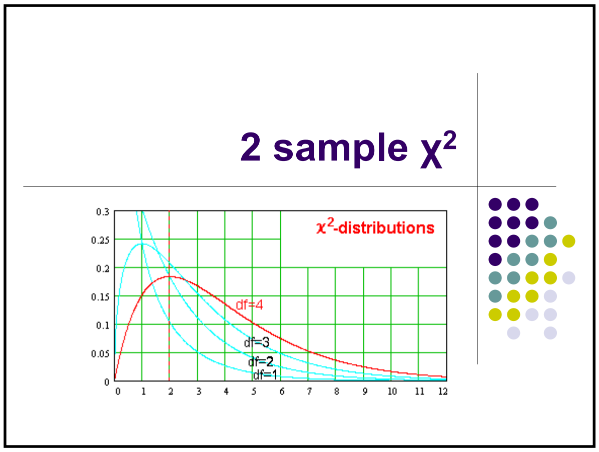
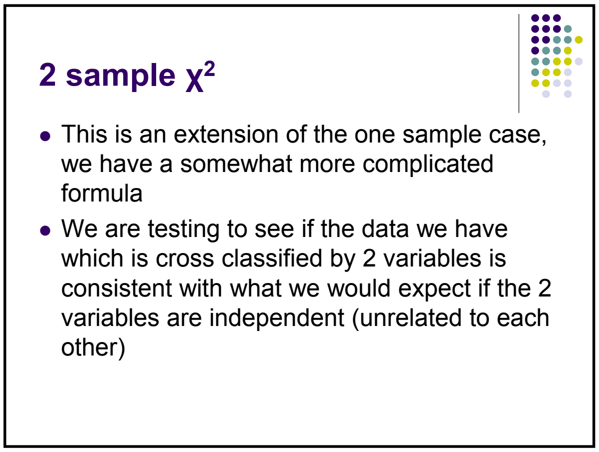
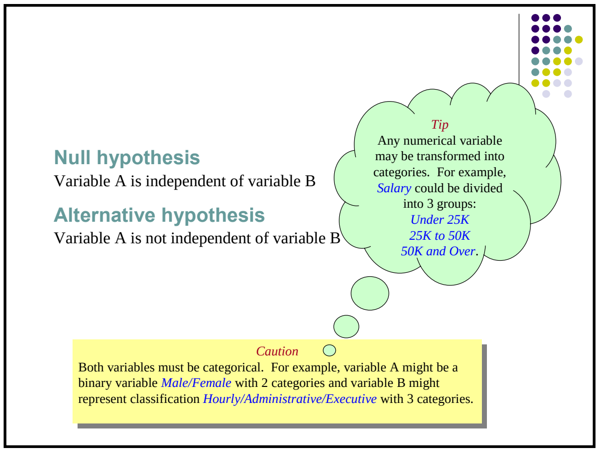


2 sample χ^2



2 sample χ^2

- This is an extension of the one sample case, we have a somewhat more complicated formula
- We are testing to see if the data we have which is cross classified by 2 variables is consistent with what we would expect if the 2 variables are independent (unrelated to each other)



Null hypothesis

Variable A is independent of variable B

Alternative hypothesis

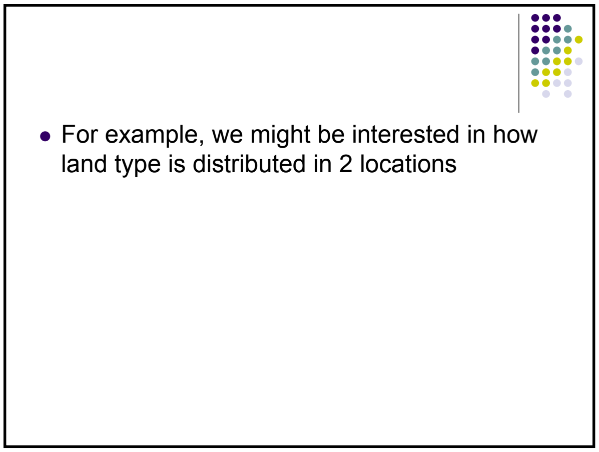
Variable A is not independent of variable B

Tip

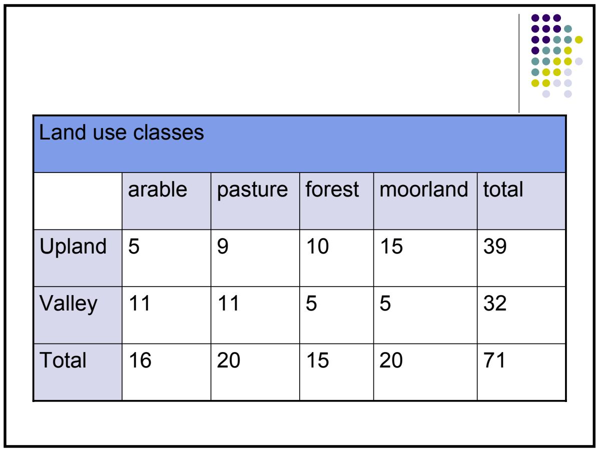
Any numerical variable may be transformed into categories. For example, *Salary* could be divided into 3 groups:
Under 25K
25K to 50K
50K and Over.

Caution

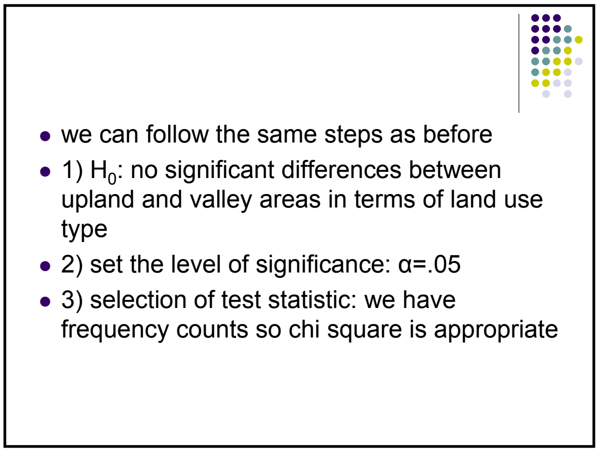
Both variables must be categorical. For example, variable A might be a binary variable *Male/Female* with 2 categories and variable B might represent classification *Hourly/Administrative/Executive* with 3 categories.



- For example, we might be interested in how land type is distributed in 2 locations



Land use classes					
	arable	pasture	forest	moorland	total
Upland	5	9	10	15	39
Valley	11	11	5	5	32
Total	16	20	15	20	71



- we can follow the same steps as before
- 1) H_0: no significant differences between upland and valley areas in terms of land use type
- 2) set the level of significance: α=.05
- 3) selection of test statistic: we have frequency counts so chi square is appropriate

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{k} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

r rows
k columns in contingency table whose members are O_{ij}s

$$E_{ij} = \frac{\sum_{i=1}^{r} O_i \sum_{j=1}^{k} O_j}{\sum_{i=1}^{r} \sum_{j=1}^{k} O_{ij}}$$

- row total times column total / overall total
- E_{ij}s
- (39 x 16) /71 = 8.8
- (39 x 20)/71 = 11.0 2 cells with this
- (39 x 15)/71 = 8.2
- (32 x 16)/71= 7.2
- (32 x 20)/71=9.0 2 cells with this
- (32 x 15)/71=6.8

- Expected values

upland	8.8	11.0	8.2	11.0
valley	7.2	9.0	6.8	9.0

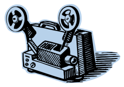
df=(2-1)(4-1)=(1)(3)=3 (r-1)(k-1)

4) compute the test statistic

upland	$(5-8.8)^2/8.8=1.64$	$(9-11)^2/11=.36$	$(10-8.2)^2/8.2=.40$	$(15-11)^2/11=1.45$
valley	$(11-7.2)^2/7.2=2.0$	$(11-9)^2/9=.44$	$(5-6.8)^2/6.8=.48$	$(5-9)^2/9=1.78$

- χ^2=1.64+.36+.40+1.45+2.0+.44+.48+1.78=8.55
- 5) determine rejection value
- χ^2_3=7.82, α=0.05
- 6) the calculated χ^2 exceeds the critical value
- 7) we reject H_0

- 2 way table video
- To see the video online go to
  - http://www.learner.org/resources/series65.html
  - Title: Inference for Two-Way Tables



Why the formula

- Let’s take a look a a hypothetical distribution of geography students by gender

	Honours	Other	Total
Males	73	12	85
Females	43	39	82
Total	116	51	167

Why the formula?

- To calculate the expected frequency of the first cell in the table, first calculate the proportion of students in honours without considering their gender. The table shows that of the 167 subjects in the sample, 116 were honours students. Therefore, 116/167 were honours students.

- If the null hypothesis were true, the expected frequency for the first cell would equal the product of the number of males (85) and the proportion of people in honours (116/167). This is equal to (85)(116)/167 = 59.042. Therefore, the expected frequency for this cell is 59.042.

special case of χ^2 (2 x 2 table)

- for a 2 x 2 table χ^2 is calculated by

$$\chi^2 = \frac{n(|AD - BC| - n/2)^2}{(A+B)(C+D)(A+C)(B+D)}$$

df=(number of rows-1)(number of columns-1)=1
n = total number of individuals

Example

A	B
C	D

15 A	11 B
5 C	12 D

$$\chi^2 = \frac{43(|180 - 55| - 43/2)^2}{(26)(17)(20)(23)} = \frac{43(125 - 21.5)^2}{203320} = 2.26$$

- $\alpha=0.05$, df=1, $\chi^2_c=3.84$ MB1
- we accept H_0

Combining cells

- remember the test requires expected cell frequency to be at least 5, if less than 5 the usual practice is to combine cells
- 2 basic options
- combine each end column with the one next to it - only do this if it makes sense, ie. Combining strongly disagree with somewhat disagree
- combine 2 end columns to get a single column - combining the extreme positions on a question

- suspiciously small
  - values close to zero are suspicious
  - so consider whether data has been doctored or small calculation error has occurred
- the additive property of χ^2
  - sometimes an experiment is performed more than once
  - its okay to sum the values of χ^2 obtained from each performance of the experiment, to sum the df and then test the significance

statistics available in SPSS

- Likelihood ratio chi-square is an alternative to test the hypothesis of no association of columns and rows in nominal-level tabular data.
  - based on maximum likelihood estimation. Though computed differently, likelihood ratio chi-square is interpreted the same way.
- Linear by Linear Association chi-square is a version of chi-square for ordinal data. It assumes equally ordered (interval or near interval) data.

Mantel-Haenszel chi-square

- also called the *Mantel-Haenszel test for linear association*, unlike ordinary and likelihood ratio chi-square, is an ordinal measure of significance. It is preferred when testing the significance of linear relationship between two ordinal variables. If found significant, the interpretation is that increases in one variable are associated with increases (or decreases for negative relationships) in the other greater than would be expected by chance of random sampling.

- Like other chi-square statistics, M-H chi-square should not be used with tables with small cell counts.

Fisher Exact Test of Significance

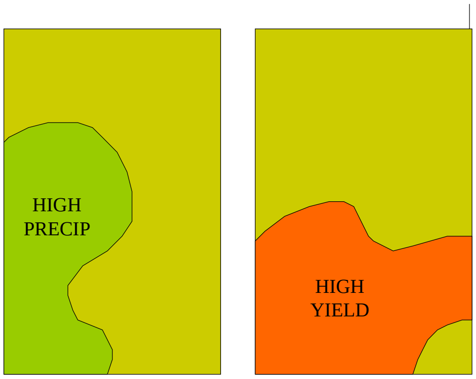
- The Fisher exact test of significance is used in place of the chi-square test in small 2-by-2 tables. It tests the probability of getting a table as strong as the observed or stronger simply due to the chance of sampling, where "strong" is defined by the proportion of cases on the diagonal with the most cases.

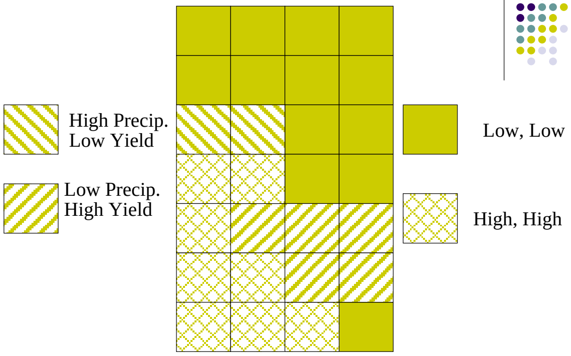
Slide 19

MB1 Milford Green, 7/8/2004

Chi-Square Statistic

- Measures the independence of association between two distributions
- Examples
  - Relationship between wheat yield and precipitation
  - Two maps showing high and low yields and high and low precipitation





Chi-Square

- By combining distribution on one map we can better understand the relationship between the two distributions
- In this example we are using a grid
  - The finer the grid, the more precise the measurement
- Four possibilities exist
  - Low rainfall, low yield
  - Low rainfall, high yield
  - High rainfall, low yield
  - High rainfall, high yield

Chi-Square

- Record the total number of occurrences into a table of observed frequencies

		WHEAT	
		High	Low
PRECIP.	High	8	2
	Low	5	13

Compute Chi-Square

- Therefore, in our example we have

Observed

	High	Low
High	8	2
Low	5	13

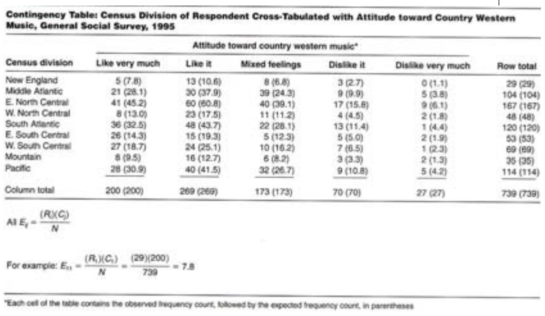
Expected

	High	Low
High	5	5
Low	8	10

X^2=6.2

Is it significant?

- df =2, p=.05
- Critical value=5.99
- Observed X^2=6.2



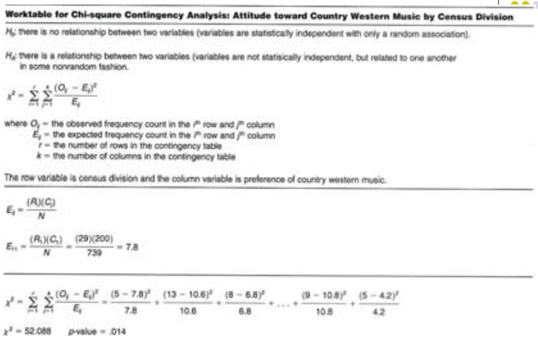
Contingency Table: Census Division of Respondent Cross-Tabulated with Attitude toward Country Western Music, General Social Survey, 1995

	Attitude toward country western music*					
Census division	Like very much	Like it	Mixed feelings	Dislike it	Dislike very much	Row total
New England	5 (7.8)	13 (10.6)	8 (6.8)	3 (2.7)	0 (1.1)	29 (29)
Middle Atlantic	21 (28.1)	30 (37.9)	39 (24.3)	9 (9.9)	5 (3.8)	104 (104)
E. North Central	41 (45.2)	60 (60.8)	40 (39.1)	17 (15.8)	9 (6.1)	167 (167)
W. North Central	8 (13.0)	23 (17.5)	11 (11.2)	4 (4.5)	2 (1.8)	48 (48)
South Atlantic	36 (32.5)	48 (43.7)	22 (28.1)	13 (11.4)	1 (4.4)	120 (120)
E. South Central	26 (14.3)	15 (19.3)	5 (12.3)	5 (5.0)	2 (1.9)	53 (53)
W. South Central	27 (18.7)	24 (25.1)	10 (16.2)	7 (6.5)	1 (2.3)	69 (69)
Mountain	8 (9.5)	16 (12.7)	6 (8.2)	3 (3.3)	2 (1.3)	35 (35)
Pacific	28 (30.9)	40 (41.5)	32 (26.7)	9 (10.8)	5 (4.2)	114 (114)
Column total	200 (200)	269 (269)	173 (173)	70 (70)	27 (27)	739 (739)

All $E_{ij} = \frac{(R_i)(C_j)}{N}$

For example: $E_{11} = \frac{(R_1)(C_1)}{N} = \frac{(29)(200)}{739} = 7.8$

*Each cell of the table contains the observed frequency count, followed by the expected frequency count, in parentheses



Worktable for Chi-square Contingency Analysis: Attitude toward Country Western Music by Census Division

H_0: there is no relationship between two variables (variables are statistically independent with only a random association).

H_A: there is a relationship between two variables (variables are not statisically independent, but related to one another in some nonrandom fashion.

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{k} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where O_{ij} = the observed frequency count in the i^{th} row and j^{th} column
E_{ij} = the expected frequency count in the i^{th} row and j^{th} column
r = the number of rows in the contingency table
k = the number of columns in the contingency table

The row variable is census division and the column variable is preference of country western music.

$$E_{ij} = \frac{(R_i)(C_j)}{N}$$

$$E_{11} = \frac{(R_1)(C_1)}{N} = \frac{(29)(200)}{739} = 7.8$$

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{k} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \frac{(5 - 7.8)^2}{7.8} + \frac{(13 - 10.6)^2}{10.6} + \frac{(8 - 6.8)^2}{6.8} + \ldots + \frac{(9 - 10.8)^2}{10.8} + \frac{(5 - 4.2)^2}{4.2}$$

$\chi^2 = 52.088$ p-value = .014