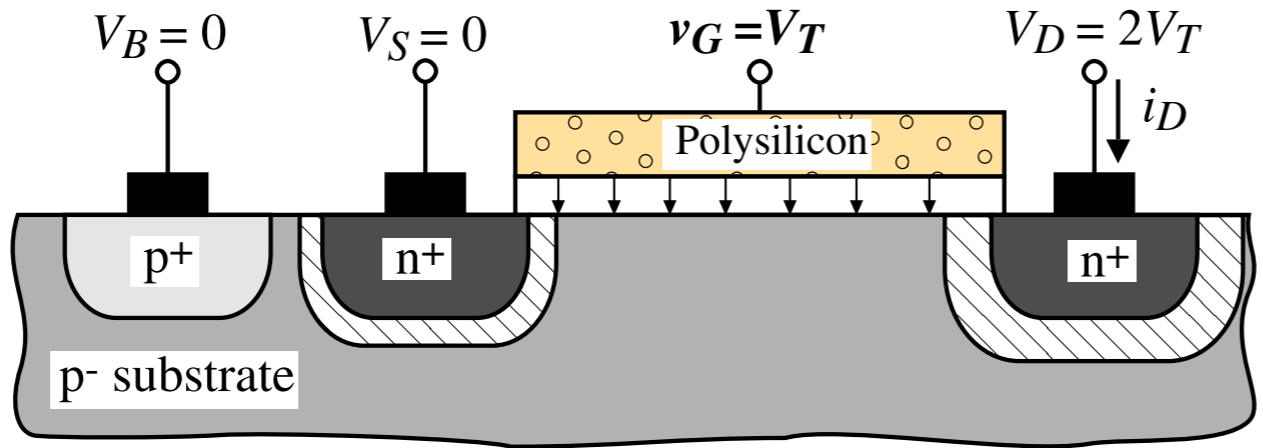


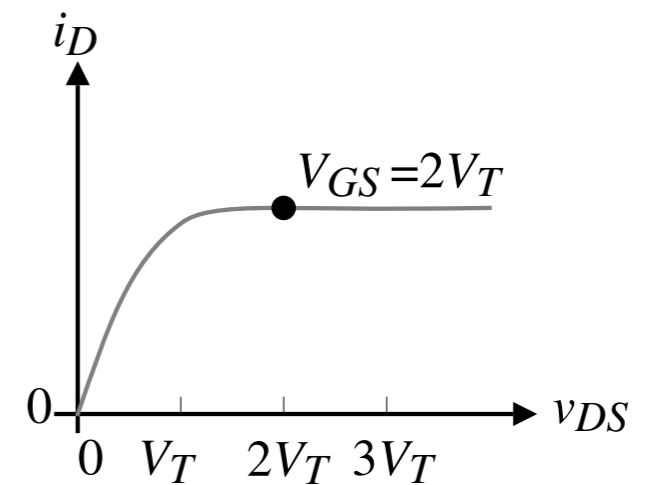
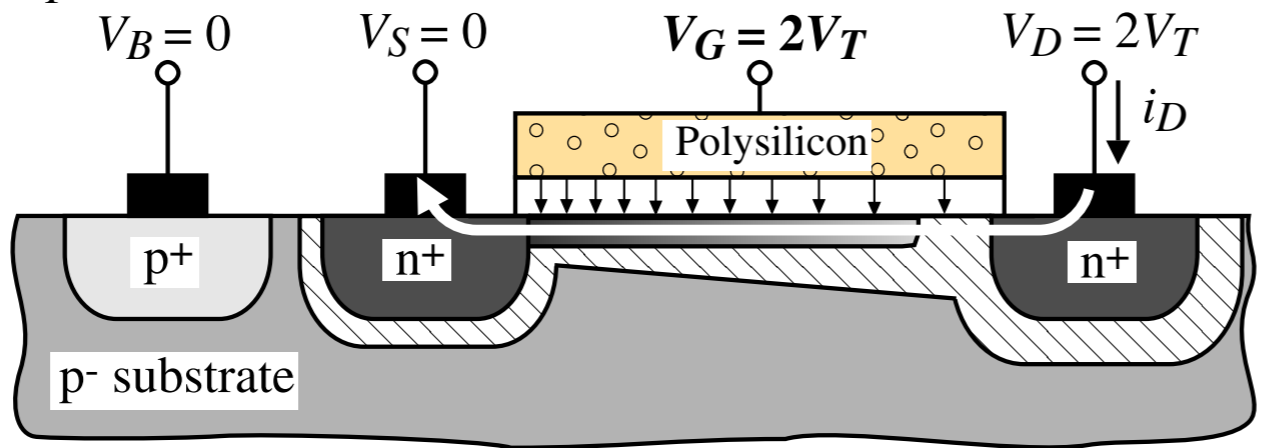
Fig.3.1-04

# Output Characteristics of the Enhanced NMOS when $v_{DS} = 2V_T$

$V_{GS} = V_T$ :



$V_{GS} = 2V_T$ :



$V_{GS} = 3V_T$ :

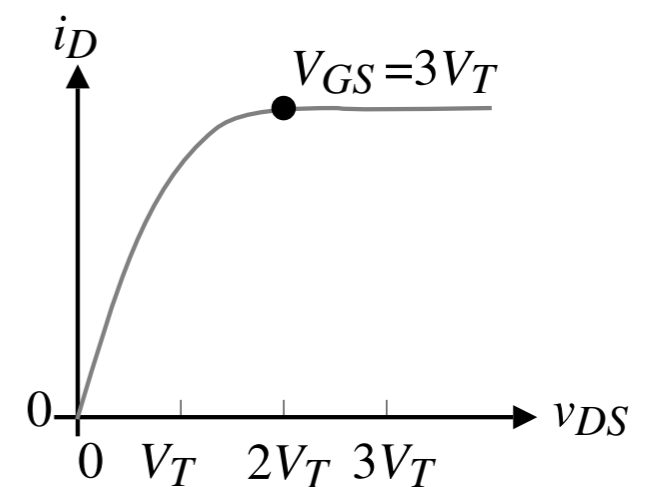
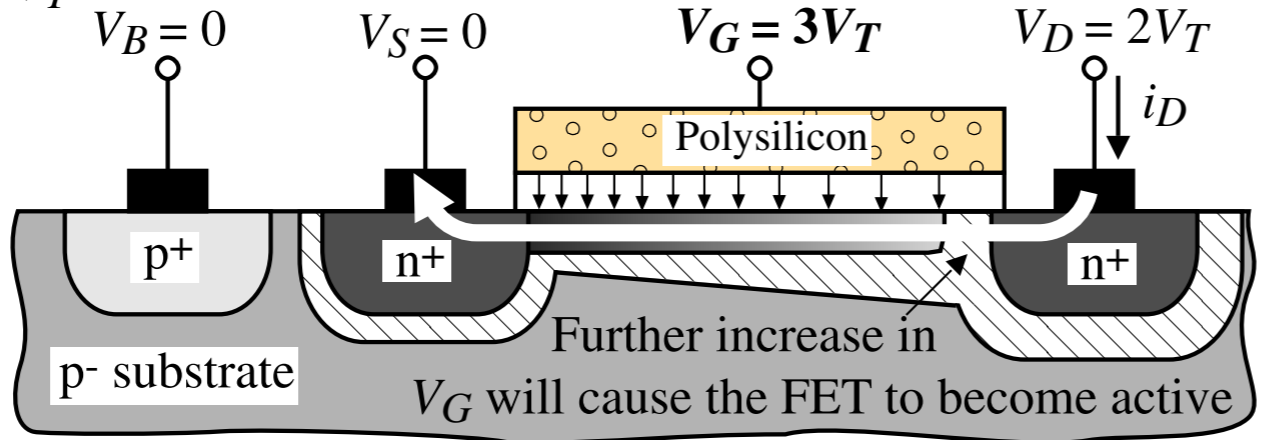


Fig.3.1-05

## Output Characteristics of an Enhancement NMOS Transistor

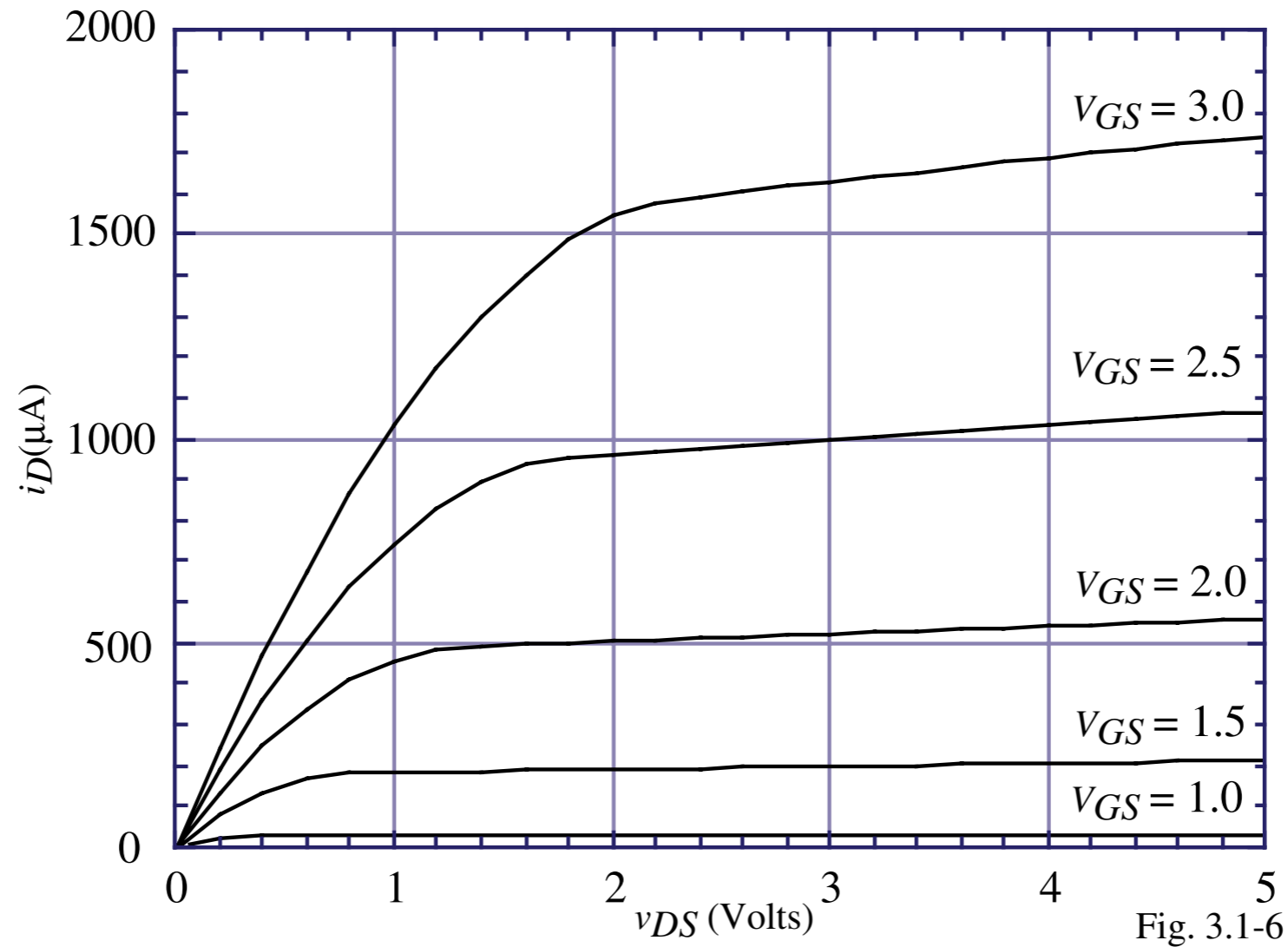


Fig. 3.1-6

### SPICE Input File:

```

Output Characteristics for NMOS
M1 6 1 0 0 MOS1 w=5u l=1.0u
VGS1 1 0 1.0
M2 6 2 0 0 MOS1 w=5u l=1.0u
VGS2 2 0 1.5
M3 6 3 0 0 MOS1 w=5u l=1.0u
VGS3 3 0 2.0
M4 6 4 0 0 MOS1 w=5u l=1.0u
VGS4 4 0 2.5

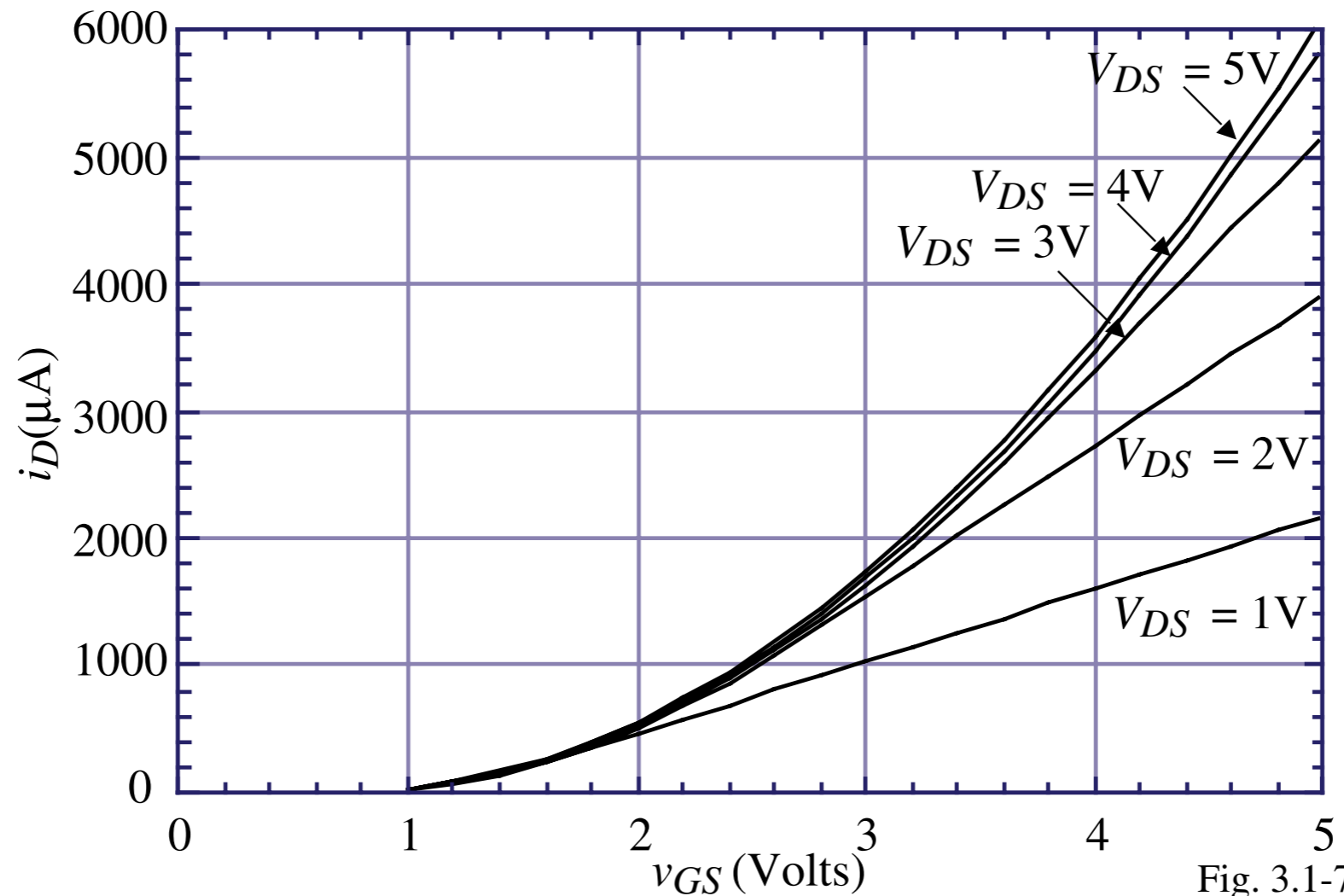
```

```

M5 6 5 0 0 MOS1 w=5u l=1.0u
VGS5 5 0 3.0
VDS 6 0 5
.model mos1 nmos (vto=0.7 kp=110u
+gamma=0.4 +lambda=.04 phi=.7)
.dc vds 0 5 .2
.print dc ID(M1), ID(M2), ID(M3), ID(M4),
ID(M5)
.end

```

## Transconductance Characteristics of an Enhancement NMOS Transistor



### SPICE Input File:

```

Transconductance Characteristics for NMOS
M1 1 6 0 0 MOS1 w=5u l=1.0u
VDS1 1 0 1.0
M2 2 6 0 0 MOS1 w=5u l=1.0u
VDS2 2 0 2.0
M3 3 6 0 0 MOS1 w=5u l=1.0u
VDS3 3 0 3.0
M4 4 6 0 0 MOS1 w=5u l=1.0u
VDS4 4 0 4.0

```

```

M5 5 6 0 0 MOS1 w=5u l=1.0u
VDS5 5 0 5.0
VGS 6 0 5
.model mos1 nmos (vto=0.7 kp=110u
+gamma=0.4 lambda=.04 phi=.7)
.dc vgs 0 5 .2
.print dc ID(M1), ID(M2), ID(M3), ID(M4),
ID(M5)
.probe
.end

```

## 3.2 - LARGE SIGNAL FET MODEL FOR HAND CALCULATIONS

### Large Signal Model Derivation

Derivation-

1.) Let the charge per unit area in the channel inversion layer be

$$Q_I(y) = -C_{ox}[v_{GS} - v(y) - V_T] \quad (\text{coul./cm}^2)$$

2.) Define sheet conductivity of the inversion layer per square as

$$\sigma_S = \mu_o Q_I(y) \left( \frac{\text{cm}^2}{\text{v}\cdot\text{s}} \right) \left( \frac{\text{coulombs}}{\text{cm}^2} \right) = \frac{\text{amps}}{\text{volt}} = \frac{1}{\Omega/\text{sq.}}$$

3.) Ohm's Law for current in a sheet is

$$J_S = \frac{i_D}{W} = -\sigma_S E_y = -\sigma_S \frac{dv}{dy} \rightarrow dv = \frac{-i_D}{\sigma_S W} dy = \frac{-i_D dy}{\mu_o Q_I(y) W} \rightarrow i_D dy = -W \mu_o Q_I(y) dv$$

4.) Integrating along the channel for 0 to  $L$  gives

$$\int_0^L i_D dy = - \int_0^{v_{DS}} W \mu_o Q_I(y) dv = \int_0^{v_{DS}} W \mu_o C_{ox} [v_{GS} - v(y) - V_T] dv$$

5.) Evaluating the limits gives

$$i_D = \frac{W \mu_o C_{ox}}{L} \left[ (v_{GS} - V_T) v(y) - \frac{v^2(y)}{2} \right]_0^{v_{DS}} \rightarrow \boxed{i_D = \frac{W \mu_o C_{ox}}{L} \left[ (v_{GS} - V_T) v_{DS} - \frac{v_{DS}^2}{2} \right]}$$

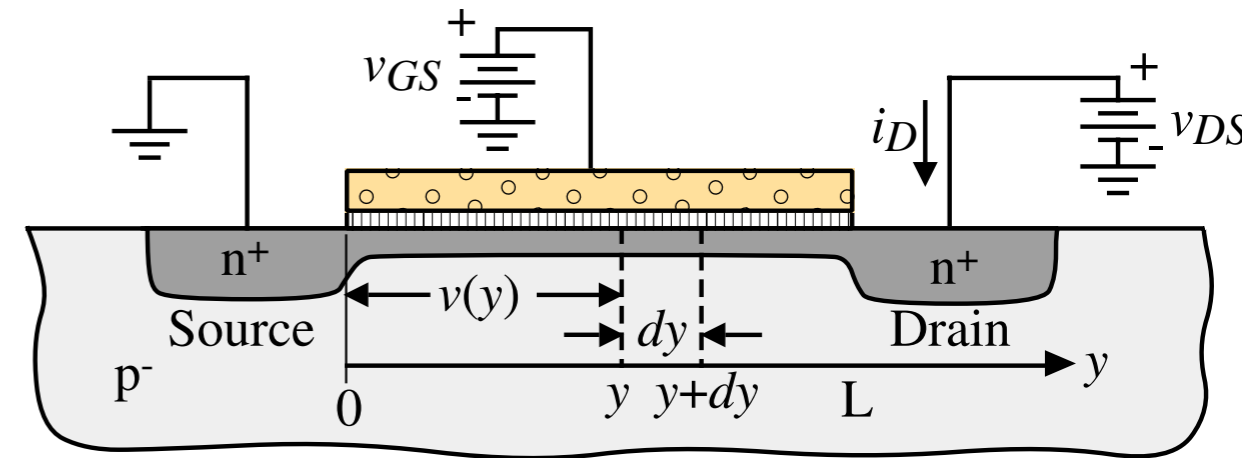


Fig.110-03



## Saturation Voltage - $V_{DS}(\text{sat})$

Interpretation of the large signal model:

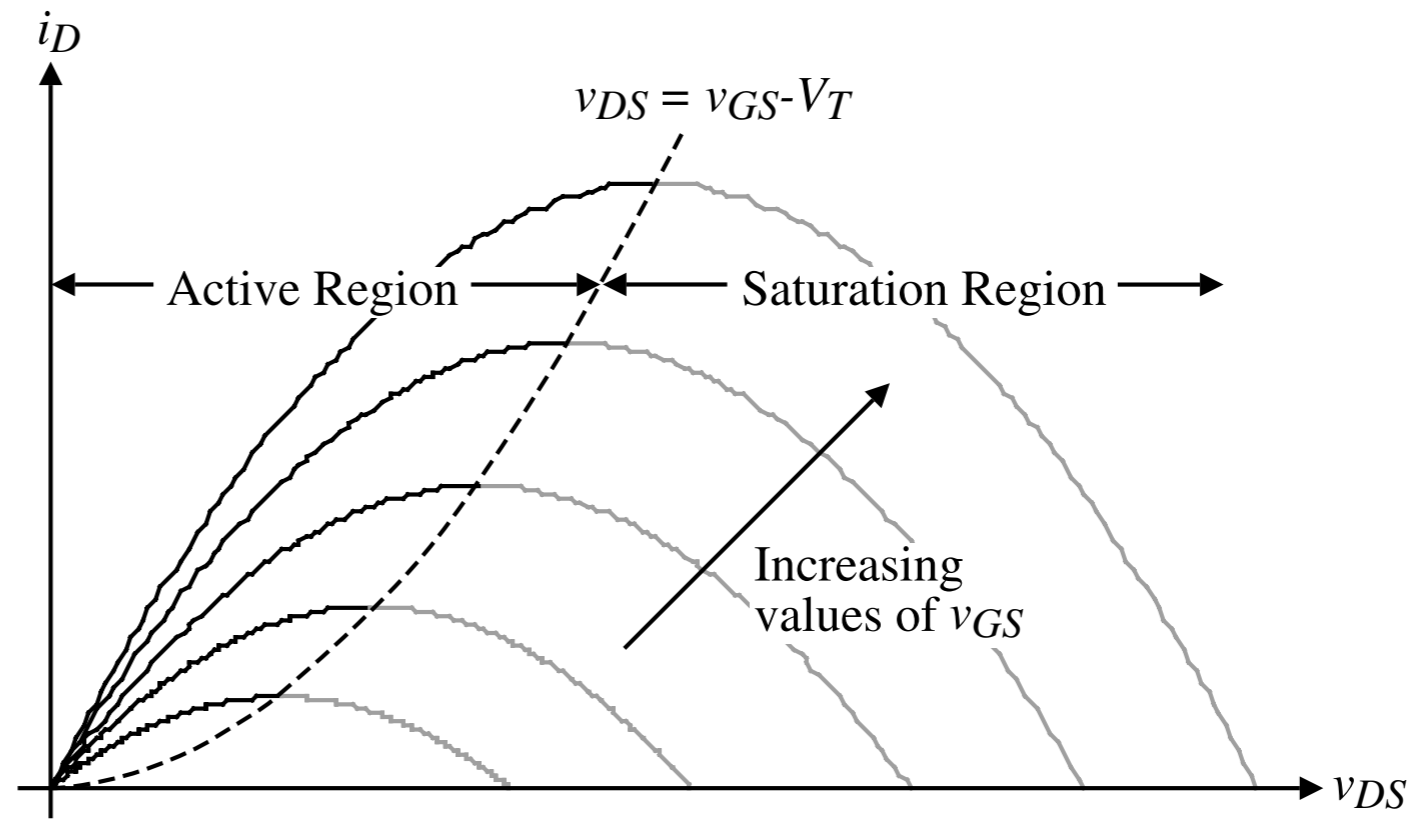


Fig. 110-04

The saturation voltage for MOSFETs is the value of drain-source voltage at the peak of the inverted parabolas.

$$\frac{di_D}{dv_{DS}} = \frac{\mu_o C_{ox} W}{L} [(v_{GS} - V_T) - v_{DS}] = 0$$

$$\boxed{v_{DS}(\text{sat}) = v_{GS} - V_T}$$

Useful definitions:

$$\frac{\mu_o C_{ox} W}{L} = \frac{K' W}{L} = \beta$$

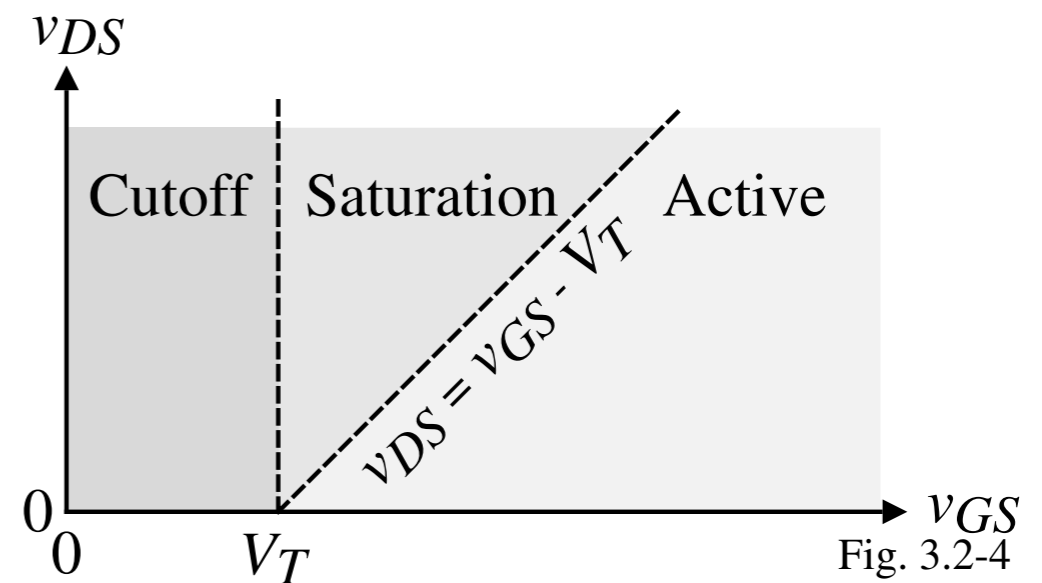


Fig. 3.2-4

## The Simple Large Signal MOSFET Model

### Regions of Operation of the MOS Transistor:

#### 1.) Cutoff Region:

$$v_{GS} - V_T < 0$$

$$i_D = 0$$

(Ignores subthreshold currents)

#### 2.) Active Region

$$0 < v_{DS} < v_{GS} - V_T$$

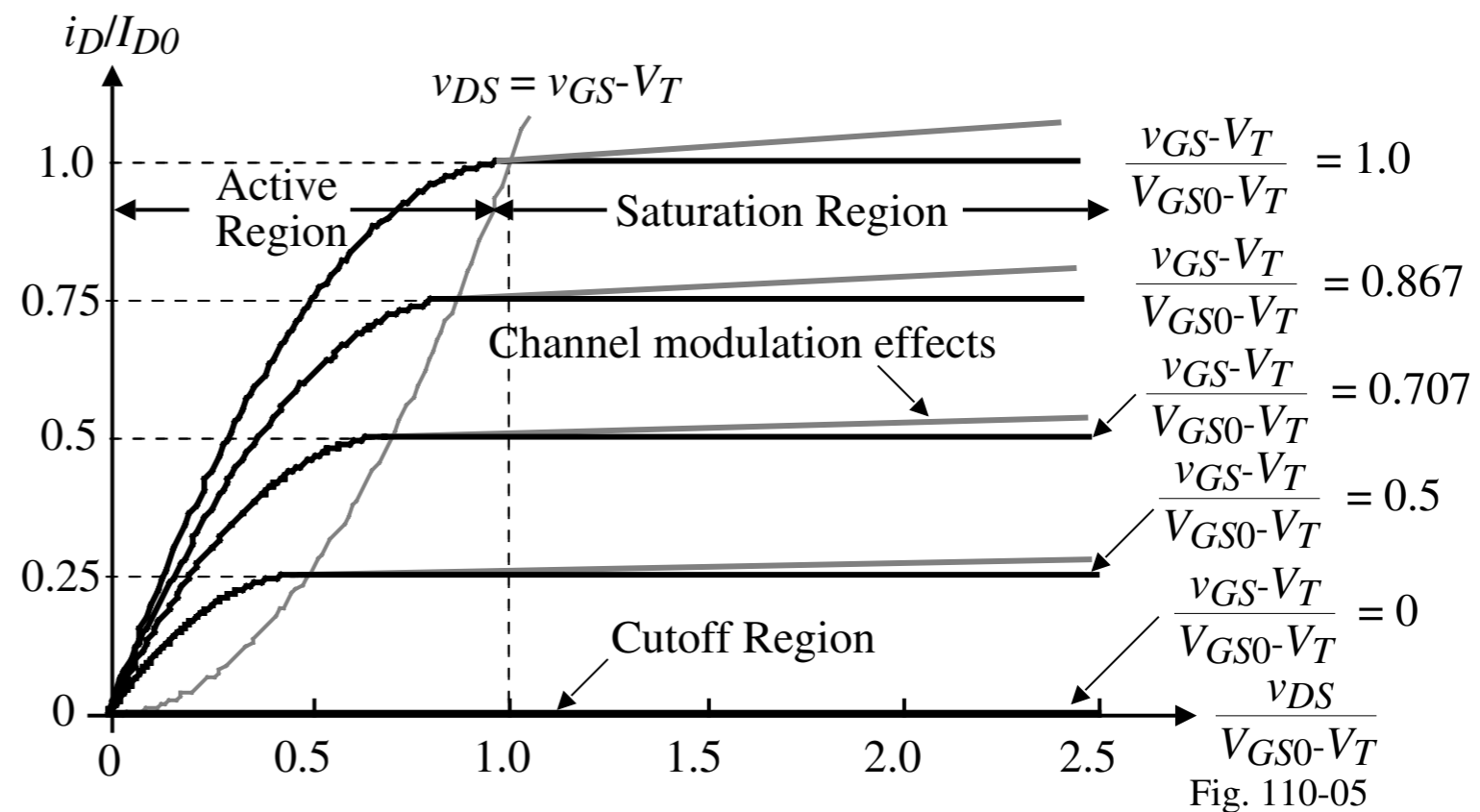
$$i_D = \frac{\mu_o C_{ox} W}{2L} [2(v_{GS} - V_T) - v_{DS}] v_{DS}$$

#### 3.) Saturation Region

$$0 < v_{GS} - V_T < v_{DS}$$

$$i_D = \frac{\mu_o C_{ox} W}{2L} (v_{GS} - V_T)^2$$

### Output Characteristics of the MOSFET:



## Influence of $V_{DS}$ on the Output Characteristics

Channel modulation effect:

As the value of  $v_{DS}$  increases, the effective  $L$  decreases causing the current to increase.

Illustration:

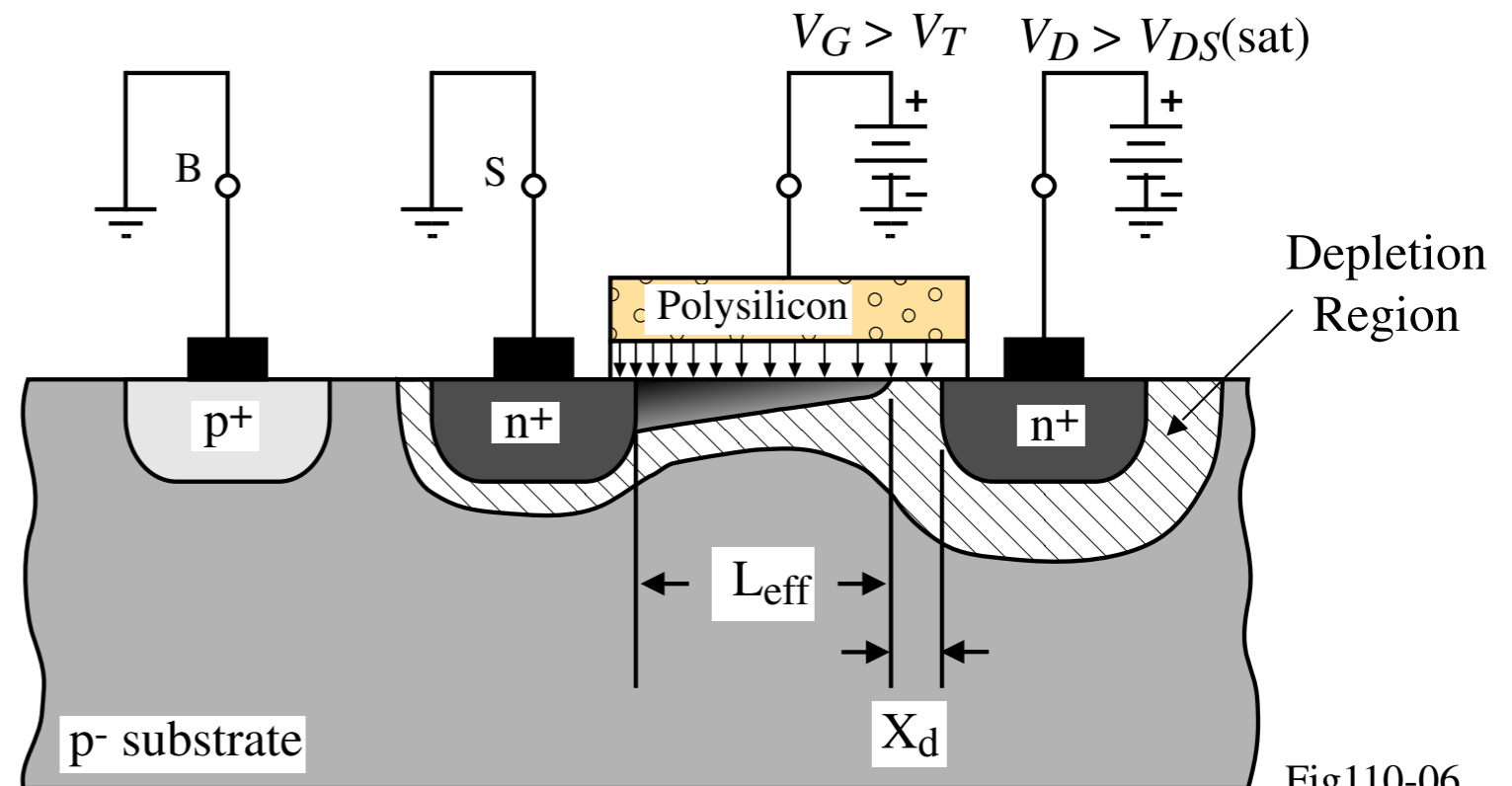
Note that  $L_{\text{eff}} = L - X_d$

Therefore the model in saturation becomes,

$$i_D = \frac{K'W}{2L_{\text{eff}}} (v_{GS} - V_T)^2 \rightarrow \frac{di_D}{dv_{DS}} = -\frac{K'W}{2L_{\text{eff}}^2} (v_{GS} - V_T)^2 \frac{dL_{\text{eff}}}{dv_{DS}} = \frac{i_D}{L_{\text{eff}}} \frac{dX_d}{dv_{DS}} \equiv \lambda i_D$$

Therefore, a good approximation to the influence of  $v_{DS}$  on  $i_D$  is

$$i_D \approx i_D(\lambda = 0) + \frac{di_D}{dv_{DS}} v_{DS} = i_D(\lambda = 0)(1 + \lambda v_{DS}) = \frac{K'W}{2L} (v_{GS} - V_T)^2 (1 + \lambda v_{DS})$$



## Channel Length Modulation Parameter, $\lambda$

Assume the MOS transistor is saturated-

$$\therefore i_D = \frac{\mu C_{ox} W}{2L} (v_{GS} - V_T)^2 (1 + \lambda v_{DS})$$

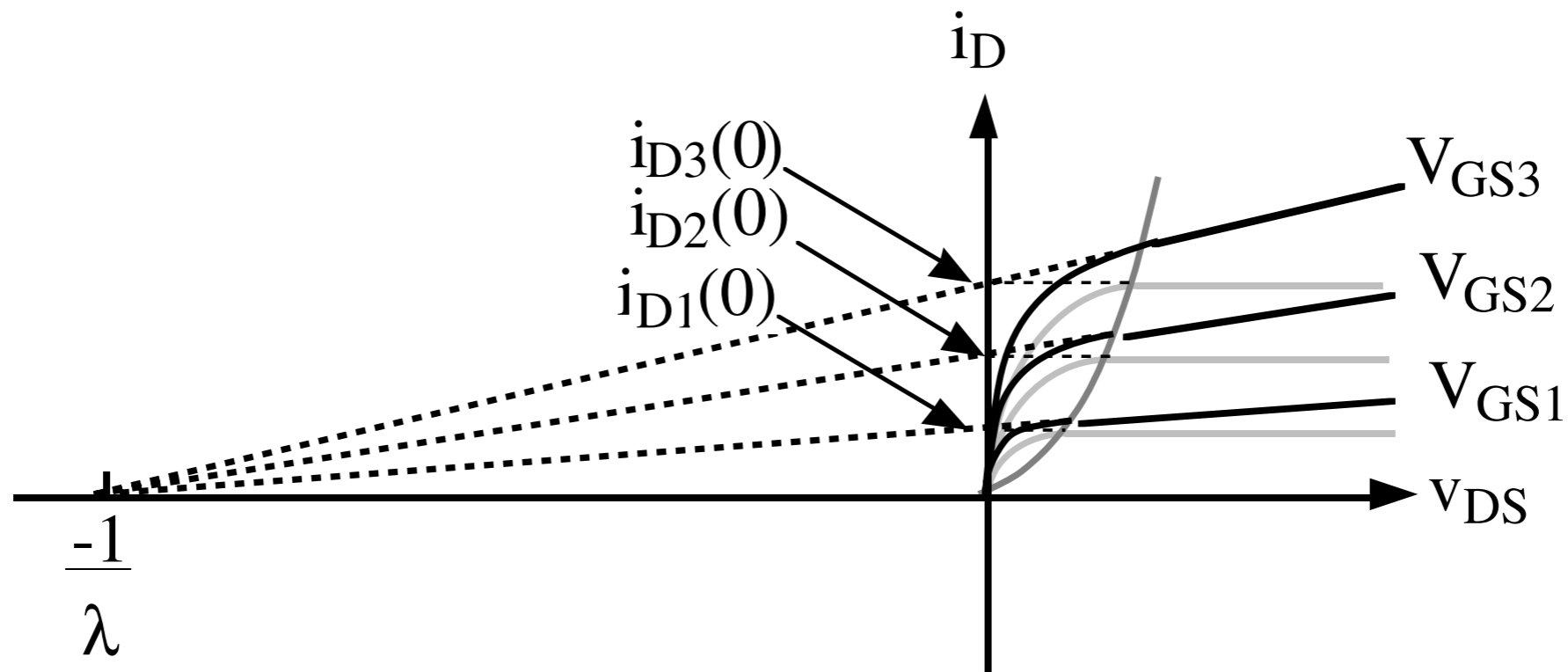
Define  $i_D(0) = i_D$  when  $v_{DS} = 0V$ .

$$\therefore i_D(0) = \frac{\mu C_{ox} W}{2L} (v_{GS} - V_T)^2$$

Now,

$$i_D = i_D(0)[1 + \lambda v_{DS}] = i_D(0) + \lambda i_D(0) v_{DS}$$

Matching with  $y = mx + b$  gives the value of  $\lambda$



## Influence of the Bulk Voltage on the Large Signal MOSFET Model

Illustration of the influence of the bulk:

$V_{SB0} = 0V$ :

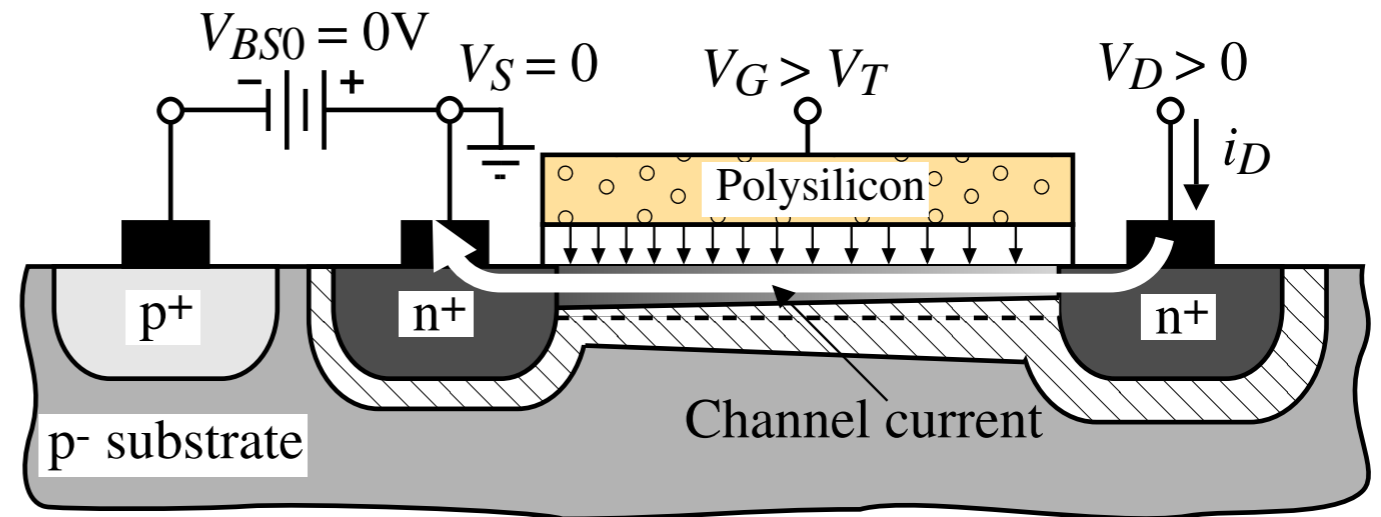


Fig.110-07A

$V_{SB1} > 0V$ :

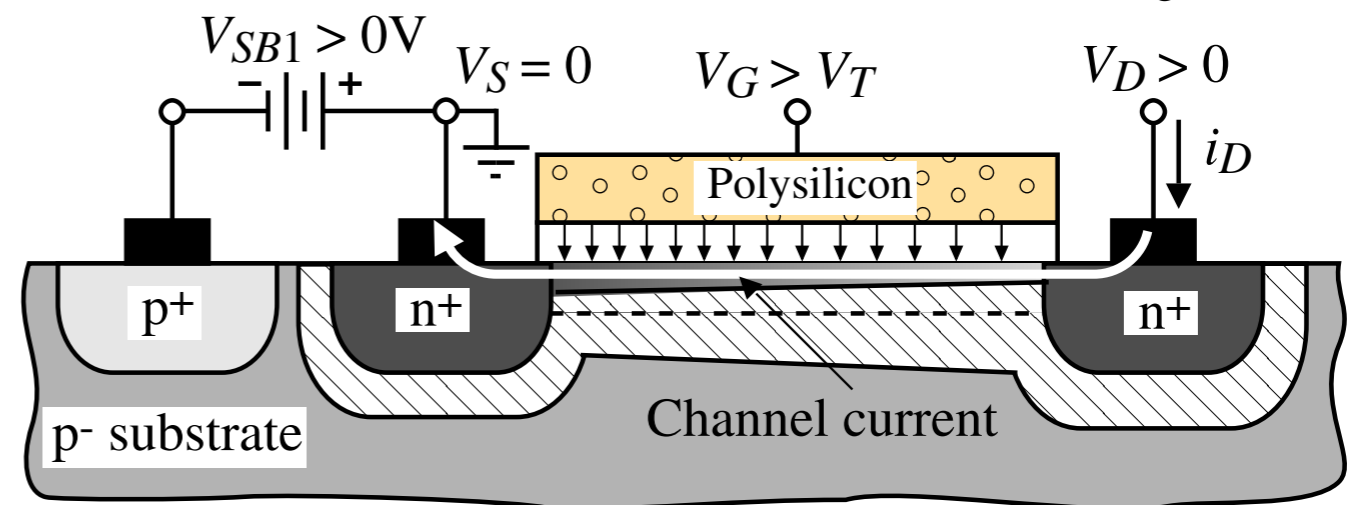


Fig.110-07B

$V_{SB2} > V_{SB1}$ :

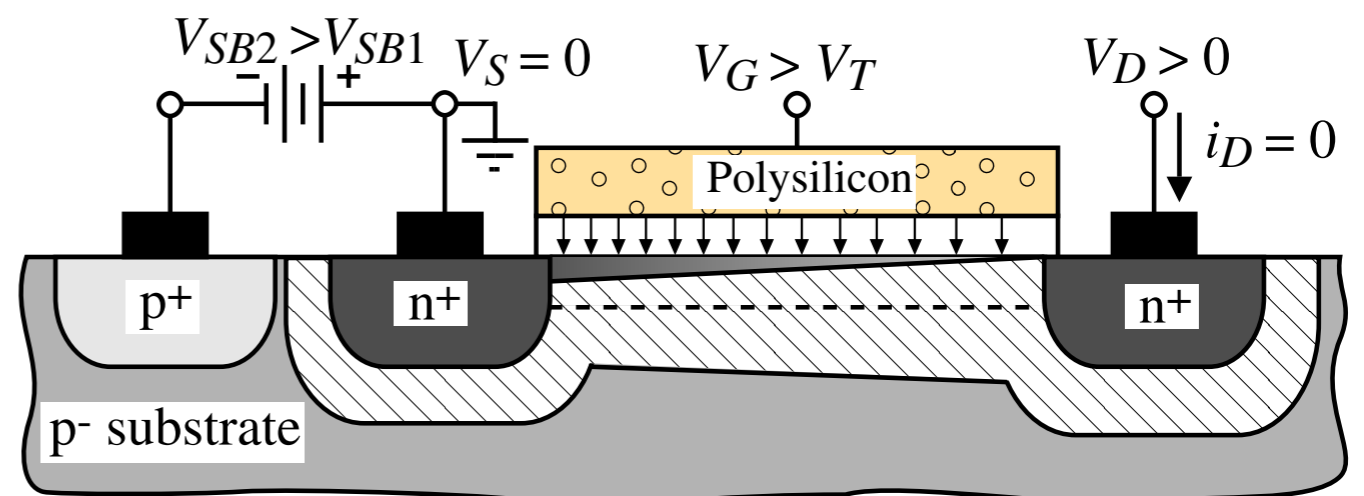


Fig.110-07C

## Influence of the Bulk Voltage on the Large Signal MOSFET Model - Continued

Bulk-Source ( $v_{BS}$ ) influence on the transconductance characteristics-

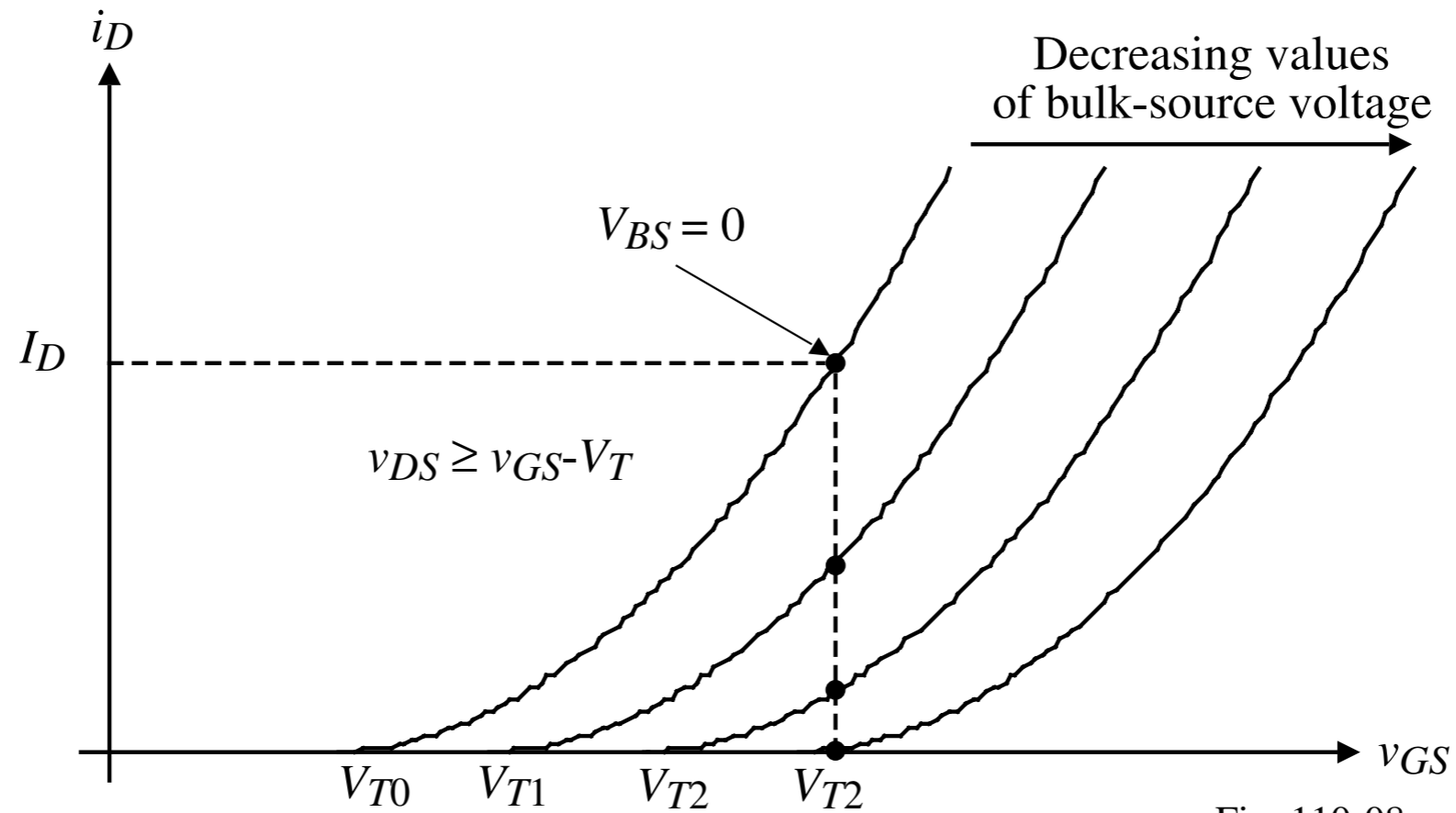


Fig. 110-08

In general, the simple model incorporates the bulk effect into  $V_T$  by the previously developed relationship:

$$V_T(v_{BS}) = V_{T0} + \gamma \sqrt{2|\phi_f| + |v_{BS}|} - \gamma \sqrt{2|\phi_f|}$$

## Summary of the Simple Large Signal MOSFET Model

N-channel reference convention:

Non-saturation-

$$i_D = \frac{W\mu_o C_{ox}}{L} \left[ (v_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2} \right] (1 + \lambda v_{DS})$$

Saturation-

$$i_D = \frac{W\mu_o C_{ox}}{L} \left[ (v_{GS} - V_T)v_{DS}(\text{sat}) - \frac{v_{DS}(\text{sat})^2}{2} \right] (1 + \lambda v_{DS}) = \frac{W\mu_o C_{ox}}{2L} (v_{GS} - V_T)^2 (1 + \lambda v_{DS})$$

where:

$\mu_o$  = zero field mobility (cm<sup>2</sup>/volt·sec)

$C_{ox}$  = gate oxide capacitance per unit area (F/cm<sup>2</sup>)

$\lambda$  = channel-length modulation parameter (volts<sup>-1</sup>)

$V_T = V_{T0} + \gamma(\sqrt{2|\phi_f| + |v_{BS}|} - \sqrt{2|\phi_f|})$

$V_{T0}$  = zero bias threshold voltage

$\gamma$  = bulk threshold parameter (volts<sup>-0.5</sup>)

$2|\phi_f|$  = strong inversion surface potential (volts)

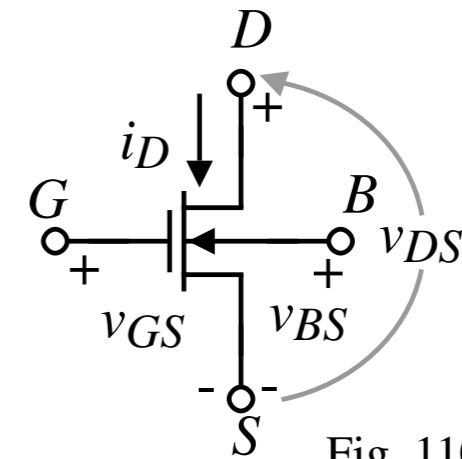


Fig. 110-10

For p-channel MOSFETs, use n-channel equations with p-channel parameters and invert current.

## Silicon Constants

Constant Symbol	Constant Description	Value	Units
$V_G$	Silicon bandgap (27°C)	1.205	V
$k$	Boltzmann's constant	$1.381 \times 10^{-23}$	J/K
$n_i$	Intrinsic carrier concentration (27°C)	$1.45 \times 10^{10}$	cm <sup>-3</sup>
$\epsilon_0$	Permittivity of free space	$8.854 \times 10^{-14}$	F/cm
$\epsilon_{si}$	Permittivity of silicon	$11.7 \epsilon_0$	F/cm
$\epsilon_{ox}$	Permittivity of SiO <sub>2</sub>	$3.9 \epsilon_0$	F/cm



## MOSFET Parameters

Model Parameters for a Typical CMOS Bulk Process (0.8 $\mu\text{m}$  CMOS n-well):

Parameter Symbol	Parameter Description	Typical Parameter Value		Units
		N-Channel	P-Channel	
$V_{T0}$	Threshold Voltage ( $V_{BS} = 0$ )	$0.7 \pm 0.15$	$-0.7 \pm 0.15$	V
$K'$	Transconductance Parameter (in saturation)	$110.0 \pm 10\%$	$50.0 \pm 10\%$	$\mu\text{A}/\text{V}^2$
$\gamma$	Bulk threshold parameter	0.4	0.57	(V) <sup>1/2</sup>
$\lambda$	Channel length modulation parameter	0.04 (L=1 $\mu\text{m}$ ) 0.01 (L=2 $\mu\text{m}$ )	0.05 (L=1 $\mu\text{m}$ ) 0.01 (L=2 $\mu\text{m}$ )	(V) <sup>-1</sup>
$2 \phi_F $	Surface potential at strong inversion	0.7	0.8	V

## **3.3 - LARGE SIGNAL MODEL EXTENSIONS TO SHORT-CHANNEL MOSFETS**

### **Extensions**

- Velocity saturation
- Weak inversion (subthreshold)
- Substrate currents

### **Substrate Interference**

- Problems of mixed signal circuits on the same substrate
- Modeling and potential solutions

## VELOCITY SATURATION

### What is Velocity Saturation?

The most important short-channel effect in MOSFETs is the velocity saturation of carriers in the channel. A plot of electron drift velocity versus electric field is shown below.

An expression for the electron drift velocity as a function of the electric field is,

$$v_d \approx \frac{\mu_n E}{1 + E/E_c}$$

where

$v_d$  = electron drift velocity (m/s)

$\mu_n$  = low-field mobility ( $\approx 0.07\text{m}^2/\text{V}\cdot\text{s}$ )

$E_c$  = critical electrical field at which velocity saturation occurs

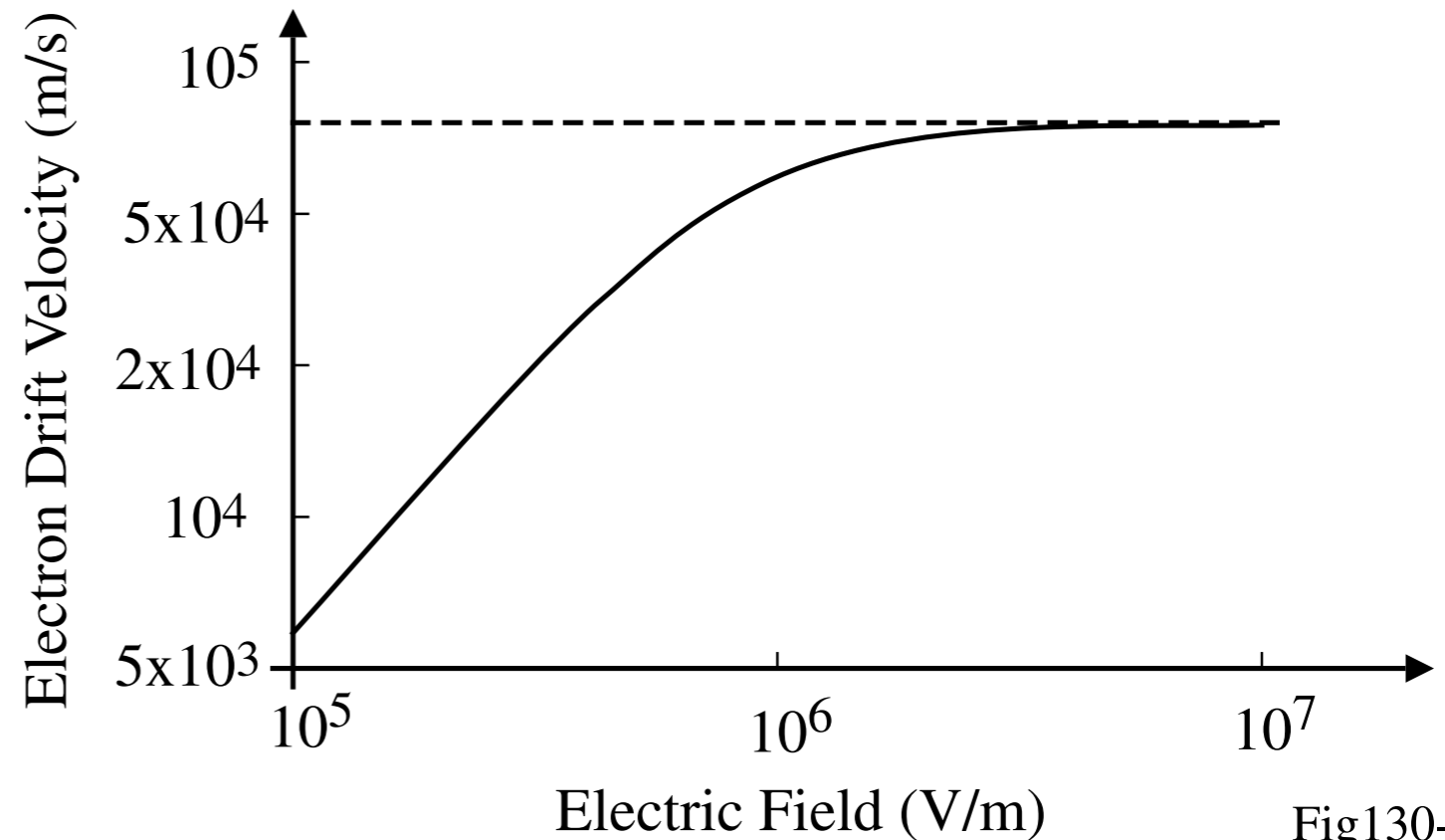


Fig130-1

## Important Short Channel Effects

- 1.) An approximate plot of the  $n$  as a function of channel length is shown below where

$$i_D \propto (v_{GS} - V_T)^n$$

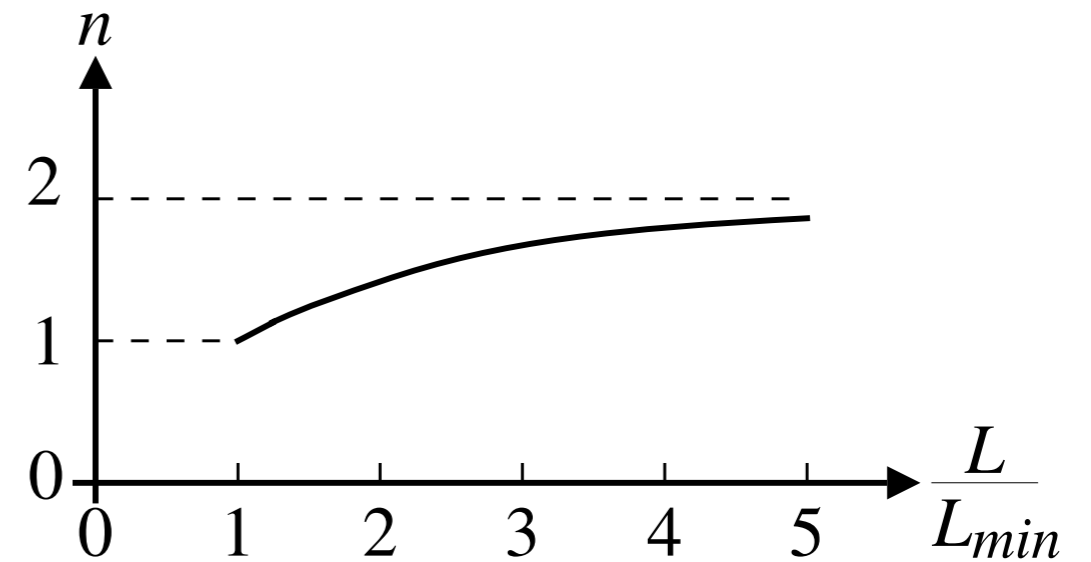


Fig.130-5

- 2.) Note that the value of  $\lambda$  varies with channel length,  $L$ . The data below is from a  $0.25\mu\text{m}$  CMOS technology.

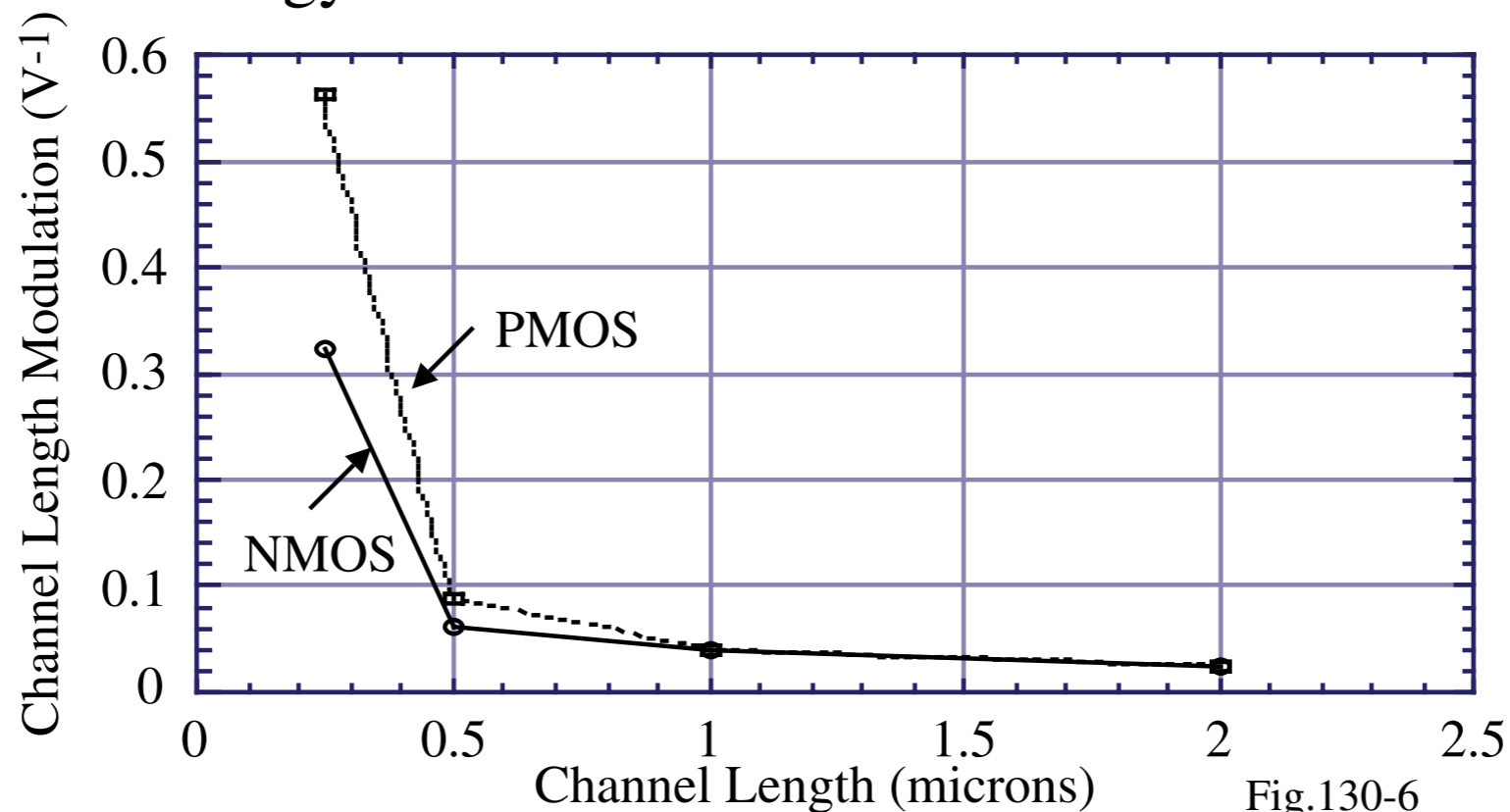


Fig.130-6

## SUBTHRESHOLD MOSFET MODEL

### What is Weak Inversion Operation?

Weak inversion operation occurs when the applied gate voltage is below  $V_T$  and pertains to when the surface of the substrate beneath the gate is weakly inverted.

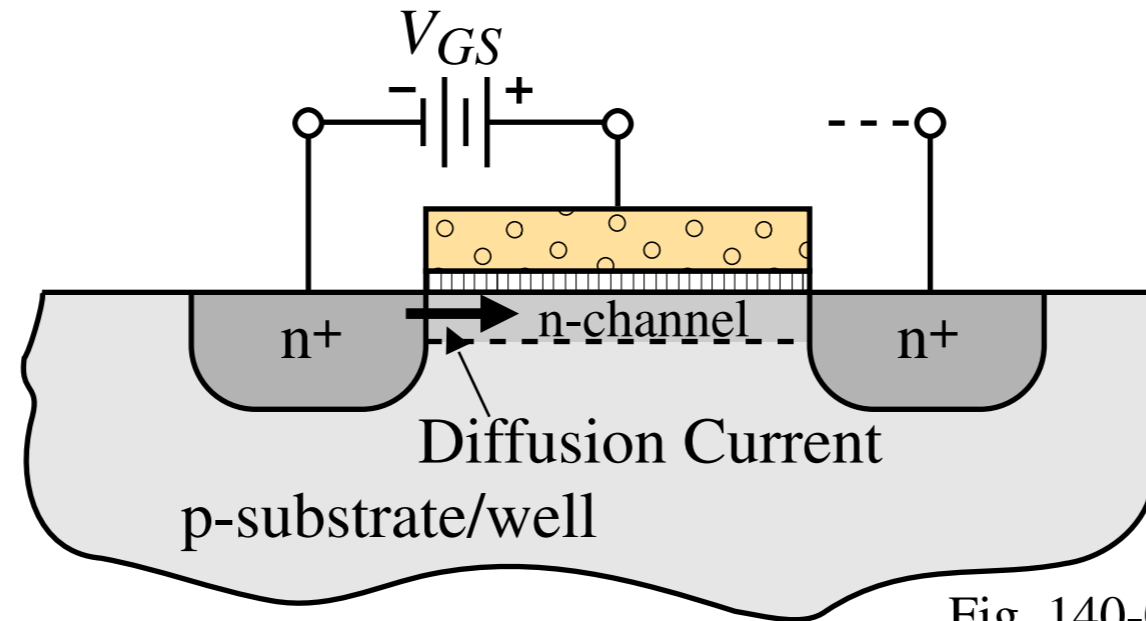


Fig. 140-01

Regions of operation according to the surface potential,  $\phi_S$  (or  $\psi_S$ )

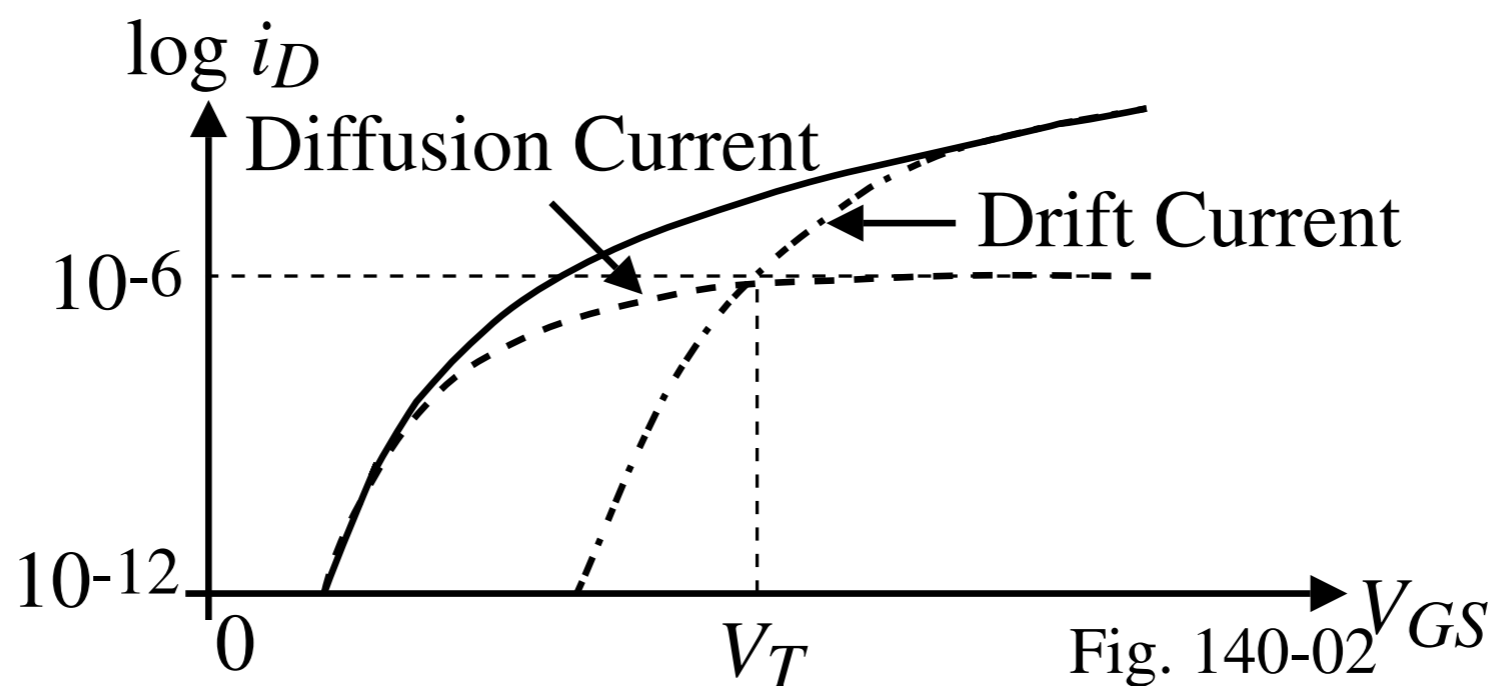
- |                               |  |
|-------------------------------|--|
| $\phi_S < \phi_F :$           | Substrate not inverted                         |
| $\phi_F < \phi_S < 2\phi_F :$ | Channel is weakly inverted (diffusion current) |
| $2\phi_F < \phi_S :$          | Strong inversion (drift current)               |

## Drift versus Diffusion Current

- 1.) For strong inversion, the gate voltage controls the charge in the inverted region but not in the depletion region. The concentration of charge across the channel is approximately constant and the current is drift caused by electric field.
- 2.) For weak inversion, the charge in channel is much less than that in the depletion region and drift current decreases. However, there is a concentration gradient in the channel, that causes diffusion current.

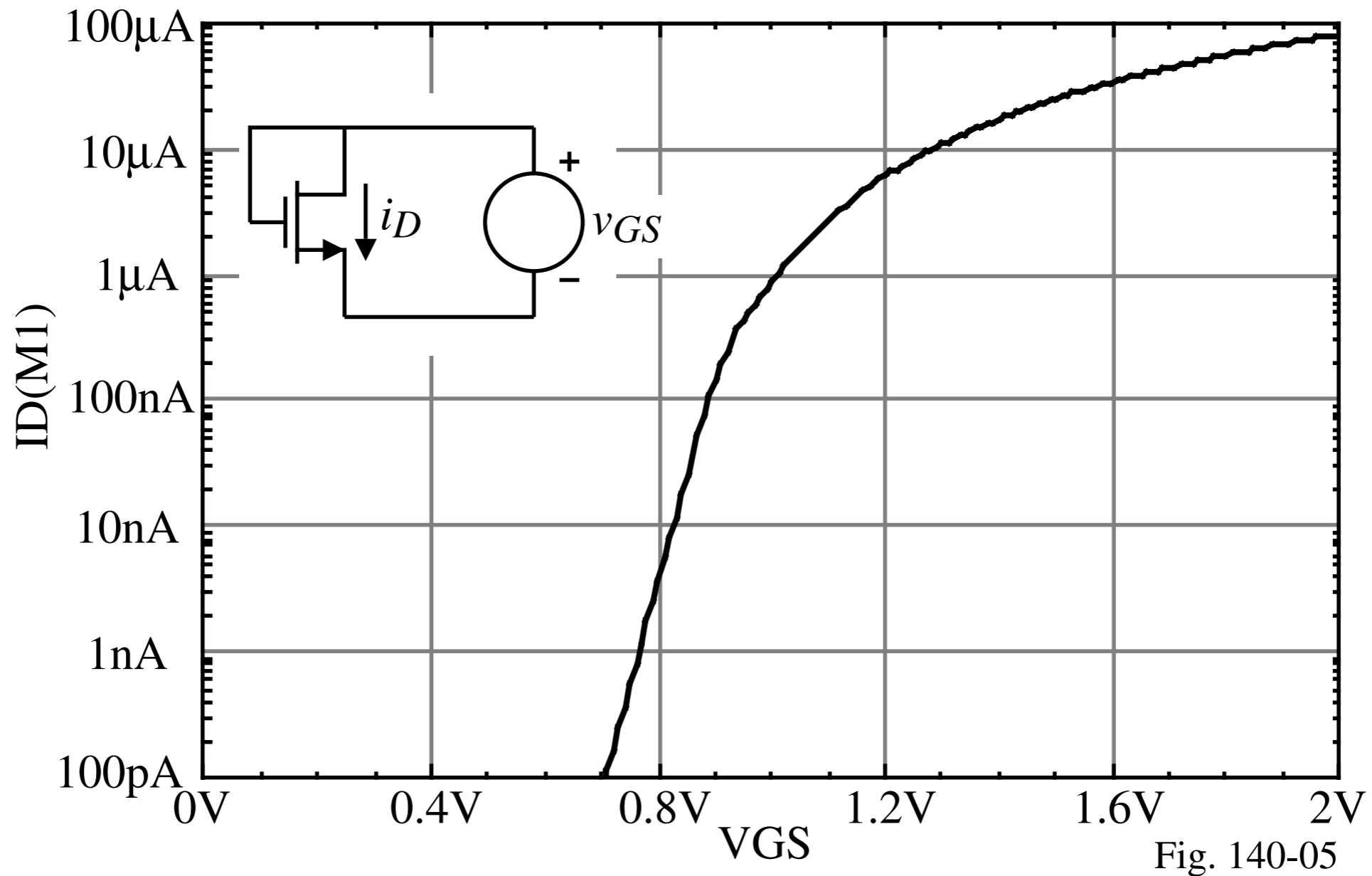
*The n-channel MOSFET acts like a NPN BJT: the emitter is the source, the base is the substrate and the collector is the drain.*

Illustration:



## Simulation of an n-channel MOSFET in both Weak and Strong Inversion

Uses the BSIM model.<sup>†</sup>



<sup>†</sup> Y. Cheng and C. Hu, *MOSFET Modeling & BSIM3 User's Guide*, Kluwer Academic Publishers, Boston, 1999.

## SUBSTRATE CURRENT FLOW IN MOSFETS

### Impact Ionization

Impact Ionization:

Occurs because high electric fields cause an impact which generates a hole-electron pair. The electrons flow out the drain and the holes flow into the substrate causing a substrate current flow.

Illustration:

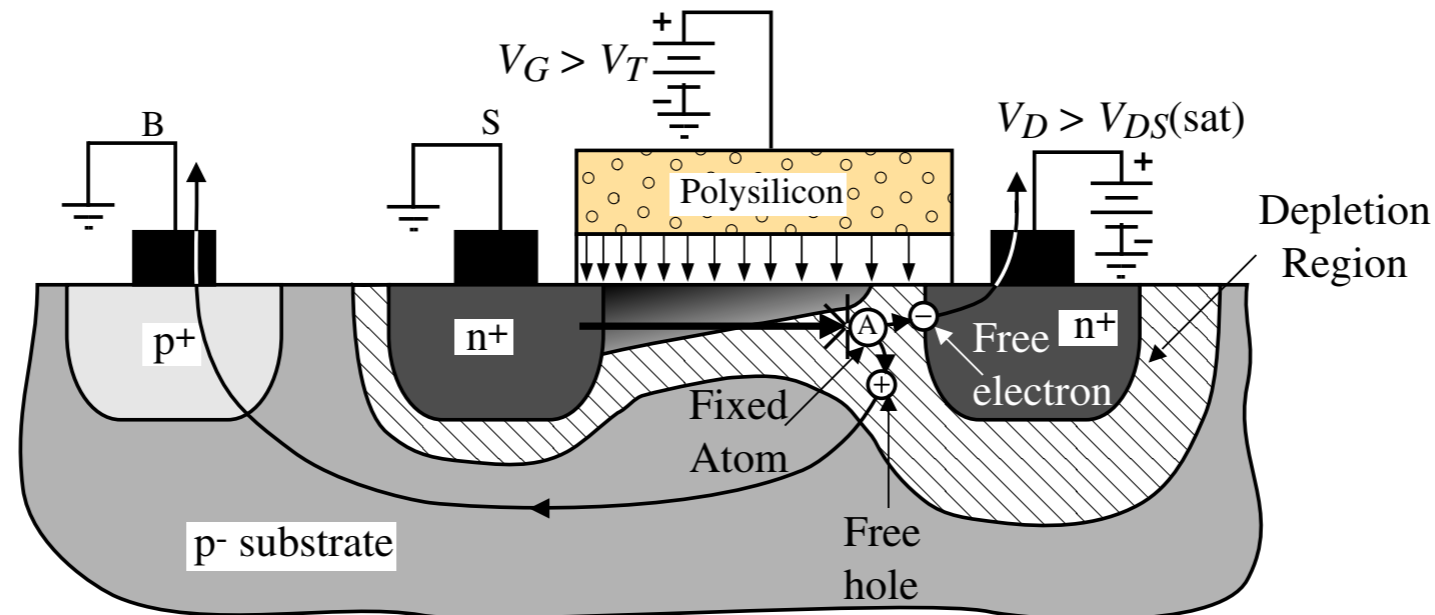


Fig130-7



## **SUBSTRATE INTERFERENCE IN CMOS CIRCUITS**

### **How Do Carriers Get Injected into the Substrate?**

- 1.) Hot carriers (substrate current)
- 2.) Electrostatic coupling (across depletion regions and other dielectrics)
- 3.) Electromagnetic coupling (parallel conductors)

### **Why is this a Problem?**

With decreasing channel lengths, more circuitry is being integrated on the same substrate. The result is that noisy circuits (circuits with rapid transitions) are beginning to adversely influence sensitive circuits (such as analog circuits).

### **Present Solution**

Keep circuit separate by using multiple substrates and put the multiple substrates in the same package.

# Hot Carrier Injection in CMOS Technology without an Epitaxial Region

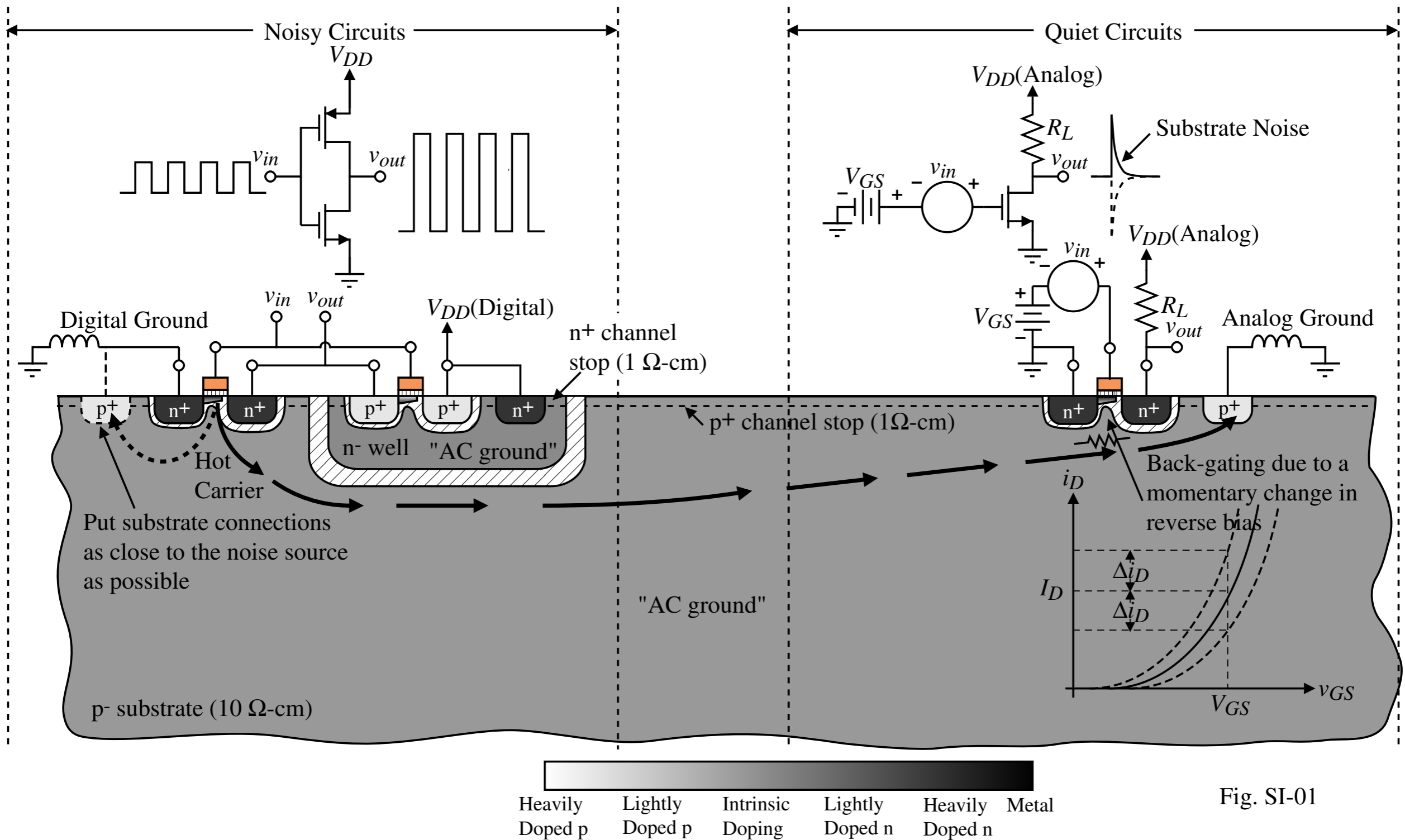


Fig. SI-01

# Hot Carrier Injection in CMOS Technology with an Epitaxial Region

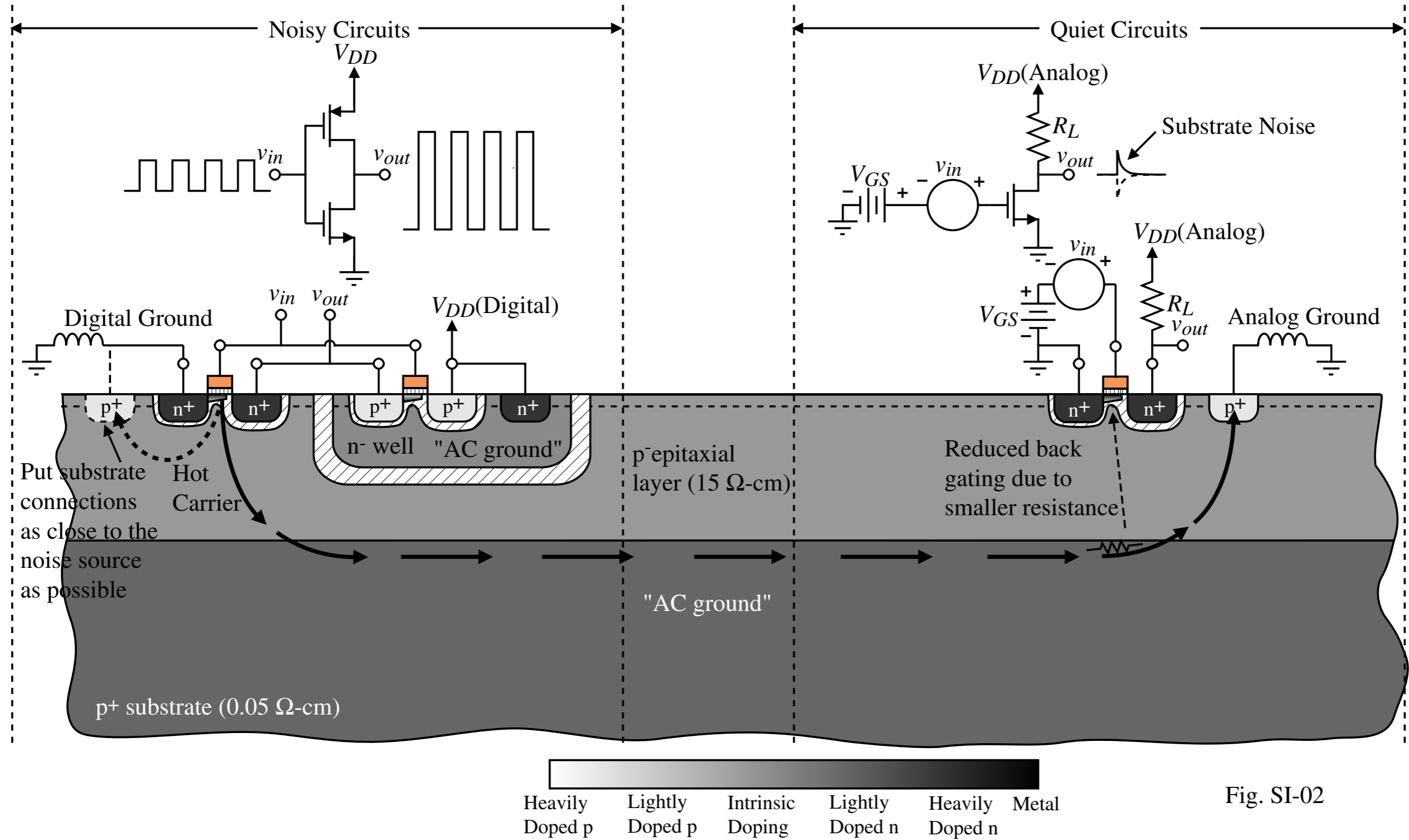
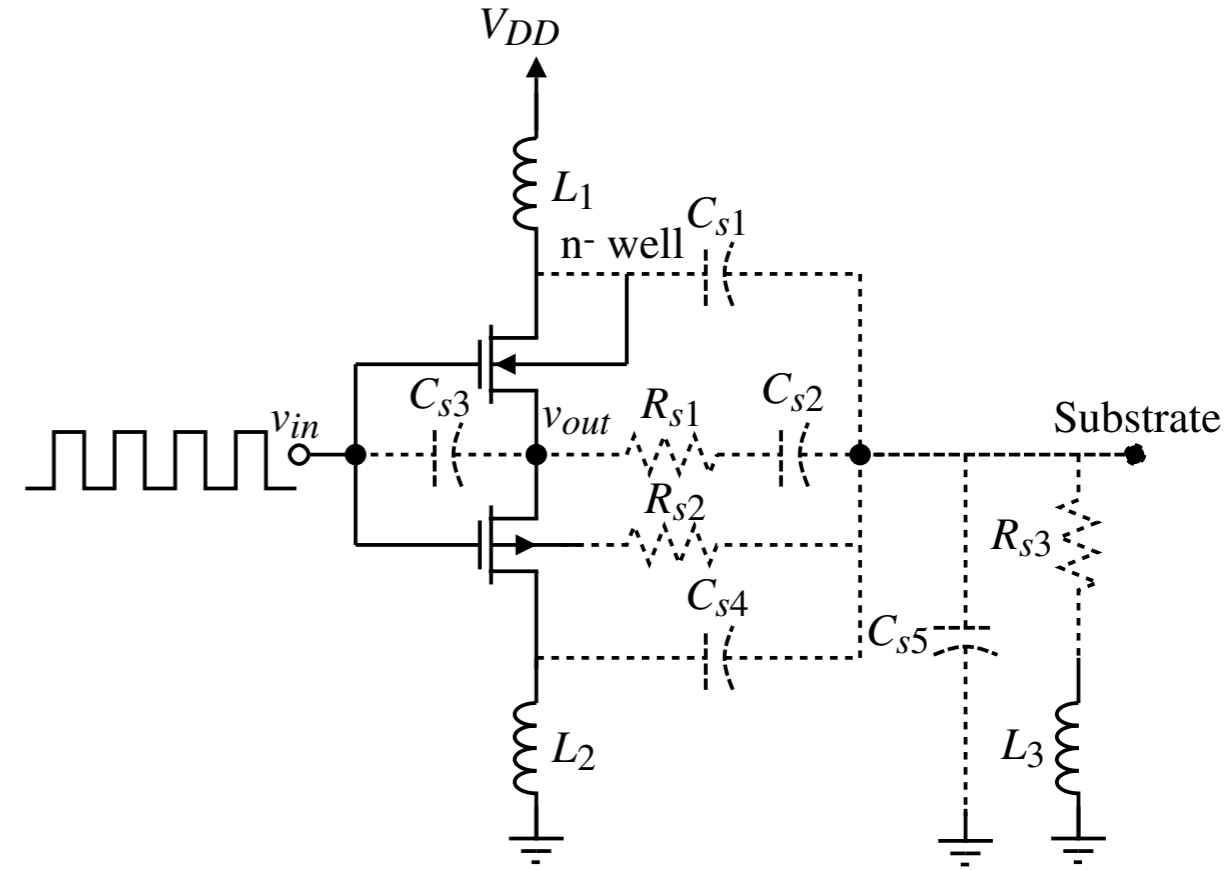
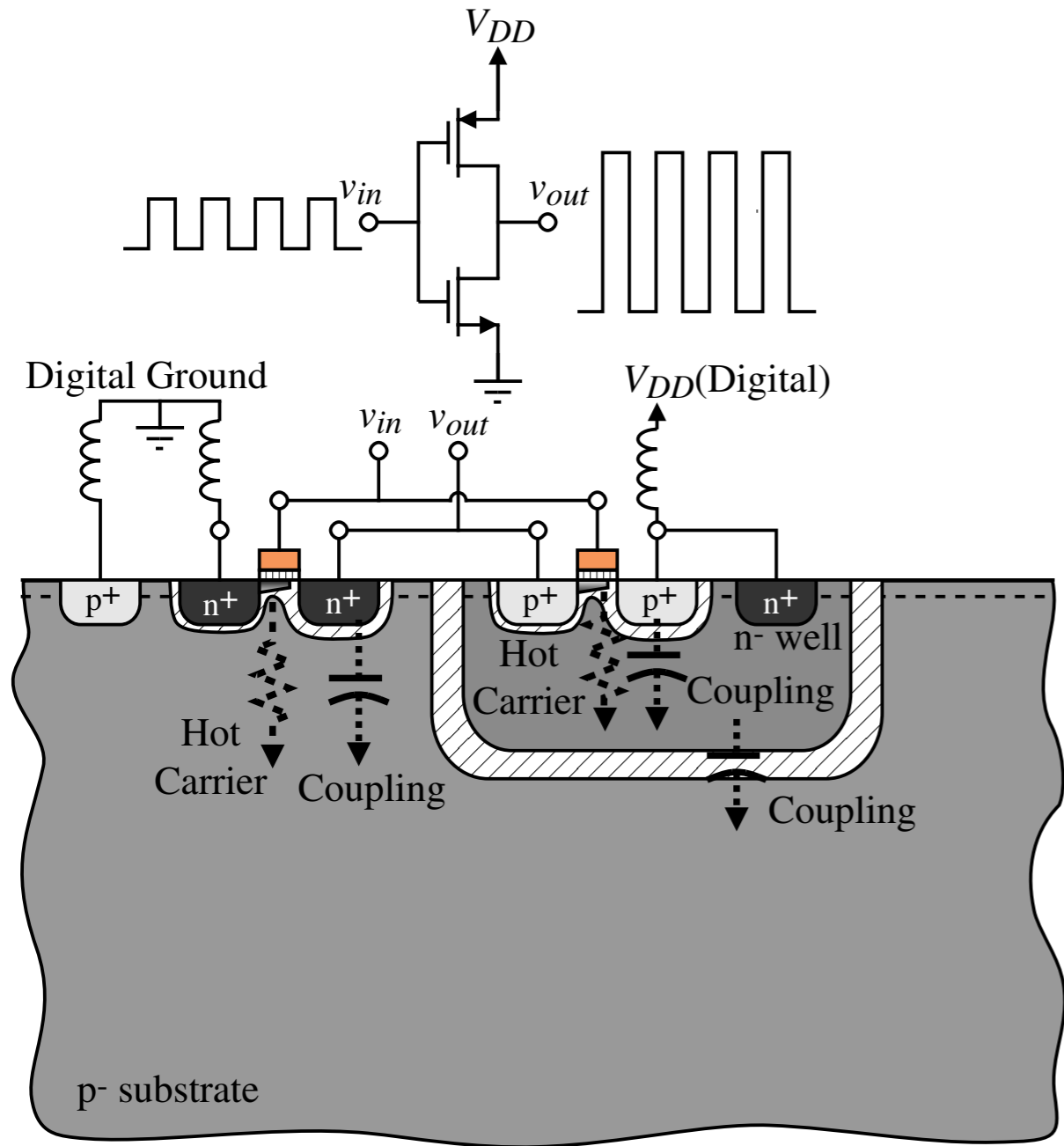


Fig. SI-02

# Computer Model for Substrate Interference Using SPICE Primitives

## Noise Injection Model:



- $C_{s1}$  = Capacitance between n-well and substrate
- $C_{s2}, C_{s3}$  and  $C_{s4}$  = Capacitances between interconnect lines (including bond pads) and substrate
- $C_{s5}$  = All capacitance between the substrate and ac ground
- $R_{s1}, R_{s2}$  and  $R_{s3}$  = Bulk resistances in n-well and substrate
- $L_1, L_2$  and  $L_3$  = Inductance of the bond wires and package leads

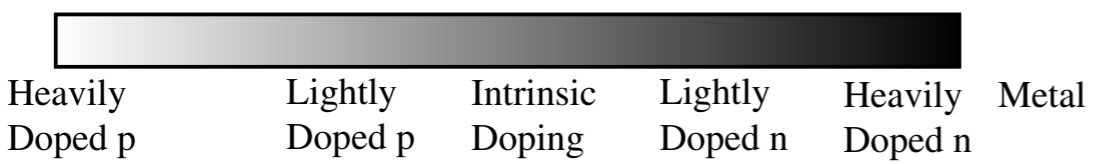
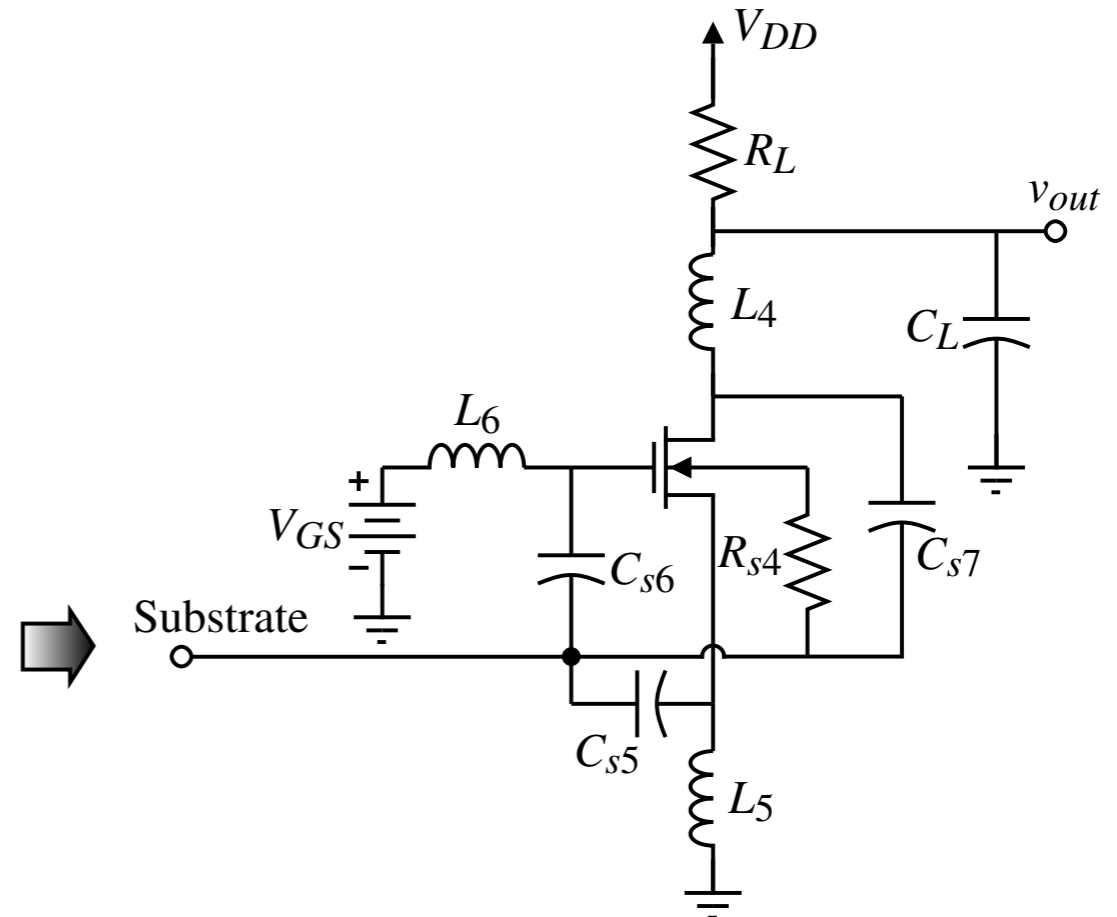
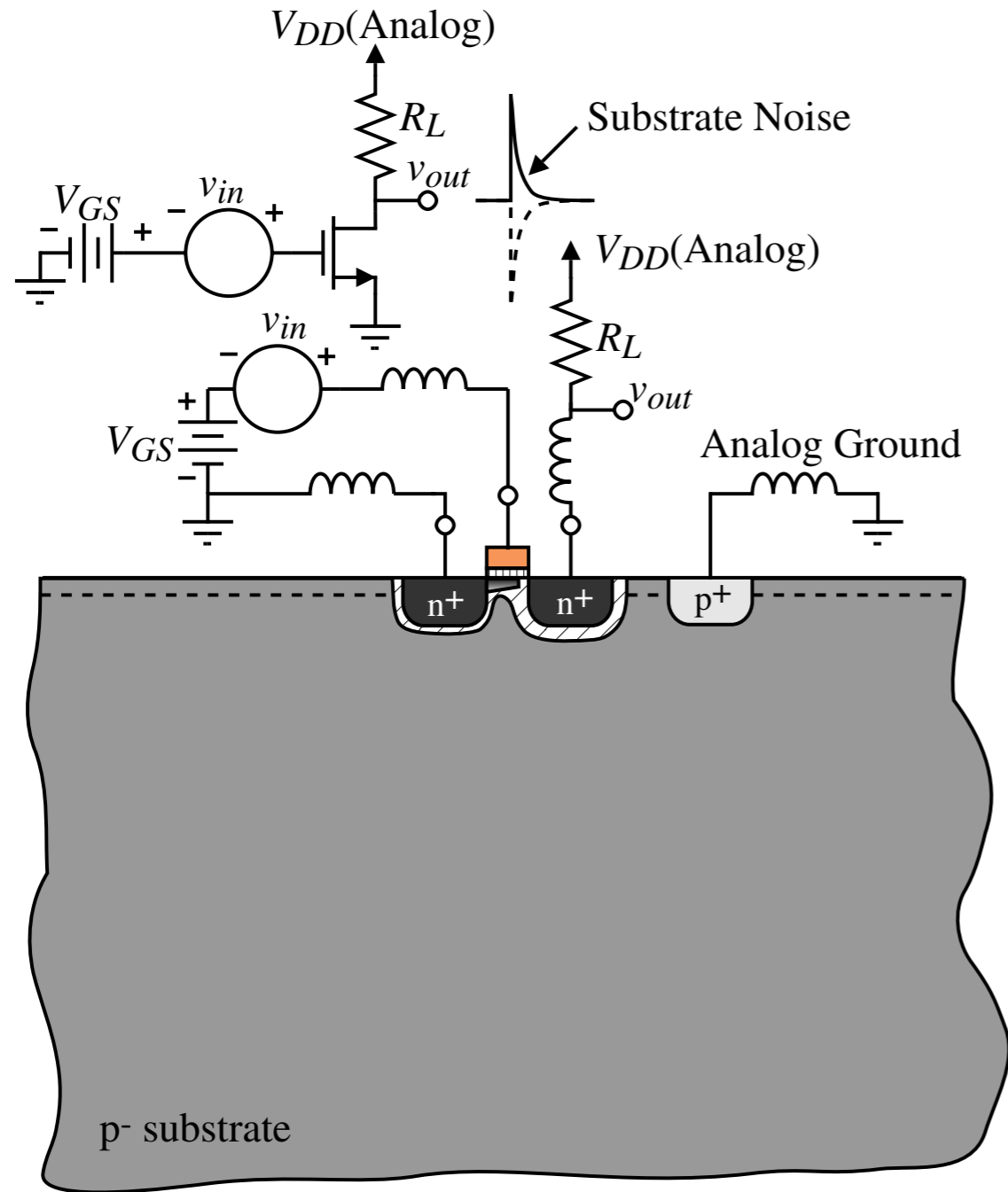


Fig. SI-06

# Computer Model for Substrate Interference Using SPICE Primitives

## Noise Detection Model:



$C_{s5}, C_{s6}$  and  $C_{s7}$  = Capacitances between interconnect lines (including bond pads) and substrate  
 $R_{s4}$  = Bulk resistance in the substrate  
 $L_4, L_5$  and  $L_6$  = Inductance of the bond wires and package leads

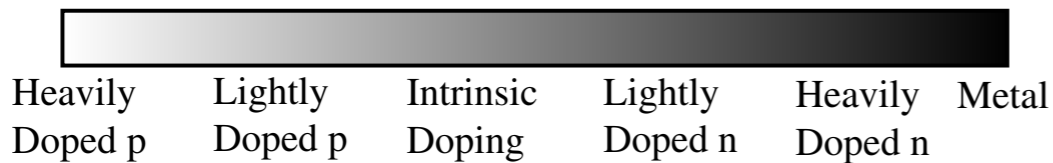


Fig. SI-07

## Other Sources of Substrate Injection

(We do it to ourselves and can't blame the digital circuits.)

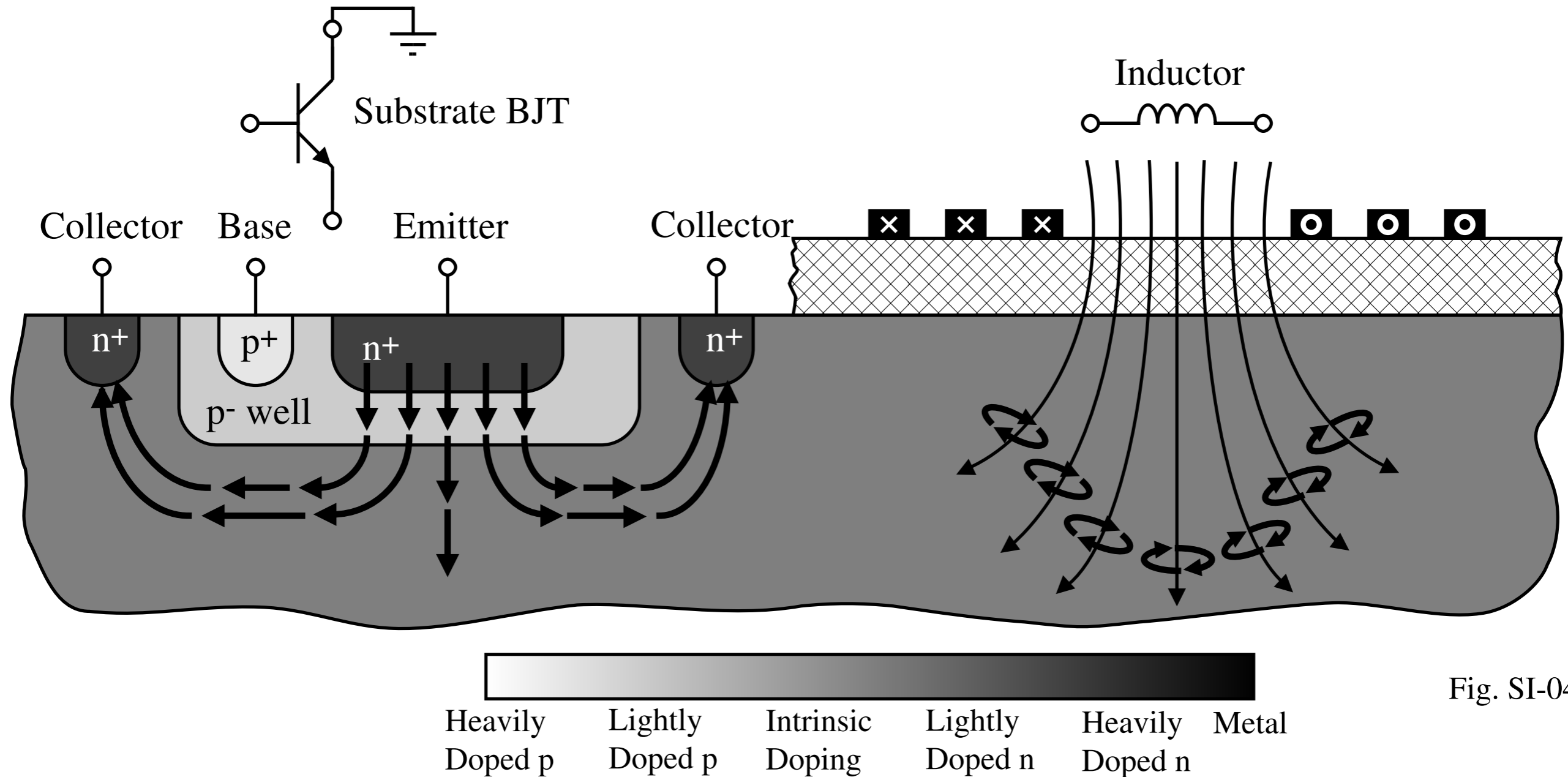


Fig. SI-04

Also, there is coupling from power supplies and clock lines to other adjacent signal lines.

## What is a Good Ground?

- On-chip, it is a region with very low bulk resistance.  
It is best accomplished by connecting metal to the region at as many points as possible.
- Off-chip, it is all determined by the connections or bond wires.

The inductance of the bond wires is large enough to create significant ground potential changes for fast current transients.

$$v = L \frac{di}{dt}$$

Use multiple bonding wires to reduce the ground noise caused by inductance.

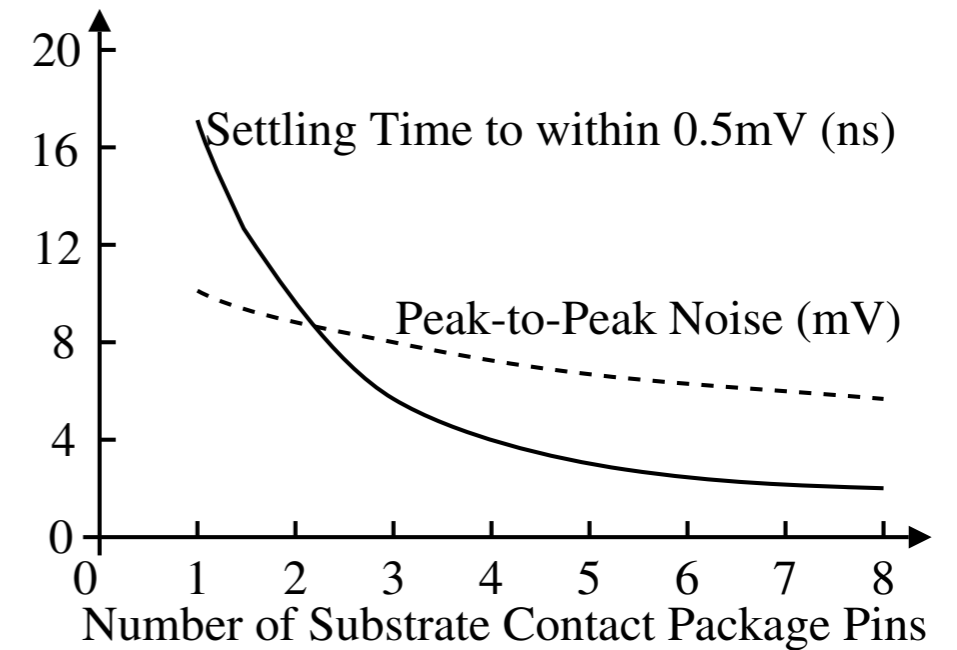
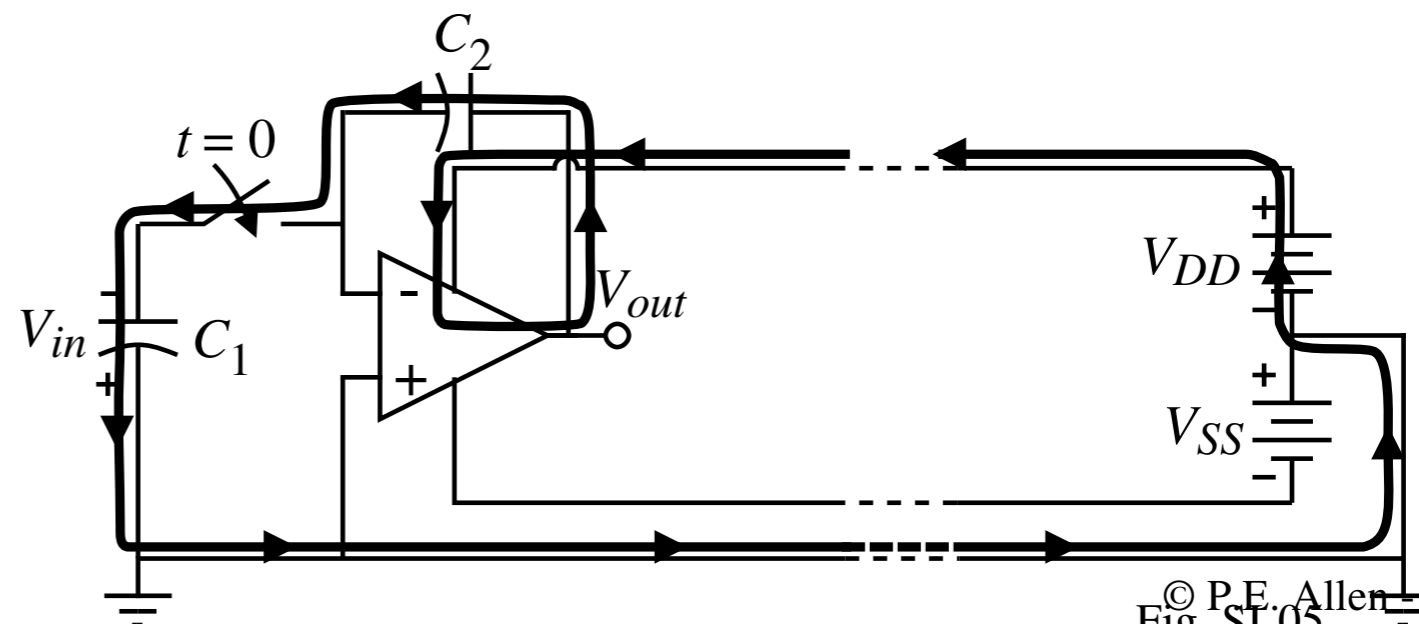


Fig. SI-08

- Fast changing signals have part of their path (circuit through ground and power supplies). Therefore bypass the off-chip power supplies to ground as close to the chip as possible.

© P.E. Allen, 2002  
Fig. SI-05

## Summary of Substrate Interference

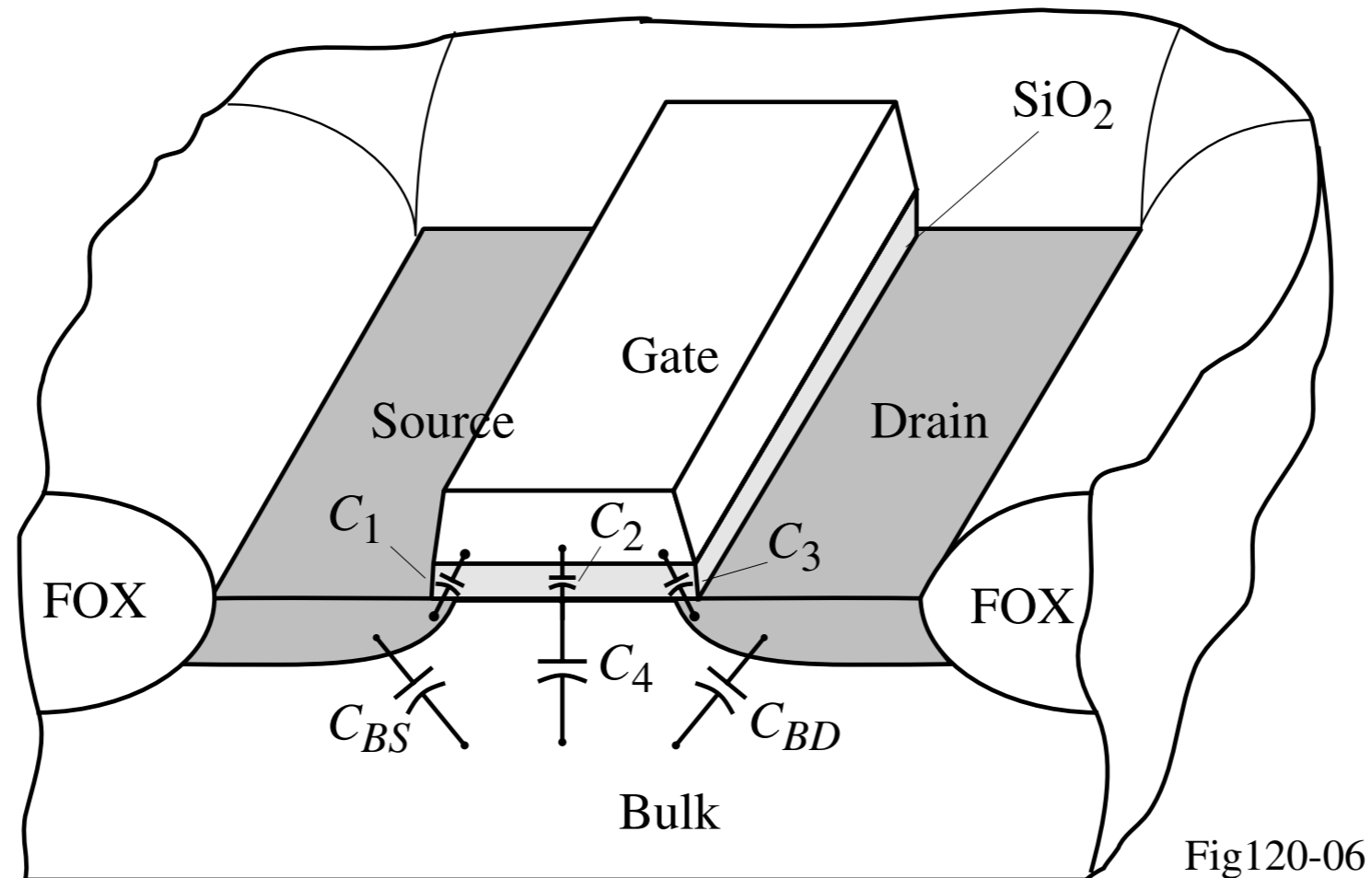
- Methods to reduce substrate noise
  - 1.) Physical separation
  - 2.) Guard rings placed close to the sensitive circuits with dedicated package pins.
  - 3.) Reduce the inductance in power supply and ground leads (best method)
  - 4.) Connect regions of constant potential (wells and substrate) to metal with as many contacts as possible.
- Noise Insensitive Circuit Design Techniques
  - 1.) Design for a high power supply rejection ratio (PSRR)
  - 2.) Use multiple devices spatially distinct and average the signal and noise.
  - 3.) Use “quiet” digital logic (power supply current remains constant)
  - 4.) Use differential signal processing techniques.
- Some references
  - 1.) D.K. Su, M.J. Loinaz, S. Masui and B.A. Wooley, “Experimental Results and Modeling Techniques for Substrate Noise in Mixed-Signal IC’s,” *J. of Solid-State Circuits*, vol. 28, No. 4, April 1993, pp. 420-430.
  - 2.) K.M. Fukuda, T. Anbo, T. Tsukada, T. Matsuura and M. Hotta, “Voltage-Comparator-Based Measurement of Equivalently Sampled Substrate Noise Waveforms in Mixed-Signal ICs,” *J. of Solid-State Circuits*, vol. 31, No. 5, May 1996, pp. 726-731.
  - 3.) X. Aragonés, J. Gonzalez and A. Rubio, *Analysis and Solutions for Switching Noise Coupling in Mixed-Signal ICs*, Kluwer Academic Publishers, Boston, MA, 1999.



## 3.4 - CAPACITANCES OF THE MOSFET

### Types of Capacitance

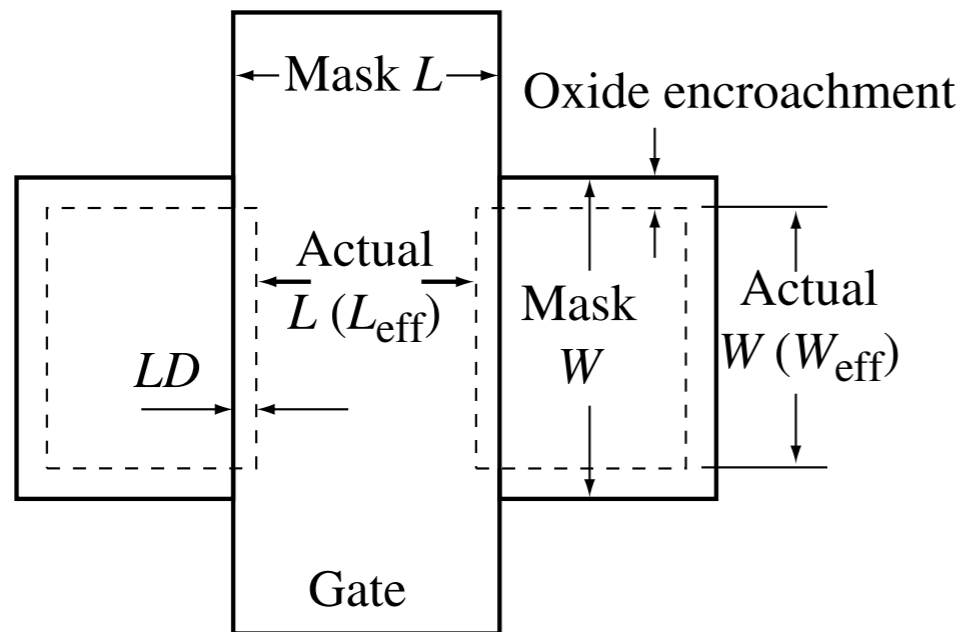
Physical Picture:



MOSFET capacitors consist of:

- Depletion capacitances
- Charge storage or parallel plate capacitances

## Charge Storage (Parallel Plate) MOSFET Capacitances - $C_1$ , $C_2$ , $C_3$ and $C_4$



Overlap capacitances:

$$C_1 = C_3 = LD \cdot W_{\text{eff}} \cdot C_{ox} = C_{GS} \text{ or } C_{GD}$$

(LD  $\approx$  0.015  $\mu\text{m}$  for LDD structures)

Channel capacitances:

$$C_2 = \text{gate-to-channel} = C_{ox} W_{\text{eff}} \cdot (L - 2LD) = C_{ox} W_{\text{eff}} \cdot L_{\text{eff}}$$

$C_4$  = voltage dependent channel-bulk/substrate capacitance

Source-gate overlap capacitance  $C_{GS}$  ( $C_1$ )

Drain-gate overlap capacitance  $C_{GD}$  ( $C_3$ )

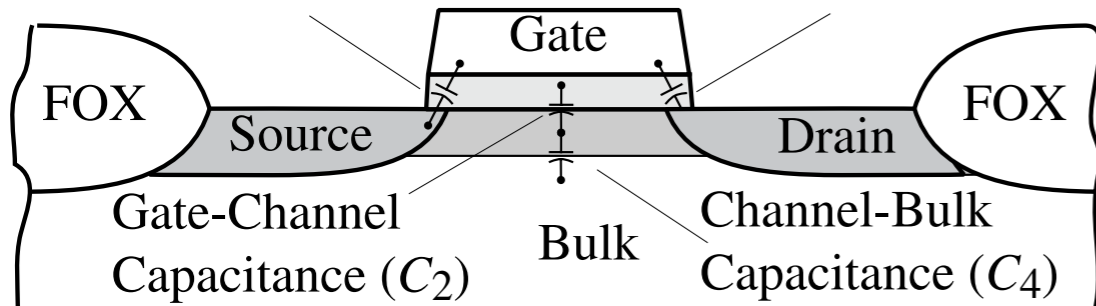


Fig. 120-09

## Charge Storage (Parallel Plate) MOSFET Capacitances - $C_5$

View looking down the channel from source to drain

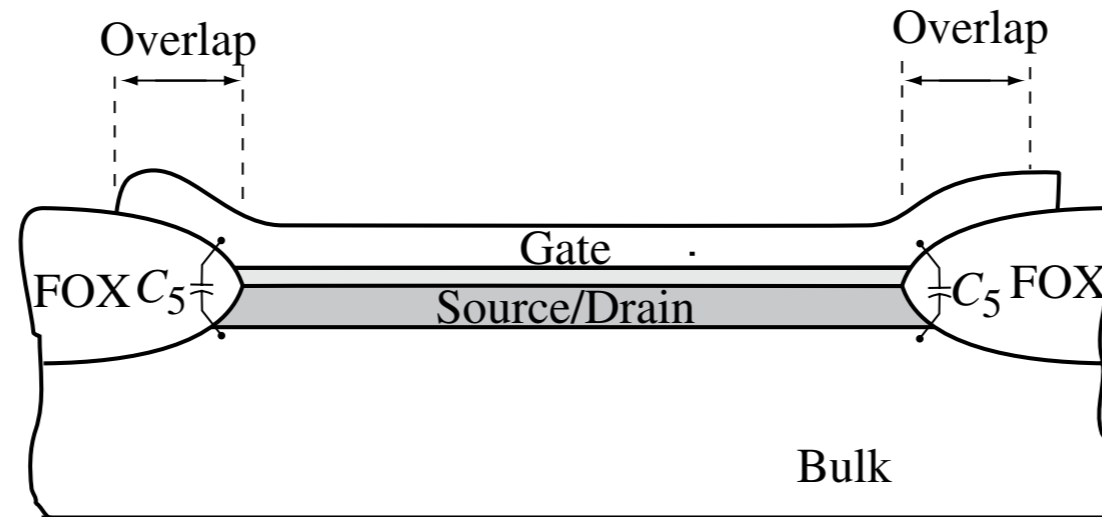


Fig120-10

$$C_5 = CGBO$$

Capacitance values based on an oxide thickness of  $140 \text{ \AA}$  or  $C_{OX} = 24.7 \times 10^{-4} \text{ F/m}^2$ :

Type	P-Channel	N-Channel	Units
CGSO	$220 \times 10^{-12}$	$220 \times 10^{-12}$	F/m
CGDO	$220 \times 10^{-12}$	$220 \times 10^{-12}$	F/m
CGBO	$700 \times 10^{-12}$	$700 \times 10^{-12}$	F/m
CJ	$560 \times 10^{-6}$	$770 \times 10^{-6}$	F/m <sup>2</sup>
CJSW	$350 \times 10^{-12}$	$380 \times 10^{-12}$	F/m
MJ	0.5	0.5	
MJSW	0.35	0.38	

## Illustration of $C_{GD}$ , $C_{GS}$ and $C_{GB}$

Comments on the variation of  $C_{BG}$  in the cutoff region:

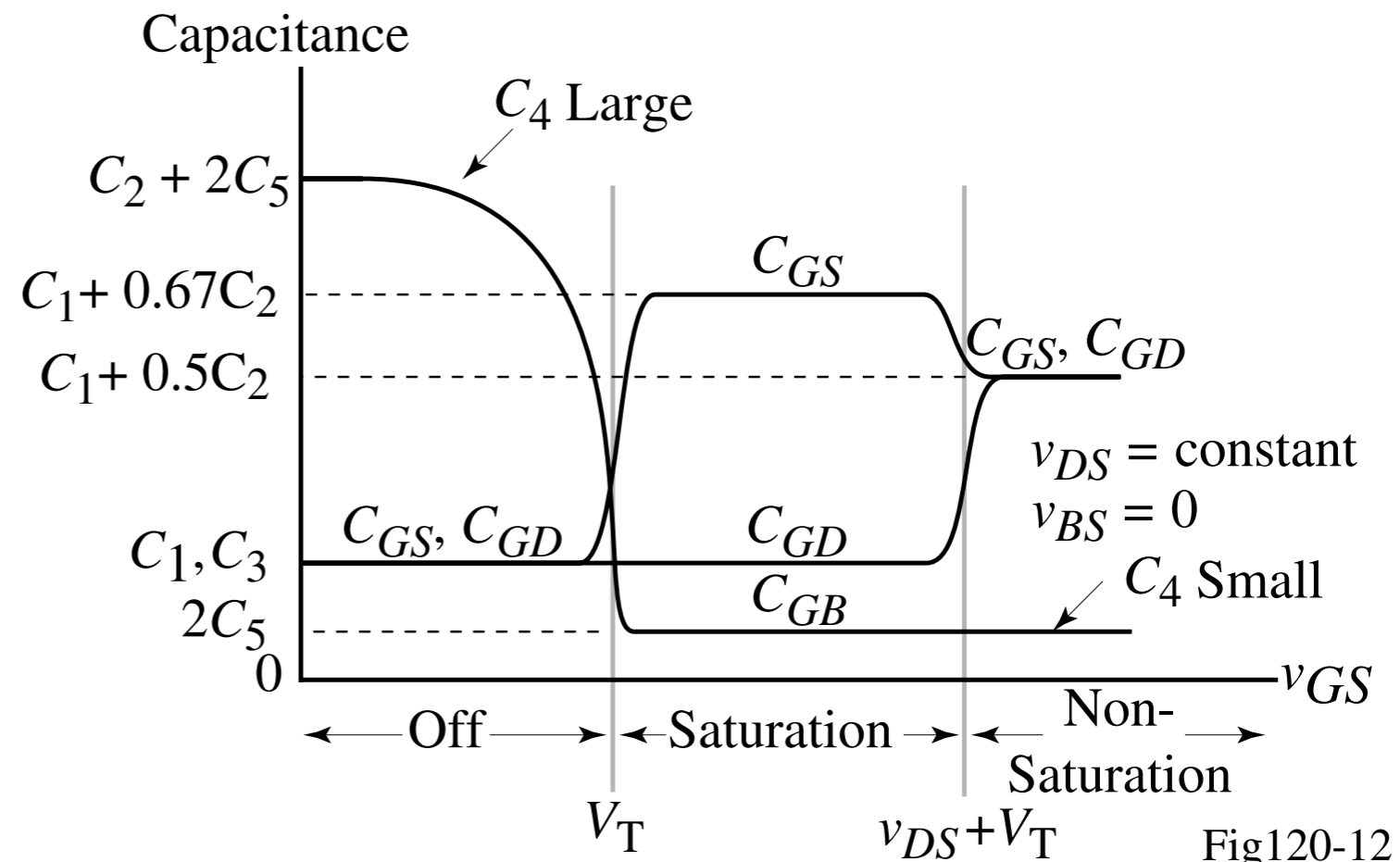
$$C_{BG} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_4}} + 2C_5$$

1.) For  $v_{GS} \approx 0$ ,  $C_{GB} \approx C_2 + 2C_5$

( $C_4$  is large because of the thin inversion layer in weak inversion where  $V_{GS}$  is slightly less than  $V_T$ )

2.) For  $0 < v_{GS} \leq V_T$ ,  $C_{GB} \approx 2C_5$

( $C_4$  is small because of the thicker inversion layer in strong inversion)



## 3.6 - TEMPERATURE AND NOISE MODELS FOR THE MOSFET

### Large Signal Temperature Model

Transconductance parameter:

$$K'(T) = K'(T_0) (T/T_0)^{-1.5} \quad (\text{Exponent becomes } +1.5 \text{ below } 77^\circ\text{K})$$

Threshold Voltage:

$$V_T(T) = V_T(T_0) + \alpha(T-T_0) + \dots$$

Typically  $\alpha_{NMOS} = -2\text{mV}/^\circ\text{C}$  to  $-3\text{mV}/^\circ\text{C}$  from  $200^\circ\text{K}$  to  $400^\circ\text{K}$  (PMOS has a + sign)

### Example

Find the value of  $I_D$  for a NMOS transistor at  $27^\circ\text{C}$  and  $100^\circ\text{C}$  if  $V_{GS} = 2\text{V}$  and  $W/L = 5\mu\text{m}/1\mu\text{m}$  if  $K'(T_0) = 110\mu\text{A}/\text{V}^2$  and  $V_T(T_0) = 0.7\text{V}$  and  $T_0 = 27^\circ\text{C}$  and  $\alpha_{NMOS} = -2\text{mV}/^\circ\text{C}$ .

### Solution

At room temperature, the value of drain current is,

$$I_D(27^\circ\text{C}) = \frac{110\mu\text{A}/\text{V}^2 \cdot 5\mu\text{m}}{2 \cdot 1\mu\text{m}} (2-0.7)^2 = 465\mu\text{A}$$

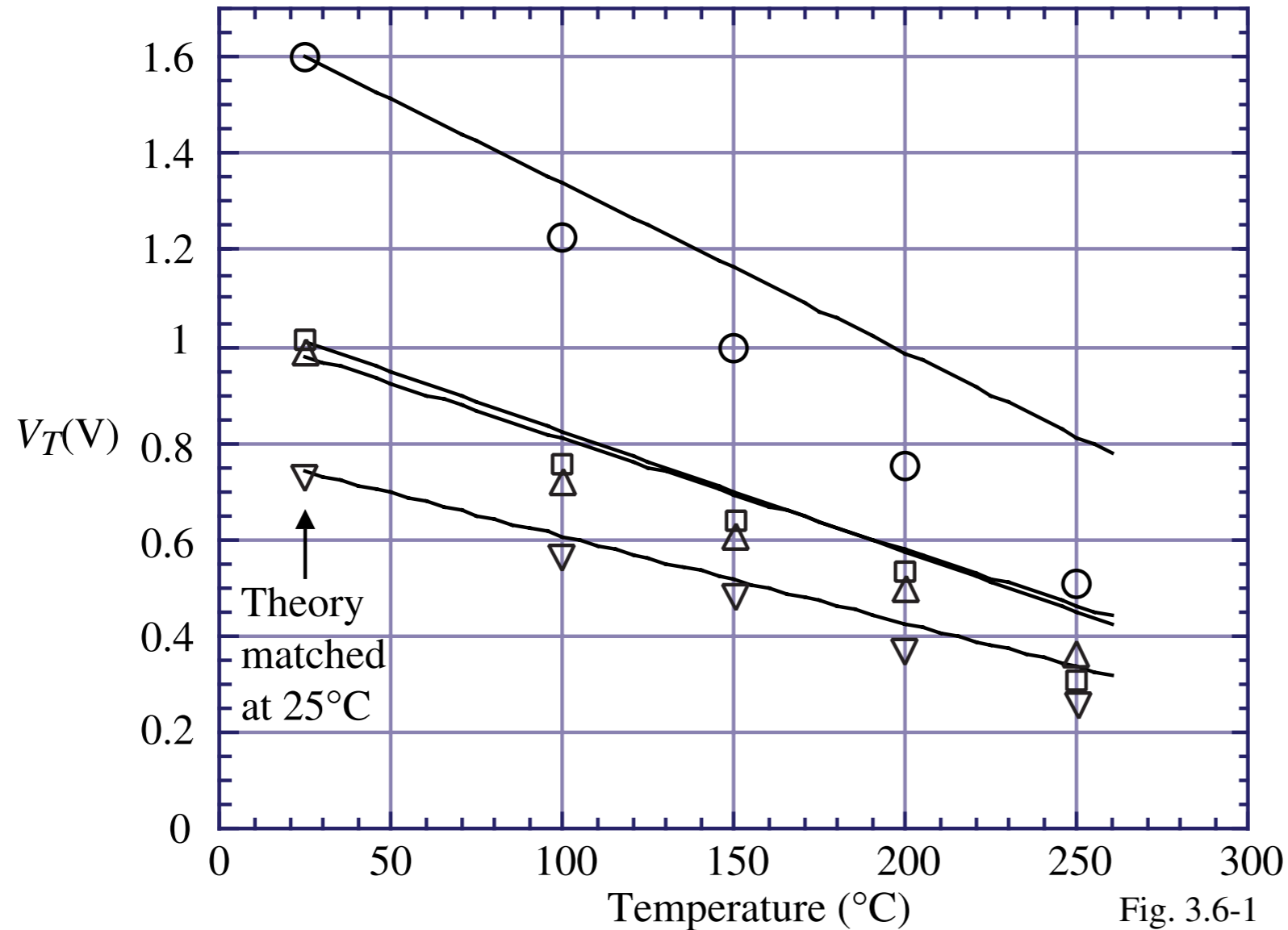
At  $T = 100^\circ\text{C}$  ( $373^\circ\text{K}$ ),  $K'(100^\circ\text{C}) = K'(27^\circ\text{C}) (373/300)^{-1.5} = 110\mu\text{A}/\text{V}^2 \cdot 0.72 = 79.3\mu\text{A}/\text{V}^2$

and  $V_T(100^\circ\text{C}) = 0.7 - (.002)(73^\circ\text{C}) = 0.554\text{V}$

$$\therefore I_D(100^\circ\text{C}) = \frac{79.3\mu\text{A}/\text{V}^2 \cdot 5\mu\text{m}}{2 \cdot 1\mu\text{m}} (2-0.554)^2 = 415\mu\text{A} \quad (\text{Repeat with } V_{GS} = 1.5\text{V})$$

## Experimental Verification of the MOSFET Temperature Dependence

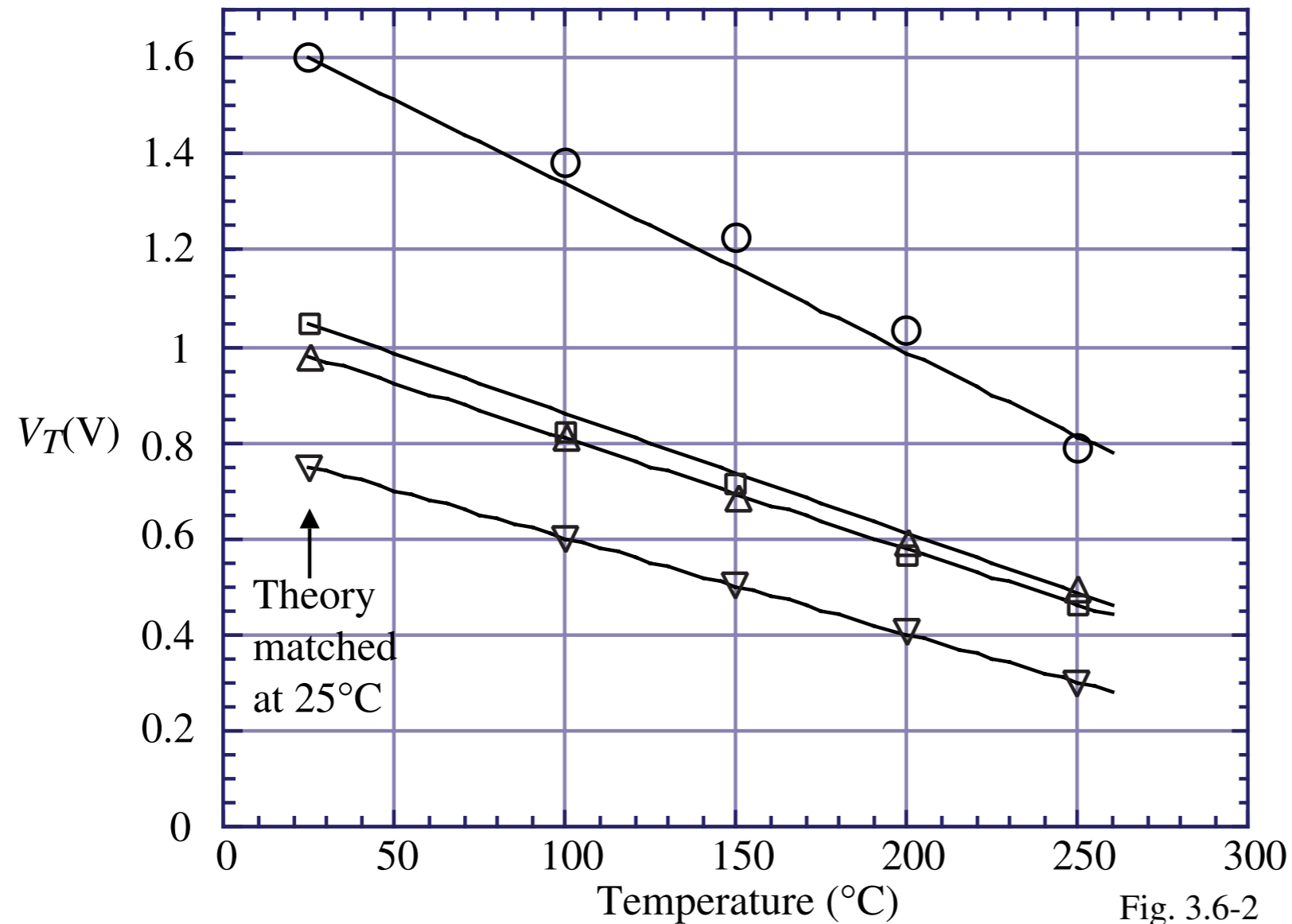
NMOS Threshold:



Symbol	Min. L	$N_A$ (cm <sup>-3</sup> )	$t_{ox}$ (Å)	$\alpha$ (mV/°C)
O	6 $\mu$ m	2x10 <sup>16</sup>	1000	-3.5
□	5 $\mu$ m	1x10 <sup>16</sup>	650	-2.5
Δ	4 $\mu$ m	2x10 <sup>16</sup>	500	-2.3
▽	2 $\mu$ m	3.3x10 <sup>16</sup>	275	-1.8

## Experimental Verification of the MOSFET Temperature Dependence

PMOS Threshold:



Symbol	Min. L	$N_A$ ( $\text{cm}^{-3}$ )	$T_{ox}$ ( $\text{\AA}$ )	$\alpha$ ( $\text{mV}/^{\circ}\text{C}$ )
O	$6\mu\text{m}$	$2 \times 10^{15}$	1000	+3.5
$\square$	$5\mu\text{m}$	$2 \times 10^{15}$	650	+2.5
$\Delta$	$4\mu\text{m}$	$2 \times 10^{16}$	500	+2.3
$\nabla$	$2\mu\text{m}$	$1.1 \times 10^{16}$	275	+2.0

## Zero Temperature Coefficient (ZTC) Point for MOSFETs

For a given value of gate-source voltage, the drain current of the MOSFET will be independent of temperature. Consider the following circuit:

Assume that the transistor is saturated and that:

$$\mu = \mu_0 \left( \frac{T}{T_0} \right)^{-1.5} \quad \text{and} \quad V_T(T) = V_T(T_0) + \alpha(T - T_0)$$

where  $\alpha = -0.0023 \text{ V}/^\circ\text{C}$  and  $T_0 = 27^\circ\text{C}$

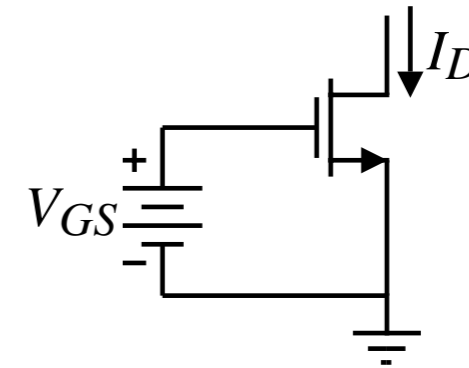


Fig. 4.5-12

$$\therefore I_D(T) = \frac{\mu_0 C_{ox} W}{2L} \left( \frac{T}{T_0} \right)^{-1.5} [V_{GS} - V_{T0} - \alpha(T - T_0)]^2$$

$$\frac{dI_D}{dT} = \frac{-1.5\mu_0 C_{ox}}{2T_0} \left( \frac{T}{T_0} \right)^{-2.5} [V_{GS} - V_{T0} - \alpha(T - T_0)]^2 + \alpha\mu_0 C_{ox} \left( \frac{T}{T_0} \right)^{-1.5} [V_{GS} - V_{T0} - \alpha(T - T_0)] = 0$$

$$\therefore V_{GS} - V_{T0} - \alpha(T - T_0) = \frac{-4T\alpha}{3} \quad \Rightarrow \quad \boxed{V_{GS}(\text{ZTC}) = V_{T0} - \alpha T_0 - \frac{\alpha T}{3}}$$

Let  $K' = 10 \mu\text{A}/\text{V}^2$ ,  $W/L = 5$  and  $V_{T0} = 0.71 \text{ V}$ .

At  $T = 27^\circ\text{C}$  ( $300^\circ\text{K}$ ),  $V_{GS}(\text{ZTC}) = 0.71 - (-0.0023)(300^\circ\text{K}) - (0.333)(-0.0023)(300^\circ\text{K}) = 1.63 \text{ V}$

At  $T = 27^\circ\text{C}$  ( $300^\circ\text{K}$ ),  $I_D = (10 \mu\text{A}/\text{V}^2)(5/2)(1.63 - 0.71)^2 = 21.2 \mu\text{A}$

At  $T = 200^\circ\text{C}$  ( $473^\circ\text{K}$ ),  $V_{GS}(\text{ZTC}) = 0.71 - (-0.0023)(300^\circ\text{K}) - (0.333)(-0.0023)(473^\circ\text{K}) = 1.76 \text{ V}$



## Experimental Verification of the ZTC Point

The data below is for a  $5\mu\text{m}$  n-channel MOSFET with  $W/L=50\mu\text{m}/10\mu\text{m}$ ,  $N_A=10^{16}\text{ cm}^{-3}$ ,  $t_{ox} = 650\text{\AA}$ ,  $u_0 C_{ox} = 10\mu\text{A}/\text{V}^2$ , and  $V_{T0} = 0.71\text{V}$ .

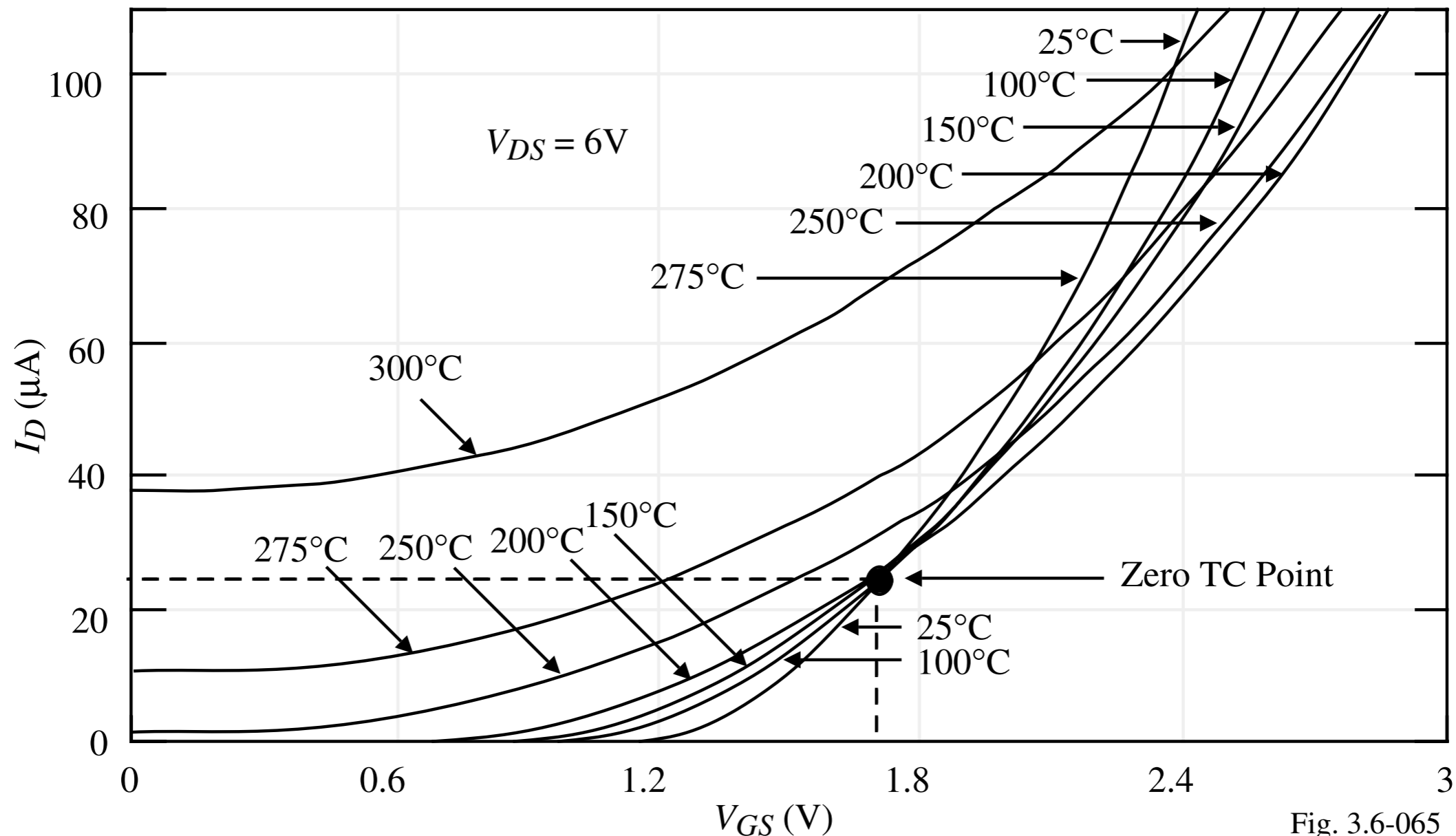


Fig. 3.6-065

## ZTC Point for PMOS

The data is for a  $5\mu\text{m}$  p-channel MOSFET with  $W/L=50\mu\text{m}/10\mu\text{m}$ ,  $N_D=2\times 10^{15}\text{cm}^{-3}$ , and  $t_{ox} = 650\text{\AA}$ .

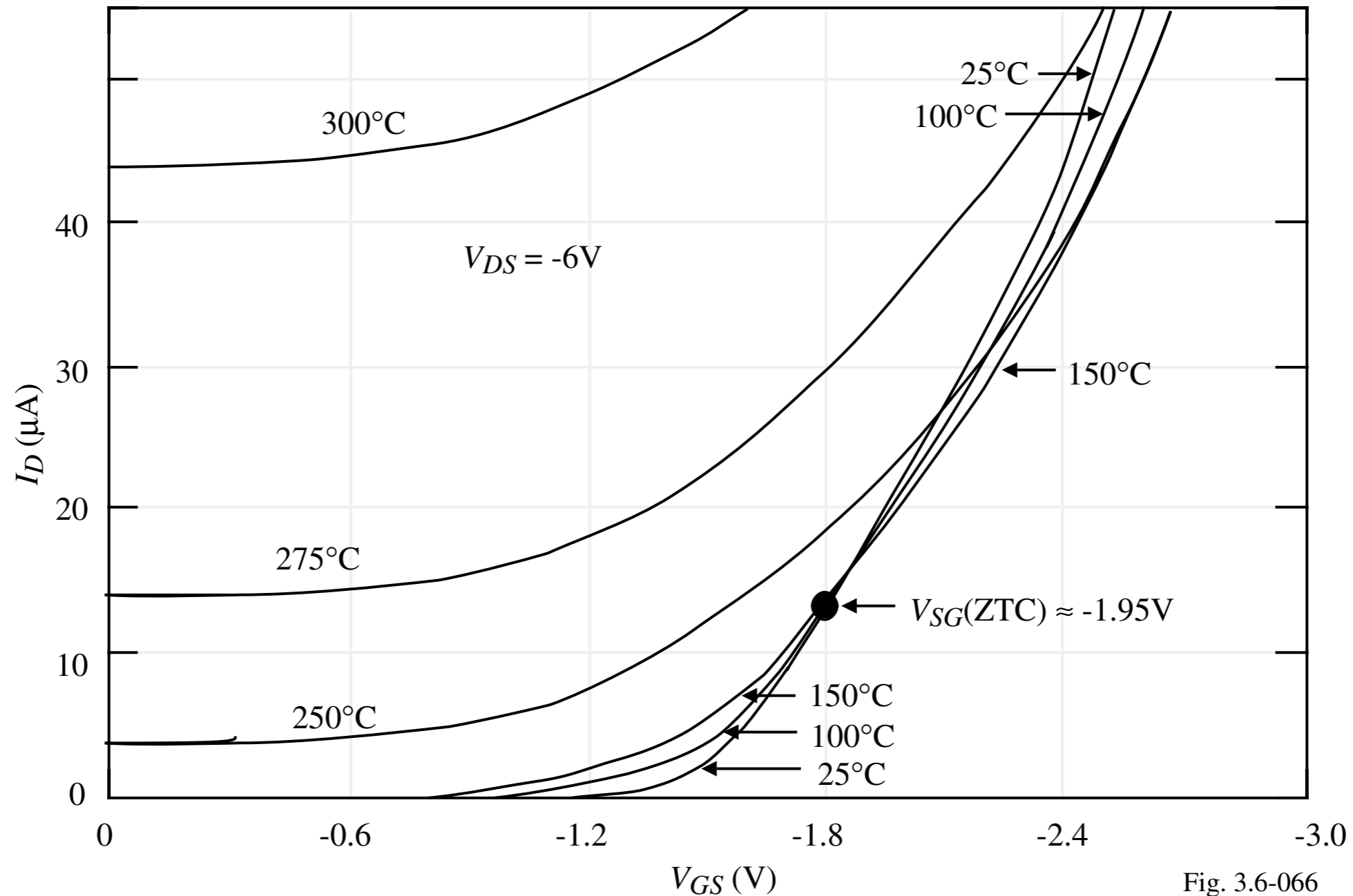
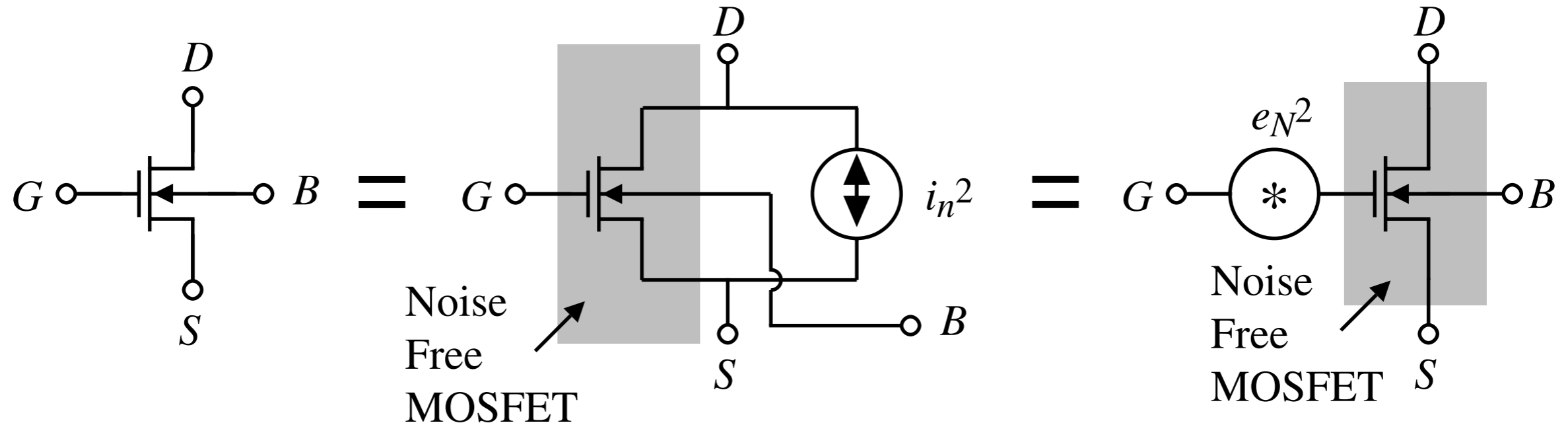


Fig. 3.6-066

Zero temperature coefficient will occur for every MOSFET up to about  $200^\circ\text{C}$ .

## MOSFET NOISE

### MOS Device Noise at Low Frequencies



where

$$i_n^2 = \left[ \frac{8kTg_m(1+\eta)}{3} + \frac{KF I_D}{f^S C_{ox} L^2} \right] \Delta f \quad (\text{amperes}^2)$$

$\Delta f$  = bandwidth at a frequency,  $f$

$$\eta = \frac{g_{mbs}}{g_m}$$

$k$  = Boltzmann's constant

$KF$  = Flicker noise coefficient

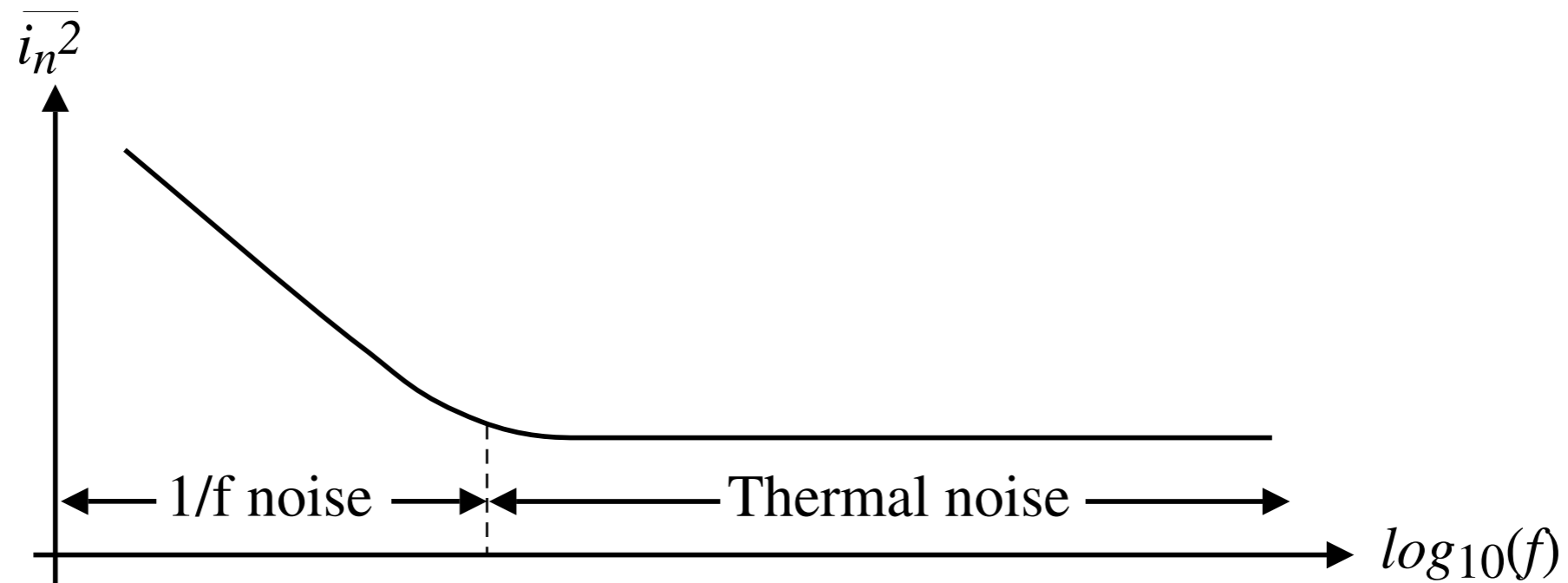
$S$  = Slope factor of the 1/f noise

## Reflecting the MOSFET Noise to the Gate

Dividing  $i_n^2$  by  $g_m^2$  gives

$$e_n^2 = \frac{i_n^2}{g_m^2} = \left[ \frac{8kT(1+\eta)}{3g_m} + \frac{KF}{2fC_{ox}WLK'} \right] \Delta f \quad (\text{volts}^2)$$

It will be convenient to use  $B = \frac{KF}{2C_{ox}K'}$  for model simplification.



## SEC. 3.9 – MODELS FOR SIMULATION OF MOS CIRCUITS

### FET Model Generations

- First Generation – Physically based analytical model including all geometry dependence.
- Second Generation – Model equations became subject to mathematical conditioning for circuit simulation. Use of empirical relationships and parameter extraction.
- Third Generation – A return to simpler model structure with reduced number of parameters which are physically based rather than empirical. Uses better methods of mathematical conditioning for simulation including more specialized smoothing functions.

Performance Comparison of Models (from Cheng and Hu, *MOSFET Modeling & BSIM3 Users Guide*)

Model	Minimum L ( $\mu\text{m}$ )	Minimum Tox (nm)	Model Continuity	$i_D$ Accuracy in Strong Inversion	$i_D$ Accuracy in Subthreshold	Small signal parameter	Scalability
MOS1	5	50	Poor	Poor	Not Modeled	Poor	Poor
MOS2	2	25	Poor	Poor	Poor	Poor	Fair
MOS3	1	20	Poor	Fair	Poor	Poor	Poor
BSIM1	0.8	15	Fair	Good	Fair	Poor	Fair
BSIM2	0.35	7.5	Fair	Good	Good	Fair	Fair
BSIM3v2	0.25	5	Fair	Good	Good	Good	Good
BSIM3v3	0.15	4	Good	Good	Good	Good	Good

## First Generation Models

### Level 1 (MOS1)

- Basic square law model based on the gradual channel approximation and the square law for saturated drain current.
- Good for hand analysis.
- Needs improvement for deep-submicron technology (must incorporate the square law to linear shift)

### Level 2 (MOS2)

- First attempt to include small geometry effects
- Inclusion of the channel-bulk depletion charge results in the familiar  $3/2$  power terms
- Introduced a simple subthreshold model which was not continuous with the strong inversion model.
- Model became quite complicated and probably is best known as a “developing ground” for better modeling techniques.

### Level 3 (MOS3)

- Used to overcome the limitations of Level 2. Made use of a semi-empirical approach.
- Added DIBL and the reduction of mobility by the lateral field.
- Similar to Level 2 but considerably more efficient.
- Used binning but was poorly implemented.

## Second Generation Models

### BSIM (Berkeley Short-Channel IGFET Model)

- Emphasis is on mathematical conditioning for circuit simulation
- Short channel models are mostly empirical and shifts the modeling to the parameter extraction capability
- Introduced a more detailed subthreshold current model with good continuity
- Poor modeling of channel conductance

### HSPICE Level 28

- Based on BSIM but has been extensively modified.
- More suitable for analog circuit design
- Uses model binning
- Model parameter set is almost entirely empirical
- User is locked into HSPICE
- Model is proprietary

### BSIM2

- Closely based on BSIM
- Employs several expressions developed from two dimensional analysis
- Makes extensive modifications to the BSIM model for mobility and the drain current
- Uses a new subthreshold model
- Output conductance model makes the model very suitable for analog circuit design
- The drain current model is more accurate and provides better convergence
- Becomes more complex with a large number of parameters
- No provisions for variations in the operating temperature

## Third Generation Models

### BSIM3

- This model has achieved stability and is being widely used in industry for deep submicron technology.
- Initial focus of simplicity was not realized.

### MOS Model 9

- Developed at Philips Laboratory
- Has extensive heritage of industrial use
- Model equations are clean and simple – should be efficient

### Other Candidates

- EKV (Enz-Krummenacher-Vittoz) – fresh approach well suited to the needs of analog circuit design



## **BSIM2 Model used in Subthreshold**

BSIM Model Parameters used in Subthreshold

VDS 1 0 DC 3.0

M1 1 1 0 0 CMOSN W=5UM L=2UM

.MODEL CMOSN NMOS LEVEL=4

+VFB=-7.92628E-01	LVFB= 1.22972E-02	WVFB=-1.00233E-01
+PHI= 7.59099E-01	LPHI= 0.00000E+00	WPHI= 0.00000E+00
+K1= 1.06705E+00	LK1= 5.08430E-02	WK1= 4.72787E-01
+K2=-4.23365E-03	LK2= 6.76974E-02	WK2= 6.27415E-02
+ETA=-4.30579E-03	LETA= 9.05179E-03	WETA= 7.33154E-03
+MUZ= 5.58459E+02	DL=6.86137E-001	DW=-1.04701E-001
+U0= 5.52698E-02	LU0= 6.09430E-02	WU0=-6.91423E-02
+U1= 5.38133E-03	LU1= 5.43387E-01	WU1=-8.63357E-02
+X2MZ= 1.45214E+01	LX2MZ=-3.08694E+01	WX2MZ= 4.75033E+01
+X2E=-1.67104E-04	LX2E=-4.75323E-03	WX2E=-2.74841E-03
+X3E= 5.33407E-04	LX3E=-4.69455E-04	WX3E=-5.26199E-03
+X2U0= 2.45645E-03	LX2U0=-1.46188E-02	WX2U0= 2.63555E-02
+X2U1=-3.80979E-04	LX2U1=-1.71488E-03	WX2U1= 2.23520E-02
+MUS= 5.48735E+02	LMUS= 3.28720E+02	WMUS= 1.35360E+02
+X2MS= 6.72261E+00	LX2MS=-3.48094E+01	WX2MS= 9.84809E+01
+X3MS=-2.79427E+00	LX3MS= 6.31555E+01	WX3MS=-1.99720E-01
+X3U1= 1.18671E-03	LX3U1= 6.13936E-02	WX3U1=-3.49351E-03
+TOX=4.03000E-002	TEMP= 2.70000E+01	VDD= 5.00000E+00
+CGDO=4.40942E-010	CGSO=4.40942E-010	CGBO=6.34142E-010
+XPART=-1.00000E+000		
+N0=1.00000E+000	LN0=0.00000E+000	WN0=0.00000E+000
+NB=0.00000E+000	LNB=0.00000E+000	WNB=0.00000E+000
+ND=0.00000E+000	LND=0.00000E+000	WND=0.00000E+000
+RSH=0 CJ=4.141500e-04	CJSW=4.617400e-10	JS=0 PB=0.8
+PBSW=0.8 MJ=0.4726	MJSW=0.3597	WDF=0 DELL=0
.DC VDS 5.0 0 0.01		
.PRINT DC ID(M1)		
.PROBE		
.END		

## **BSIM3 Model**

The background for the BSIM3 model and the equations are given in detail in the text *MOSFET Modeling & BSIM3 User's Guide*, by Y. Cheng and C. Hu, Kluwer Academic Publishers, 1999.

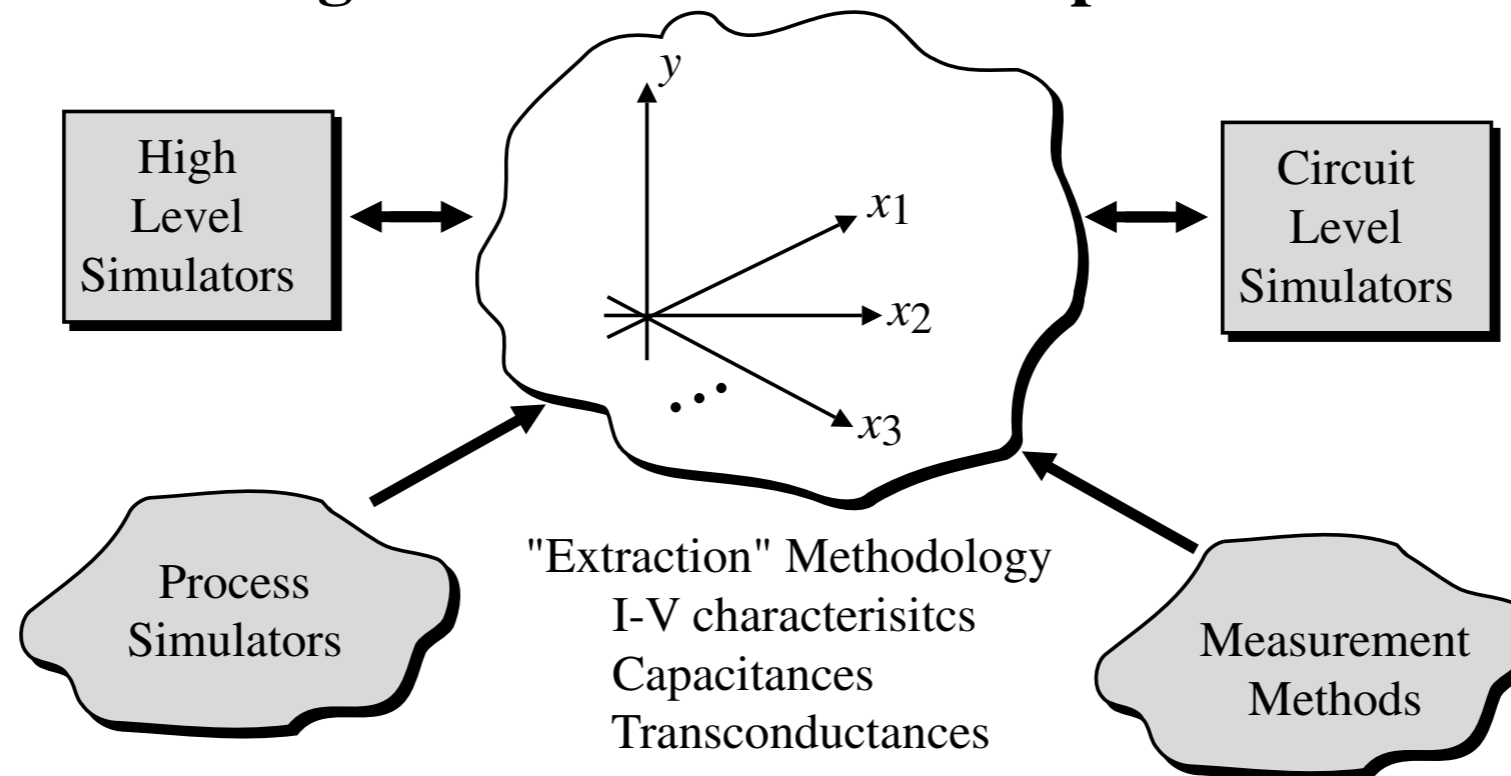
The short channel effects included in the BSIM3 model are:

- Normal and reverse short-channel and narrow-width effects on the threshold.
  - Channel length modulation (*CLM*).
  - Drain induced barrier lowering (*DIBL*).
  - Velocity saturation.
  - Mobility degradation due to the vertical electric field.
  - Impact ionization.
  - Band-to-band tunnelling.
  - Velocity overshoot.
  - Self-heating.
- 1.) Channel quantization.
  - 2.) Polysilicon depletion.

## 0.25 $\mu$ m BSIM3v3.1 NMOS Parameters

```
.MODEL CMOSN NMOS (
LEVEL = 49
+VERSION = 3.1      TNOM = 27      TOX = 5.7E-9
+XJ = 1E-7         NCH = 2.3549E17  VTH0 = 0.4273342
+K1 = 0.3922983   K2 = 0.0185825   K3 = 1E-3
+K3B = 2.0947677  W0 = 2.171779E-7  NLX = 1.919758E-7
+DVT0W = 0        DVT1W = 0          DVT2W = 0
+DVT0 = 7.137212E-3  DVT1 = 6.066487E-3  DVT2 = -0.3025397
+U0 = 403.1776038  UA = -3.60743E-12  UB = 1.323051E-18
+UC = 2.575123E-11  VSAT = 1.616298E5   A0 = 1.4626549
+AGS = 0.3136349   B0 = 3.080869E-8   B1 = -1E-7
+KETA = 5.462411E-3  A1 = 4.653219E-4   A2 = 0.6191129
+RDSW = 345.624986  PRWG = 0.3183394   PRWB = -0.1441065
+WR = 1           WINT = 8.107812E-9  LINT = 3.375523E-9
+XL = 3E-8        XW = 0          DWG = 6.420502E-10
+DWB = 1.042094E-8  VOFF = -0.1083577  NFACTOR = 1.1884386
+CIT = 0          CDSC = 2.4E-4     CDSCD = 0
+CDSCB = 0        ETA0 = 4.914545E-3  ETAB = 4.215338E-4
+DSUB = 0.0313287  PCLM = 1.2088426   PDIBLC1 = 0.7240447
+PDIBLC2 = 5.120303E-3  PDIBLCB = -0.0443076  DROUT = 0.7752992
+PSCBE1 = 4.451333E8  PSCBE2 = 5E-10     PVAG = 0.2068286
+DELTA = 0.01      MOBMOD = 1         PRT = 0
+UTE = -1.5        KT1 = -0.11        KT1L = 0
+KT2 = 0.022       UA1 = 4.31E-9       UB1 = -7.61E-18
+UC1 = -5.6E-11    AT = 3.3E4         WL = 0
+WLN = 1           WW = -1.22182E-16  WWN = 1.2127
+WWL = 0           LL = 0          LLN = 1
+LW = 0           LWN = 1          LWL = 0
+CAPMOD = 2        XPART = 0.4        CGDO = 6.33E-10
+CGSO = 6.33E-10   CGBO = 1E-11       CJ = 1.766171E-3
+PB = 0.9577677    MJ = 0.4579102     CJSW = 3.931544E-10
+PBSW = 0.99       MJSW = 0.2722644   CF = 0
+PVTH0 = -2.126483E-3  PRDSW = -24.2435379  PK2 = -4.788094E-4
+WKETA = 1.430792E-3  LKETA = -6.548592E-3 )
```

## Adjustable Precision Analog Models – Table Lookup



- Objective
  - Develop models having adjustable precision in ac and dc performance using table lookup models.
- Advantages
  - Usable at any level – device, circuit, or behavioral
  - Quickly developed from experiment or process simulators
  - Faster than analytical device models (BSIM)
- Disadvantages
  - Requires approximately 10kbytes for a typical MOS model
  - Can't be parameterized easily

## **Summary of MOSFET Models for Simulation**

- Models are much improved for efficient computer simulation
- Output conductance model is greatly improved
- Poor results for narrow channel transistors
- Can have discontinuities at bin boundaries
- Fairly complex model, difficult to understand in detail

## **SEC. 3.10 – EXTRACTION OF A LARGE SIGNAL MODEL FOR HAND CALCULATIONS**

### **Objective**

Extract a simple model that is useful for design from the computer models such as BSIM3.

### **Extraction for Short Channel Models**

Procedure for extracting short channel models:

- 1.) Extract the square-law model parameters for a transistor with length at least 10 times  $L_{min}$ .
- 2.) Using the values of  $K'$ ,  $V_T$ ,  $\lambda$ , and  $\gamma$  extract the model parameters for the following model:

$$i_D = \frac{K'}{2[1 + \theta(v_{GS} - V_T)]} \frac{W}{L} [v_{GS} - V_T]^2 (1 + \lambda v_{DS})$$

Adjust the values of  $K'$ ,  $V_T$ , and  $\lambda$  as needed.

## EXTRACTION OF THE SIMPLE, SQUARE-LAW MODEL

### Characterization of the Simple Square-Law Model

Equations for the MOSFET in strong inversion:

$$i_D = K \left( \frac{W_{\text{eff}}}{2L_{\text{eff}}} \right) (v_{GS} - V_T)^2 (1 + \lambda v_{DS}) \quad (1)$$

$$i_D = K \left( \frac{W_{\text{eff}}}{L_{\text{eff}}} \right) \left[ (v_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2} \right] (1 + \lambda v_{DS}) \quad (2)$$

where

$$V_T = V_{T0} + \gamma \left[ \sqrt{2|\phi_F| + v_{SB}} - \sqrt{2|\phi_F|} \right] \quad (3)$$

## **Extraction of Model Parameters:**

First assume that  $v_{DS}$  is chosen such that the  $\lambda v_{DS}$  term in Eq. (2) is much less than one and  $v_{SB}$  is zero, so that  $V_T = V_{T0}$ .

Therefore, Eq. (2) simplifies to

$$i_D = K' \left( \frac{W_{\text{eff}}}{2L_{\text{eff}}} \right) (v_{GS} - V_{T0})^2 \quad (6)$$

This equation can be manipulated algebraically to obtain the following

$$i_D^{1/2} = \left( \frac{K' W_{\text{eff}}}{2L_{\text{eff}}} \right)^{1/2} v_{GS} = \left( \frac{K' W_{\text{eff}}}{2L_{\text{eff}}} \right)^{1/2} V_{T0} \quad (7)$$

which has the form

$$y = mx + b \quad (8)$$

This equation is easily recognized as the equation for a straight line with  $m$  as the slope and  $b$  as the y-intercept. Comparing Eq. (7) to Eq. (8) gives

$$y = i_D^{1/2} \quad (9)$$

$$x = v_{GS} \quad (10)$$

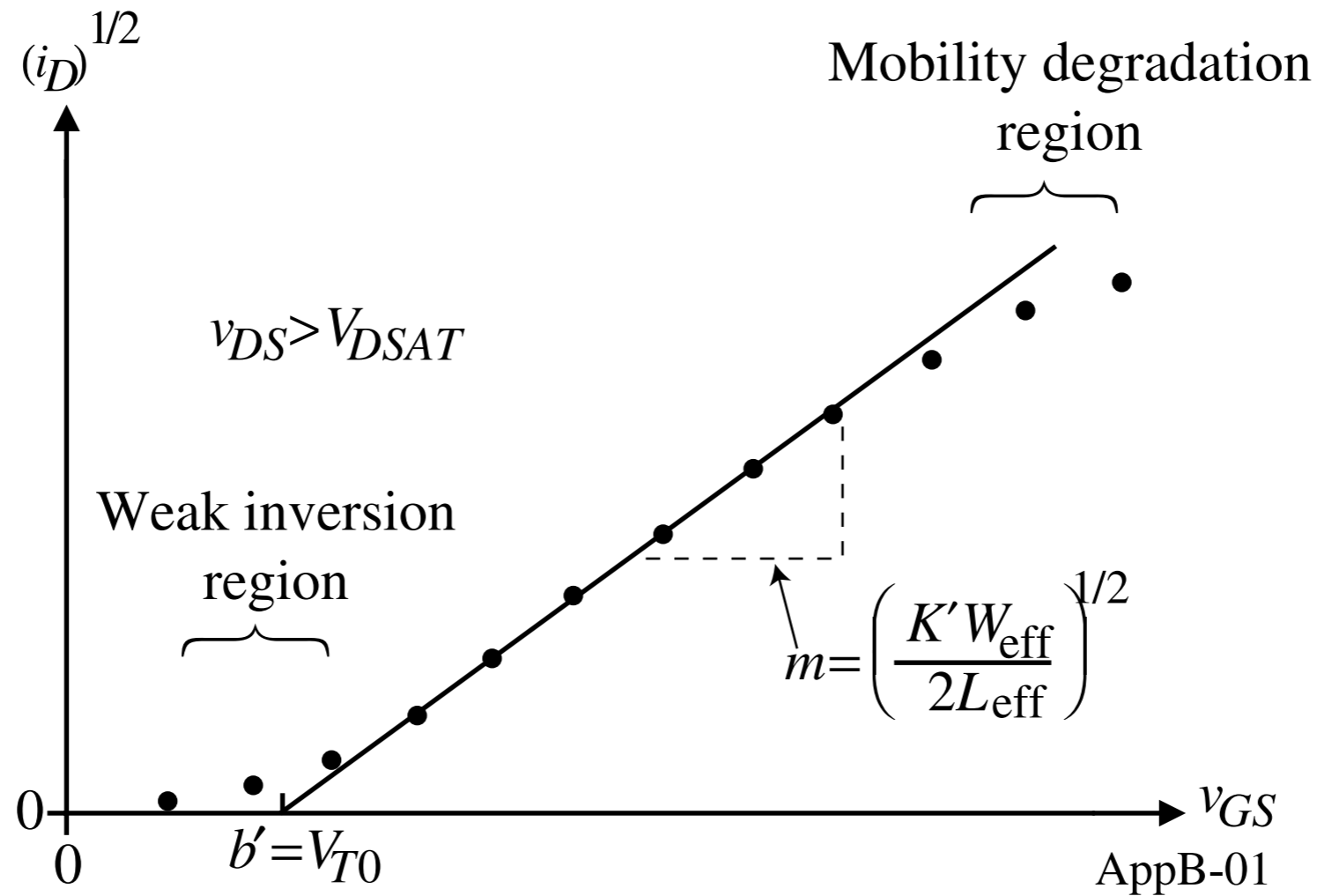
$$m = \left( \frac{K' W_{\text{eff}}}{2L_{\text{eff}}} \right)^{1/2} \quad (11)$$

and

$$b = - \left( \frac{K' W_{\text{eff}}}{2L_{\text{eff}}} \right)^{1/2} V_{T0} \quad (12)$$



## Illustration of $K'$ and $V_T$ Extraction



### Comments:

- Stay away from the extreme regions of mobility degradation and weak inversion
- Use channel lengths greater than  $L_{min}$

### **Example 3.10-1 – Extraction of $K'$ and $V_T$ Using Linear Regression**

Given the following transistor data shown in Table 3.10-1 and linear regression formulas based on the form,

$$y = mx + b \quad (13)$$

and

$$m = \frac{\sum x_i y_i - (\sum x_i \sum y_i)/n}{\sum x_i^2 - (\sum x_i)^2/n} \quad (14)$$

determine  $V_{T0}$  and  $K W/2L$ . The data in Table B-1 also give  $I_D^{1/2}$  as a function of  $V_{GS}$ .

Table 3.10-1 Data for Example 3.10-1

$V_{GS}$ (V)	$I_D$ ( $\mu$ A)	$\sqrt{I_D}$ ( $\mu$ A) <sup>1/2</sup>	$V_{SB}$ (V)
1.000	0.700	0.837	0.000
1.200	2.00	1.414	0.000
1.500	8.00	2.828	0.000
1.700	13.95	3.735	0.000
1.900	22.1	4.701	0.000

**Example 3.10-1 – Continued***Solution*

The data must be checked for linearity before linear regression is applied. Checking slopes between data points is a simple numerical technique for determining linearity. Using the formula that

$$\text{Slope} = m = \frac{\Delta y}{\Delta x} = \frac{\sqrt{I_{D2}} - \sqrt{I_{D1}}}{V_{GS2} - V_{GS1}}$$

Gives

$$m_1 = \frac{1.414 - 0.837}{0.2} = 2.885$$

$$m_2 = \frac{2.828 - 1.414}{0.3} = 4.713$$

$$m_3 = \frac{3.735 - 2.828}{0.2} = 4.535$$

$$m_4 = \frac{4.701 - 3.735}{0.2} = 4.830$$

These results indicate that the first (lowest value of  $V_{GS}$ ) data point is either bad, or at a point where the transistor is in weak inversion. This data point will not be included in subsequent analysis. Performing the linear regression yields the following results.

$$V_{T0} = 0.898 \text{ V} \quad \text{and} \quad \frac{K'W_{\text{eff}}}{2L_{\text{eff}}} = 21.92 \mu\text{A}/\text{V}^2$$

## Extraction of the Bulk-Threshold Parameter $\gamma$

Using the same techniques as before, the following equation

$$V_T = V_{T0} + \gamma [\sqrt{2|\phi_F| + v_{SB}} - \sqrt{2|\phi_F|}]$$

is written in the linear form where

$$y = V_T \tag{18}$$

$$x = \sqrt{2|\phi_F| + v_{SB}} - \sqrt{2|\phi_F|} \tag{19}$$

$$m = \gamma \tag{20}$$

$$b = V_{T0} \tag{21}$$

The term  $2|\phi_F|$  is unknown but is normally in the range of 0.6 to 0.7 volts.

Procedure:

- 1.) Pick a value for  $2|\phi_F|$ .
- 2.) Extract a value for  $\gamma$ .

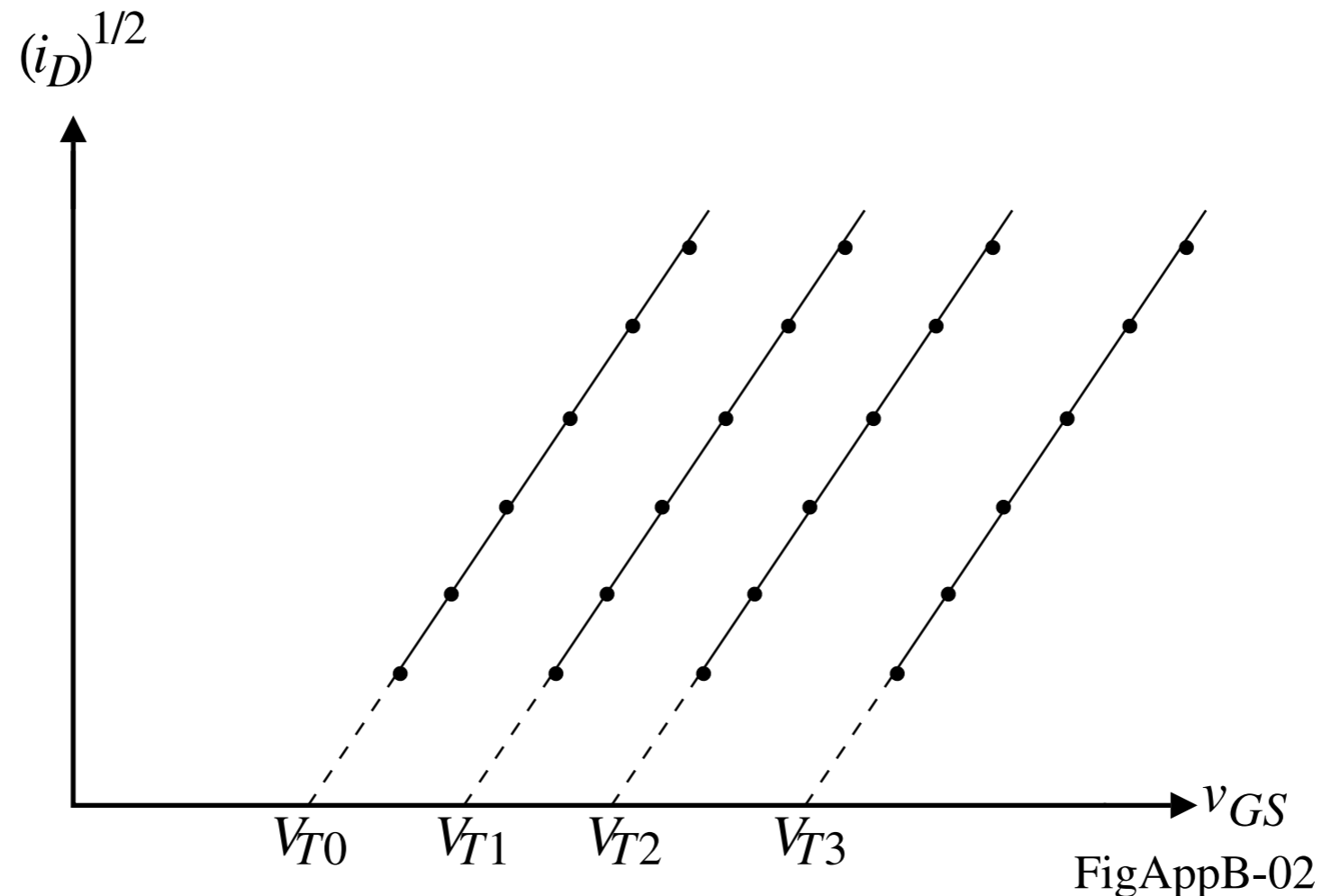
- 3.) Calculate  $N_{SUB}$  using the relationship,  $\gamma = \frac{\sqrt{2\varepsilon_{si}q N_{SUB}}}{C_{ox}}$

- 4.) Calculate  $\phi_F$  using the relationship,  $\phi_F = -\frac{kT}{q} \ln\left(\frac{N_{SUB}}{n_i}\right)$

- 5.) Iterative procedures can be used to achieve the desired accuracy of  $\gamma$  and  $2|\phi_F|$ . Generally, an approximate value for  $2|\phi_F|$  gives adequate results.

## Illustration of the Procedure for Extracting $\gamma$

A plot of  $\sqrt{i_D}$  versus  $v_{GS}$  for different values of  $v_{SB}$  used to determine  $\gamma$  is shown below.



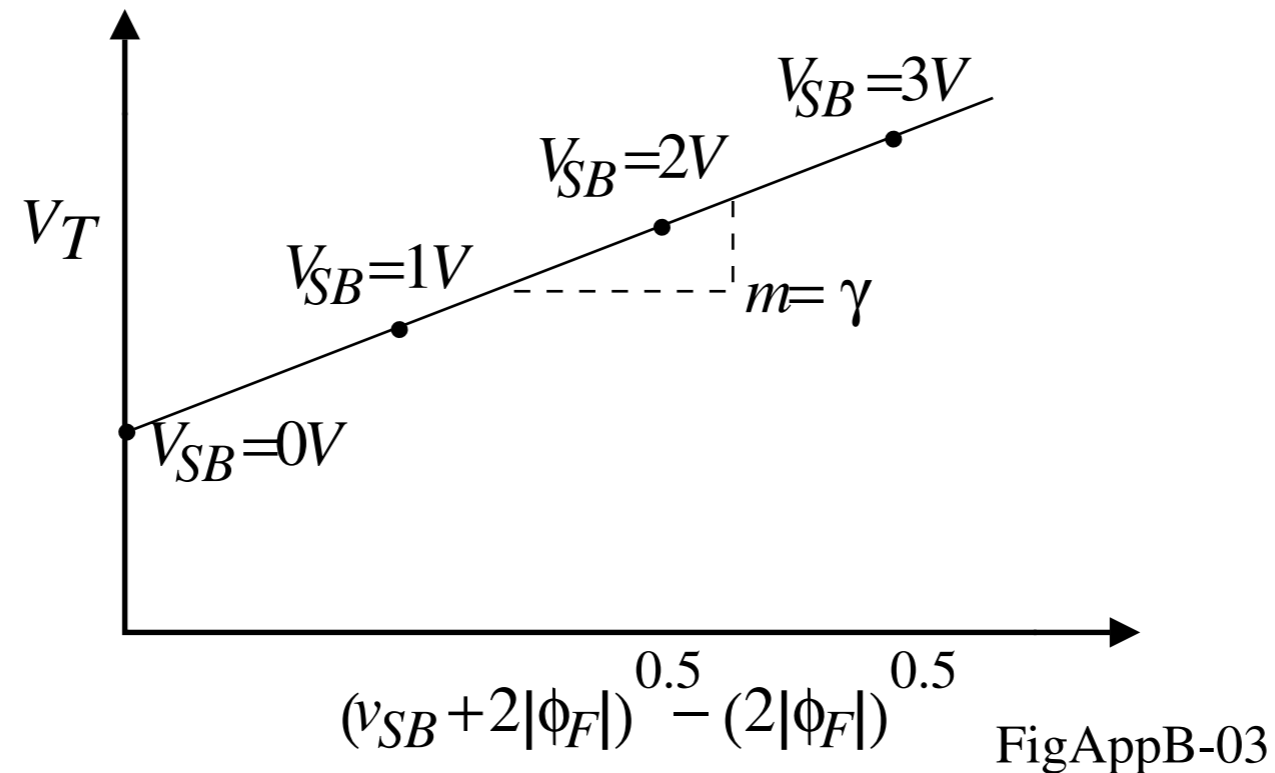
By plotting  $V_T$  versus  $x$  of Eq. (19) one can measure the slope of the best fit line from which the parameter  $\gamma$  can be extracted. In order to do this,  $V_T$  must be determined at various values of  $v_{SB}$  using the technique previously described.





## Illustration of the Procedure for Extracting $\gamma$ - Continued

Each  $V_T$  determined above must be plotted against the  $v_{SB}$  term. The result is shown below. The slope  $m$ , measured from the best fit line, is the parameter  $\gamma$ .



FigAppB-03



### **Example 3.10-2 – Extraction of the Bulk Threshold Parameter**

Using the results from Ex. B-1 and the following transistor data, determine the value of  $\gamma$  using linear regression techniques. Assume that  $2|\phi_F|$  is 0.6 volts.

Table 3.10-2 Data for Example 3.10-2.

$V_{SB}$ (V)	$V_{GS}$ (V)	$I_D$ ( $\mu$ A)
1.000	1.400	1.431
1.000	1.600	4.55
1.000	1.800	9.44
1.000	2.000	15.95
2.000	1.700	3.15
2.000	1.900	7.43
2.000	2.10	13.41
2.000	2.30	21.2

#### Solution

Table B-2 shows data for  $V_{SB} = 1$  volt and  $V_{SB} = 2$  volts. A quick check of the data in this table reveals that  $\sqrt{I_D}$  versus  $V_{GS}$  is linear and thus may be used in the linear regression analysis. Using the same procedure as in Ex. B-1, the following thresholds are determined:  $V_{T0} = 0.898$  volts (from Ex. B-1),  $V_T = 1.143$  volts (@  $V_{SB} = 1$  V), and  $V_T = 1.322$  V (@  $V_{SB} = 2$  V). Table B-3 gives the value of  $V_T$  as a function of  $[(2|\phi_F| + V_{SB})^{1/2} - (2|\phi_F|)^{1/2}]$  for the three values of  $V_{SB}$ .

**Example 3.10-2 - Continued**

Table 3.10-3 Data for Example 3.10-2.

$V_{SB}$ (V)	$V_T$ (V)	$[\sqrt{2 \phi_F  + V_{SB}} - \sqrt{2 \phi_F }]$ (V <sup>1/2</sup> )
0.000	0.898	0.000
1.000	1.143	0.490
2.000	1.322	0.838

With these data, linear regression must be performed on the data of  $V_T$  versus  $[(2|\phi_F| + V_{SB})^{0.5} - (2|\phi_F|)^{0.5}]$ . The regression parameters of Eq. (13) are

$$\sum x_i y_i = 1.668$$

$$\sum x_i^2 y_i = 4.466$$

$$\sum x_i^2 = 0.9423$$

$$(\sum x_i)^2 = 1.764$$

These values give  $m = 0.506 = \gamma$ .

## Extraction of the Channel Length Modulation Parameter, $\lambda$

The channel length modulation parameter  $\lambda$  should be determined for all device lengths that might be used. For the sake of simplicity, Eq. (2) is rewritten as

$$i_D = i'_D + \lambda' v_{DS} + i'_D \quad (22)$$

which is in the familiar linear form where

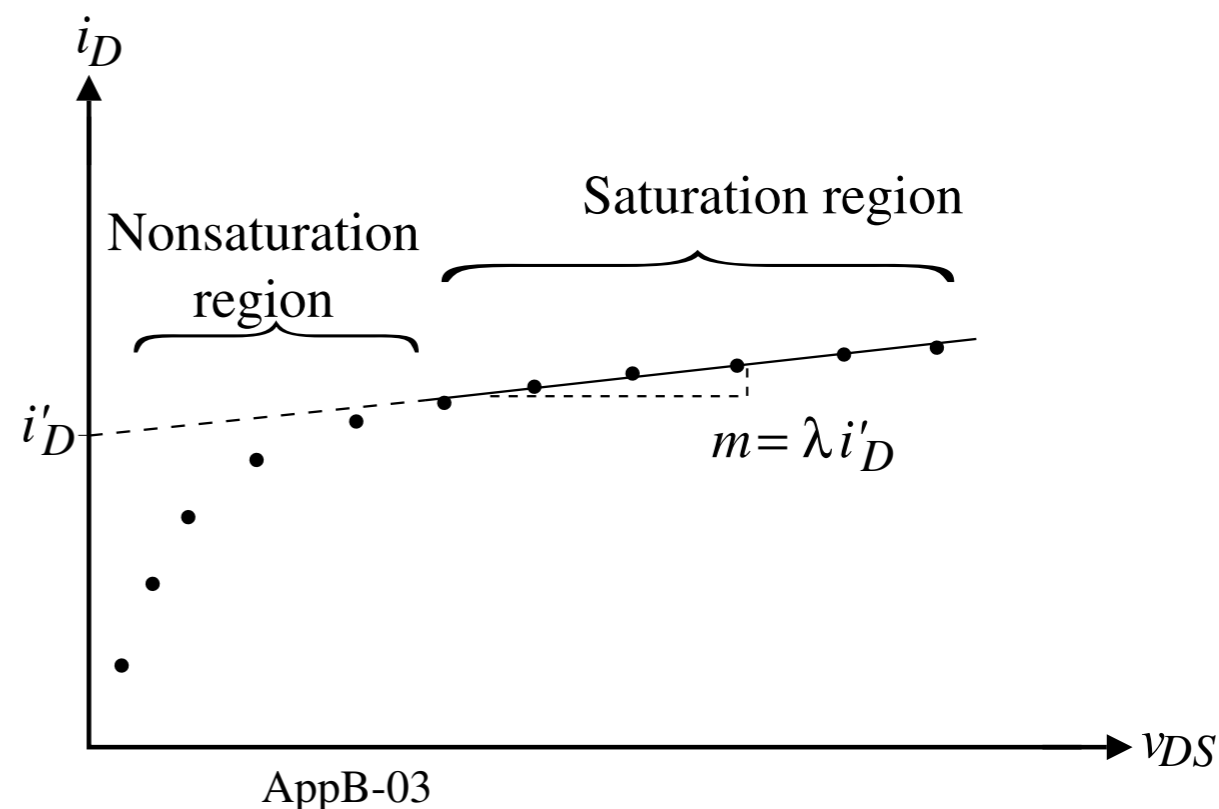
$$y = i_D \quad (\text{Eq. (2)}) \quad (23)$$

$$x = v_{DS} \quad (24)$$

$$m = \lambda i'_D \quad (25)$$

$$b = i'_D \quad (\text{Eq. (2) with } \lambda = 0) \quad (26)$$

By plotting  $i_D$  versus  $v_{DS}$ , measuring the slope of the data in the saturation region, and dividing that value by the y-intercept,  $\lambda$  can be determined. The procedure is illustrated in the figure shown.



### **Example 3.10-3 – Extraction of the Channel Length Modulation Parameter**

Given the data of  $I_D$  versus  $V_{DS}$  in Table 10.3-4, determine the parameter  $\lambda$ .

Table 10.3-4 Data for Example 3.10-3.

$I_D$ ( $\mu\text{A}$ )	39.2	68.2	86.8	94.2	95.7	97.2	98.8	100.3
$V_{DS}$ (V)	0.500	1.000	1.500	2.000	2.50	3.00	3.50	4.00

#### Solution

We note that the data of Table 3.10-4 covers both the saturation and nonsaturation regions of operation. A quick check shows that saturation is reached near  $V_{DS} = 2.0$  V. To calculate  $\lambda$ , we shall use the data for  $V_{DS}$  greater than or equal to 2.5 V. The parameters of the linear regression are

$$x_i y_i = 1277.85$$

$$\sum x_i \sum y_i = 5096.00$$

$$\sum x_i^2 = 43.5$$

$$(\sum x_i)^2 = 169$$

These values result in  $m = \lambda I'_D = 3.08$  and  $b = I'_D = 88$ , giving  $\lambda = 0.035 \text{ V}^{-1}$ .

The slope in the saturation region is typically very small, making it necessary to be careful that two data points taken with low resolution are not subtracted (to obtain the slope) resulting in a number that is of the same order of magnitude as the resolution of the data point measured. If this occurs, then the value obtained will have significant and unacceptable error.

## SEC. 3.11 - SUMMARY

- Model philosophy for analog IC design
  - Use simple models for design and sophisticated models for verification
- Models have several parts
  - Large signal static (dc variables)
  - Small signal static (midband gains, resistances)
  - Small signal dynamic (frequency response, noise)
  - Large signal dynamic (slew rate)
- In addition models may include:
  - Temperature
  - Noise
  - Process variations (Monte Carlo methods)
- Computer models
  - Must be numerically efficient
  - Quickly derived from new technology
- Analog Design “Tricks”
  - Stay away from minimum channel length if possible
    - Larger  $r_{ds}$   $\rightarrow$  larger gains
    - Better agreement
  - Don't use the computer models for design, rather verification of design