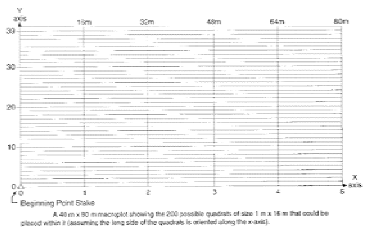
Quadrat Analyses



- A method for studying the spatial arrangement of point locations.
- It examines the frequency of points occurring in various parts of an area.
- A set of quadrats of cells (usually squares but not always) is superimposed on a study area and number of points in each cell is determined.

- By analysing the distribution of cell frequencies, the point pattern arrangement can be described.
- In nearest neighbour we look at average spacing of closest, quadrat analysis look at variability in number of points per cell.
- The absolute variability of cell frequencies can not be used because it is influenced by density of point (the mean number of point per cell)

- Recall that the coefficient of variation $CV = \frac{s}{\bar{x}}$ is used for meaningful comparisons of relative variability between distribution.
- In quadrat the index known as variance - means ratio (VMR) standardizes the degree of variability in the cell frequencies in relation to mean cell frequency:

$$VMR = \frac{VAR}{Mean}$$

- VAR = variance of the cell frequencies
- MEAN = mean cell frequency
- f_i = frequency of cells
- x_i = number per cell

- n = number of points
- m = number of cells

$$VAR = \frac{\sum f_i x_i^2 - [(\sum f_i x_i)^2 / m]}{m-1}$$

$$Mean = \frac{n}{m}$$

Interpretation

- In a dispersed set of points, the cell frequencies will be similar and the variance will be low.
- If highly clustered, the variance will be high therefore a large VAR.
- If set of points is randomly arranged, an intermediate value of variance will occur so a result near 1 suggests a random arrangement.
- In addition to being used as descriptive index, the VAR can be applied to test a distribution for randomness.

- Test statistic is t
- Another alternative is to use χ^2 where χ^2 = VMR(m-1) with df=m-1
- where m is the number of cells

$$t = \frac{VMR - 1}{\sqrt{2 / m - 1}}$$

Types of Distribution

- Five general patterns
  - **Random** any point is equally likely to occur at any location and the position of any point is not affected by the position of any other point. There is no apparent ordering of the distribution
  - **Regular** every point is as far from all of its neighbors as possible
  - **Clustered** many points are concentrated close together, and large areas that contain very few, if any, points

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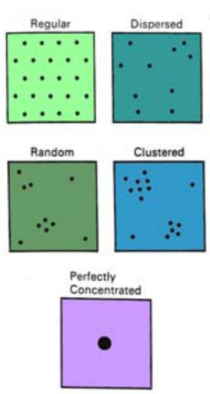
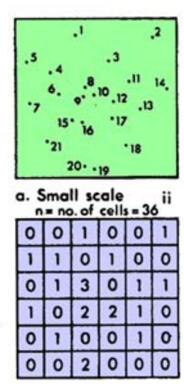
- Dispersed points are widely spread out
- Perfectly concentrated points are all in same location



Example



Example

x	f	f*x	x^2	$f*x^2$
0	20	0	0	0
1	12	12	1	12
2	3	6	4	12
3	1	3	9	9
		21		33

- MEAN = 21/36 = .583
- VMR = .593/.583 = 1.016

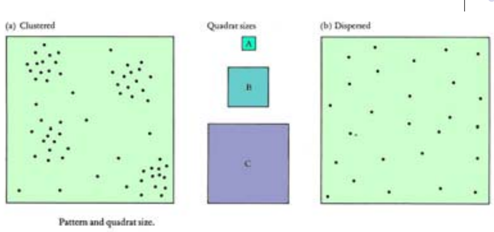
$$VAR = \frac{(33)-(441)/36}{35} = 20.75/35 = 0.593$$

- H_o: VMR = 1 (point pattern is random)
- H_1:VMR… 1 (point pattern is not random)
- α=.05, df=n-1
- t_c=2.03
- So we accept H_o
- rejection of the null hypothesis can occur if point pattern is more clustered than random (VMR>1) or more dispersed (VMR<1)

$$t = \frac{1.016-1}{\sqrt{2/(36-1)}} = 0.016/0.24 = 0.067$$

the scale problem

- the application of quadrat methods is affected by scale because selection of the size of cell is always arbitrary



- If A quadrat size is chosen we will likely produce a frequency distribution with many 0s and 1s → Poisson distribution
- If B quadrat size is chosen we would likely get a lot of variance, so a high var/mean ratio
  - → clustering
- If C quadrat size is chosen we'll get many quadrats of similar counts, so low variance
  - → dispersion

- if the hypothesis of randomness is to be accepted it must be shown that it is true at a variety of scales
- if not true then hypothesis of randomness must be rejected

- if fitting models of dependence (that is non-random) you just show that the parameter values don't vary significantly with changes in quadrat size
- otherwise scale is influencing results in some unknown manner

- the Kolmogrov - Smirnov D statistics
- an alternative to χ^2 and t tests
- measures goodness of fit by testing maximum deviation between cumulative frequency distribution and observed frequency distribution

$$D = \frac{\max|cum\ freq - obs\ freq|}{n}$$

- df = n, n = number of quadrats
- D statistic has advantage over χ^2 that the full range of values of points are considered in evaluation
- H_0: D = 0.0
- H_1: D > 0.0

Example

M	cum freq (Poisson)	obs freq	|diff|
0	20.1	20	.1
1	31.8	32	.2
2	35.2	35	.2
3	35.9	36	.1
4	36.0	36	0

- D = .2/36 = .006 α=.05
- D_c = .23
- accept H_0

- can use Poisson distribution to generate a random freq count
- λ= 21/36=.583

$$P(m) = \frac{e^{-\lambda}(\lambda^m)}{m!}$$

0	0	1	0	0	1
1	1	0	1	0	0
0	1	3	0	1	1
1	0	2	2	1	0
0	1	0	0	1	0
0	0	2	0	0	0

M	N	N x M
0	20	0
1	12	12
2	3	6
3	1	3
	36	21

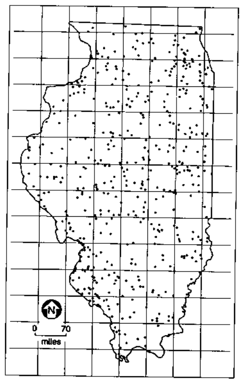
- .56 x 36 = 20.1

$$P_o = \frac{2.7183^{-.583}(0.583^0)}{0!} = 0.556$$

Example

- Using the following maps determine if the occurrence of tornado touchdowns in Illinois is random or not.
- The map contains 450 points and 63 cells.
- The observed frequency for tornados in Illinois over 54 years is:

# of tornados	obs freq	# of tornados	obs freq	# of tornados	obs freq	# of tornados	obs freq
0	0	5	10	10	3	15	1
1	1	6	5	11	3	16	0
2	2	7	8	12	0	17	0
3	7	8	6	13	0	18	1
4	4	9	8	14	4	19+	0



Illinois Tornado Touchdown Pattern with Quadrats Superimposed

- H_0: VMR = 1
- H_1: VMR … 1
- mean cell frequency = n/m
- Mean = 450/63 = 7.14
- calculate the variance
  - where f_i=frequency of cells with i tornados
  - X_i=number of tornados per cell

$$VAR = \frac{\sum f_i x_i^2 - [(\sum f_i x_i)^2 / m]}{m-1}$$

$$VAR = \frac{3986 - (450^2 / 63)}{62} = 12.45$$

- calculate variance-ratio
- VMR=VAR/MEAN=12.45/7.14=1.74
- Calculate test statistic
- χ^2= VMR(m-1) = 1.74 (62) =107.88 p=.003

Example 2

- Last year's class were seated in the following pattern on a given day
- Were they randomly distributed?

5	1	6
13	8	21
8	12	20
0	4	4
26	35	51

$$Mean = \frac{n}{m} = \frac{51}{8} = 6.4$$

$$VAR = \frac{\sum f_i x_i^2 - [(\sum f_i x_i)^2 / m]}{m-1}$$

x	f	f_x	x^2	fx^2	Σfx^2=51*51=2601
0	1	0	0	0	
1	1	1	1	1	
4	1	4	16	16	
5	1	5	25	25	
8	2	16	64	128	
12	1	12	144	144	
13	1	13	169	169	
	8	51		483	

$$VAR = \frac{483 - [2601/8]}{8-1}$$

- VAR= 157.9/7=22.56

$$VMR = \frac{VAR}{Mean}$$

- VMR= 22.56/6.4=3.5

- df=m-1

$$t = \frac{VMR - 1}{\sqrt{2/m-1}}$$

$$t = \frac{3.5-1}{\sqrt{2/(8-1)}} = \frac{2.5}{\sqrt{2/7}} = 4.67$$

- an alternative is

$$\chi^2 = VMR(m-1)$$

- χ^2= 3.5(7)=24.5 df=7 significant at α=.05
- χ^2_c = 14.07

Weakness of Quadrat Method

- Actually a measure of dispersion, and not really pattern, because it is based primarily on the density of points, and not their arrangement in relation to one another
- Results in a single measure for the entire distribution, so variations within the region are not recognized
- Grid size affects results

- Markedly different point patterns can give rise to identical frequency distributions of points by quadrats
- The potential distributions form a continuum, from the regular pattern where each point is equidistant from its neighbours, to the clustered case whose limiting example is the perfectly concentrated

- The random case is a mix of the two extremes

Advantages of Nearest Neighbor over Quadrat Analysis

- No quadrat size problem to be concerned with
- Takes distance into account
- Problems
  - Related to the entire boundary size
  - Must consider how to measure the boundary
    - Arbitrary or some natural boundary
  - May not consider a possible adjacent boundary

Quadrat Analysis

Primary Objective: Determine whether a point pattern has been generated by a random (Poisson) process

Requirements and Assumptions (when using the test inferentially):

1. Random sample of points from a population
2. Sample points are independently selected

Hypotheses:

H_0: VMR = 1 (point pattern is random)
H_A: VMR ≠ 1 (point pattern is not random)
H_A: VMR > 1 (point pattern is more clustered than random)
H_A: VMR < 1 (point pattern is more dispersed than random)

Test Statistic:

$$\chi^2 = \text{VMR}\,(m-1)$$