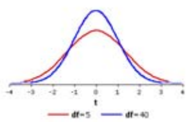
T-test



Developed 1908

William Sealy Gosset 1876-1937

T-test

- parametric tests t-test/z test
- parametric tests are more efficient/powerful than nonparametric tests but there are 3 restrictions on their use
- 1) data must be measured at the interval/ratio scale
- 2) data must be drawn from a normally distributed population

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- 3) data must be drawn in independent samples
- 4) when you have 2 or more samples, the populations from which the samples are drawn are assumed to have equal variance = homoscedasticity assumption

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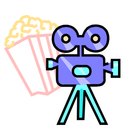
- $H_0 = \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$ the population means are equal or samples drawn from the same population or there is no significant difference between them

Video clip

- Inference about 1 mean video clip
- To view the unedited version on the web go to:
  - http://www.learner.org/resources/series65.html#



1 Sample t test

$$\underline{t} = \frac{\overline{X} - \mu}{s_{\bar{x}}}$$

$$S_{\bar{x}} = \frac{s}{\sqrt{n}}$$

df = N-1

Example of t-test

- e.g. growth rates in northern and southern Ontario
- H_0= there is no significant difference between growth rates in N and S Ontario cities t-test if n≤ 40
- northern southern
- $\bar{x}_1$ =10.6 $\bar{x}_2$ = 15.0
- s_1=11.8 s_2=9.6
  - s_i=standard deviation of ith sample

t statistic

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{SE|\bar{x}_1 - \bar{x}_2|}$$

SE is the standard error of the difference

where

$$SE|\bar{x}_1 - \bar{x}_2| = s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}\right)\left(\frac{N_1 + N_2}{N_1 N_2}\right)}$$

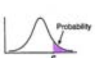
$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{11(11.8)^2 + 10(9.6)^2}{11 + 10 - 2}\right)\left(\frac{11 + 10}{11(10)}\right)}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{12911 * 191} = \sqrt{24.61} = 4.96$$

$$t = \frac{4.4}{4.96} = .887$$

t distribution

- you may see somewhat different formulas if the analyst decides to use the n-1 correction in the variance calculation of the sample
- t-tables df 2 tails
  - 10 2.228
  - 20 2.086
  - 30 2.042
  - inf 1.960



t Distribution Critical Values
(Table entry is the point *t** with given probability *p* lying above it.)

df	*p* .25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015

- as n increases $t_{critical}$ approaches 1.96 in other words a normal distribution
- for our 2 tailed test df= $(n_1-1)+(n_2-1)$=(11-1)+(10-1)=10+9=19
- at 0.05 with df=19, critical value of t=2.093 (pg 274 in textbook)
- therefore we cannot reject H_0
- conclude similar growth rates, they are not significantly different

Z test

- z test if n>40

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Z test

- e.g. clay content at 2 sites
- Site 1 site 2
  $\bar{X}_1 = 62.7$ $\bar{X}_2 = 61.8$ small difference in means
- $n_1 = 120$ $n_2 = 150$
- $s_1 = 2.50$ $s_2 = 2.62$
- Small standard deviation

Z test example

$$z = \frac{62.7 - 61.8}{\sqrt{\frac{2.50^2}{120} + \frac{2.62^2}{150}}} = 2.88$$

therefore reject H_0, there is a significant difference between sites

for z distribution with α=.05, z=1.96 (2 tailed test)

Pairwise t -test

$$\bar{d} = \sum \frac{d_j}{n}$$

where d_j is the difference between values x_{1j}, x_{2j}
if we make the assumption
of difference d_j is a random sample from a normal population

$$t = \frac{\bar{d} - 0}{s_d / \sqrt{n}}$$

$$s_d^2 = \frac{n\sum_{i=1}^{n} d_i^2 - \left(\sum_{i=1}^{n} d_i\right)^2}{n(n-1)}$$

t test formula

- we could generalize the test to allow hypotheses concerning any value for the mean difference in the population
- $\mu_d = \mu_1 - \mu_2$
- df=n-1

$$\frac{\bar{d} - \mu_{\bar{d}}}{s_d / \sqrt{n}}$$

example

- a cartographer test the time taken by intro students to perform a given set of tasks involving some extraction of information from some maps, at the end of the course this is repeated

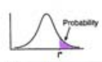
data

student	1st time	2nd time	difference
1	16	15	1
2	23	21	2
3	17	16	1
4	14	15	-1
5	16	15	1
6	21	19	2
7	19	18	1
8	24	10	14
9	26	15	11
10	19	20	-1

- $\bar{d}$ = 31/10=3.10
- s_d=5.11

$$t = \frac{3.10 - 0}{5.11\sqrt{10}} = \frac{3.10}{1.62} = 1.91$$

if α=0.05 t_c=2.262 with df=9



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- one tailed and two tailed tests
- so far we've only looked at testing against the null hypothesis, against H_1 that there is a difference between the means of the population from which the 2 samples were taken
- since we want to know if the difference lies in either direction it is a 2-tailed test

- if we want to test that there is a difference between means in a specified direction we have a 1-tailed test
- if H_1 is x >y then the null hypothesis can be rejected only if x >y and if it is significant at a chosen level